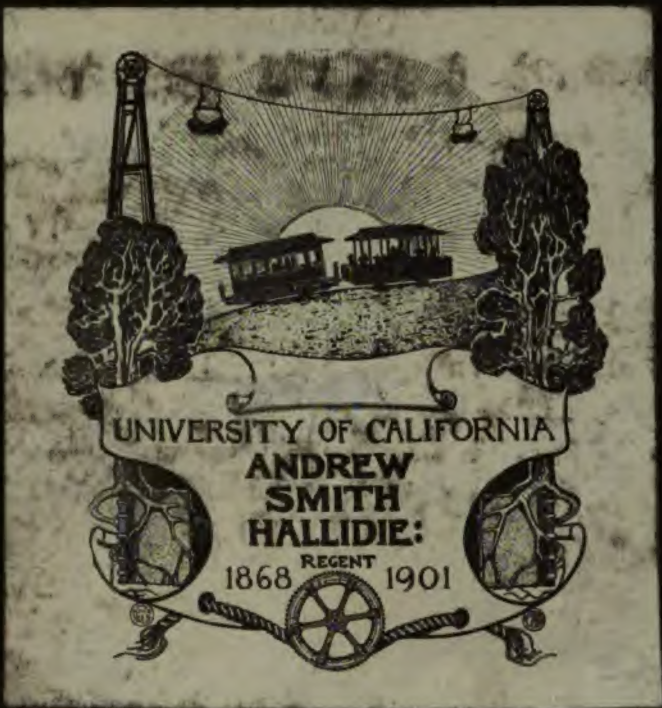
UNIVERSITY OF CALIFORNIA
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1868 1901

A TEXT-BOOK

OF

APPLIED MECHANICS

AND

MECHANICAL ENGINEERING.

VOLUME II.

Frontispiece.] (For Description, see Lecture XXVII.)

A TEXT-BOOK

OF

APPLIED MECHANICS

AND MECHANICAL ENGINEERING.

Specially Arranged
For the Use of Engineering Students Qualifying for the Institution of Civil Engineers, The Diplomas and Degrees of Technical Colleges and Universities, Advanced Science Certificates of British and Colonial Boards of Education, and Honours Certificates of the City and Guilds of London Institute, in Mechanical Engineering.

BY

ANDREW JAMIESON, M.INST.C.E.,

FORMERLY PROFESSOR OF ENGINEERING IN THE GLASGOW AND WEST OF SCOTLAND TECHNICAL COLLEGE; MEMBER OF THE INSTITUTE OF ELECTRICAL ENGINEERS; FELLOW OF THE ROYAL SOCIETY, EDINBURGH; AUTHOR OF TEXT-BOOKS ON STEAM AND STEAM ENGINES, ELEMENTARY APPLIED MECHANICS, MAGNETISM AND ELECTRICITY, ELECTRICAL RULES AND TABLES, ETC.

VOLUME II.

THIRD EDITION.

With Numerous Diagrams and Examination Questions.

LONDON:
CHARLES GRIFFIN & COMPANY, LIMITED;
EXETER STREET, STRAND.
1903.

PREFACE

TO FOURTH EDITION, VOL. I., AND THIRD EDITION, VOL. II.

THE rapidity with which Engineering Science and Practice progress, necessitates constant vigilance on the part of the Author who aims at keeping a Text-Book on Applied Mechanics and Mechanical Engineering abreast of the fast-moving times. In the many important additions to these new Editions, I have endeavoured to select useful practical examples of certain leading principles of Mechanics, as well as of the construction and action of machines which have come to the front since this book was written. I have, therefore, illustrated and described certain prominent applications of electrical machines to the driving of tools and cranes, and compared their results with what was, and still is, common practice. For, all engineers should be familiar with Electrical Transmission of Power and the circumstances under which it can be applied to the best advantage.

In addition to having had both volumes carefully studied and corrected in connection with my "Correspondence System of Teaching Engineering Science and Applications," the following additions, amongst others, have been made:—Tables of Symbols and Abbreviations: Construction, with Notes on Oiling of Heavy Shaft Bearings, and the Seigrist System of Automatic Lubrication for Large Engines, by J. L. Graham, Marine and Electric Tramway Station Engineer; Examples on Speed and Horse-power from Resistance of Models and Full-sized Steamships, by John Anderson, of Messrs. Caird & Co.'s Scientific Department, Greenock; Kynoch Roller Bearings,

Hoffman Ball Bearings, Humber Cycle Ball Bearings, and Lea-Francis Clutch Ball-race; Heenan and Froude's Water Dynamometer with Frontis-plate, together with the Applications of Froude's Dynamometer by Prof. Weighton, at Durham College of Science, and a reference to Prof. Osborne Reynolds' accurate determination by it at Owens College, Manchester, of 777 ft.-lbs. of Work, as the Mechanical Equivalent of one British Thermal Unit; Speed-reducing Worm Gear, by David Brown & Sons, Huddersfield; Specifications, Tests, and Efficiency Diagrams of the Clyde Trust Hydraulic and Electric Cranes, by George H. Baxter, M.I.M.E.; Electric Cranes for Manchester Ship Canal Warehouse Company and Electric Overhead Travelling Crane, by George Russell & Co., Ltd., Motherwell; Cable Grappling, Picking-up and Paying-out Gear; Changing Speed with Electro-Motor Connections; Electrically-driven Lathe, by James Archdale & Co., Birmingham; Machine-cut Gearing; Correct Form of Bevel Wheel Teeth; Spur, Bevel, and Worm Wheel Cutting Machines, by J. H. Gibson, David Brown & Sons, Grimshaw & Baxter; Rawhide Pinions; Wüst Double Helical Gear, by Marryat & Place, London; Speed-reducing Gear, by R. G. Ross & Sons, Glasgow; Amsler's Planimeter, as used for Measuring Engine Indicator Diagrams and other Irregular Figures; Thunderbolt's Marine and other Steam Engine Governors; Experiments upon the Action of Engine Governors, by W. G. Hibbins, A.M.Inst.C.E.; Hercules Turbine Installation and Pneumatic Tools, by John Turnbull, Jr., & Sons, Glasgow; Water Turbines and Rules for Hydraulic Motor Installations, by J. Ritchie, of Carrick & Ritchie, Edinburgh.

The half-yearly examinations of the Institution of Civil Engineers for admission of students and of associate members, which were begun in October, 1897, have now had a fair trial. The general opinion is, that these examinations have become the Standard Examinations for the Engineering Profession in Great Britain, India, and the Colonies. It was therefore gratifying to find, that this book, which had been written before these tests were instituted, was used by successful candidates

for the A.M.Inst.C.E. examinations. In the present new Editions I have, consequently, taken advantage of the courtesy of The Institution in kindly providing me with copies of the papers, to allocate each and all of the questions upon Mechanics, Structures, Strength and Elasticity of Materials, as well as Hydraulics, to their respective Lectures and Appendices. The questions which bear directly upon details of Steam Engines, as well as those upon the "Theory of Heat Engines," have been reserved for the *fourteenth* edition of my *Text-Book on Steam and Steam Engines.*

I have not forgotten the requirements of either those who desire to pass the Advanced and Honours Applied Mechanics Examinations of the Boards of Education for England, Scotland, and Ireland, or those preparing for the Mechanical Engineering Examinations of the City and Guilds of London Technological Institute, for I have also allocated their examination questions in the same way, with their 1903 Papers and Rules as guides to Teachers and Students.

In the Appendices to both volumes, I have printed extracts from the present and the prospective 1904 Rules and Syllabus of the Institution of Civil Engineers as a guide to intending candidates for admission as Associate Members.

Altogether, there will be found in Volume I. about 300 figures and diagrams, with a special new Frontis-plate and considerably over 500 questions. In Volume II. there are over 370 illustrations and 950 questions. Most of the Inst.C.E. questions will be found at the ends of the Lectures in Volume II., and future candidates will be pleased to learn, that on and after October, 1904, the subject of "*Theoretical* Mechanics" will not appear in the syllabus; whereas, the "*Theory* of Machines," including all the forms of Gearing treated of in Part II. of Volume I., will appear for the first time.

Examiners now demand a broader and more practical, although perhaps a less purely mathematical or "theoretical" expression of a young engineer's knowledge of mechanics, than was the case a few years ago. It is, therefore, to be hoped that these two volumes will meet their requirements.

I have much pleasure in thanking my senior assistant Mr. John Ramsay, for his help with the numerous additions and the very full Index, as well as Messrs. Stothart & Pitt, Bath; A. S. Vowell, C.E., London; John H. A. M'Intyre, M.I.M.E., Glasgow; Prof. David Robertson, B.Sc., Bristol; John S. Nicholson, B.Sc., Glasgow; and F. R. Stewart, B.Sc., Glasgow, for assisting me with different parts of the book.

Finally, my sincere obligations are due to Messrs. Charles Griffin & Co., the publishers, for the patience and care which they have bestowed upon the work.

ANDREW JAMIESON.

Consulting Engineer and Electrician,
16 Rosslyn Terrace, Kelvinside,
Glasgow, *October* 1903.

PREFACE TO VOLUME II.

THIS Text-Book has been written expressly for Second and Third Year Students of Applied Mechanics. It, therefore, forms a suitable companion to the Author's *Text-Book on Steam and Steam Engines.* It also forms a direct continuation of his *Elementary Manual on Applied Mechanics;* for it covers the Advanced Stage of the Science and Art Departments Examinations, and treats on many points demanded by the Honours Section. It will, moreover, be found of considerable use to those who aim at passing the Advanced and Honours Stages of the same Examinations in Machine Construction and Drawing, as well as the Examinations of the City and Guilds of London Institute in Mechanical Engineering. At the same time, the treatment of the subject is sufficiently general to satisfy the wants of other engineering students, who do not happen to have these Special Examinations in view.

The book has been divided into six parts:—

I. The Principle of Work and its Applications; Friction, Power Tests, with Efficiencies of Machines.

II. Gearing, with Applications to Machines.

III. Motion and Energy.—Governors, Flywheels, &c.

IV. Graphic Statics and Applications to Roofs, Cranes, Beams, Girders, and Bridges.

V. Strength of Materials.—Stress, Strain, Elasticity, Resilience, Cylinders, Chains, Shafts, Beams, and Girders.

VI. Hydraulics.—Hydraulic and Refrigerating Machinery.

Parts I. and II. were issued as Volume I., and the remaining Parts now form Volume II. This volume consists of Lectures XX. to XXXIV. under the following general headings:—Velocity and Acceleration—Motion and Energy—Energy of Rotation and Centrifugal Force—Engine Governors and other Applications of Centrifugal Force—Framed Structures—Roof Frames—Deficient Frames—Cranes—Beams and Girders—Stress and Strain, and Bodies under Tension—Strength of Shafts—Strength of Beams and Girders—Deflection of Beams and Girders—Hydrostatics — Hydraulic Machines — Hydrokinetics — Water Wheels and Turbines—Refrigerating Machinery.

In each Part special reference has been made to the latest and best books, and to papers read before leading Engineering Societies.

In each Part a number of examples have been fully worked out, and at the end of each Lecture a series of carefully-selected questions has been arranged, in the precise order of, and relating solely to, the subject matter of the Lecture, so that Teachers and Students may have a minimum of trouble in finding suitable examples.

Volume I. having been so kindly received, and having already passed into a Second Edition, it has been considered advisable to issue this Volume in time for the coming session, many Teachers having expressed a wish to have the continuation of the work at once. Later, the Author hopes to have the opportunity of still further amplifying and extending Part VI.

In conclusion, he has to thank many of his old Students and friends in connection with the production of the work; more especially, for the help which he received from Mr. Robert M. Anderson with Parts I. to III.; Mr. John

H. A. MacIntyre with Part IV.; and Mr. John Anderson with Part V. He has also to thank Mr. Alexander H. Weddell and Mr. John S. Nicholson for preparing numerous drawings, and the various firms who supplied illustrations of their mechanical appliances. Finally, he has received much assistance from Mr. David A. Ramsay, Mr. J. Fred. Nielson, and Mr. David Robertson, Jun., in preparing the manuscript and revising the proofs.

Great care has been taken to avoid errors, but if any should be observed by readers, the Author will be glad to have them pointed out, and to receive any suggestions tending to increase the usefulness of this book.

ANDREW JAMIESON.

THE GLASGOW AND WEST OF SCOTLAND
TECHNICAL COLLEGE.

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LECTURE XXXVII.

Refrigerating Machinery and Pneumatic Tools.

MECHANICAL ENGINEERING SYMBOLS, ABBREVIATIONS AND INDEX LETTERS

USED IN VOLUMES I. AND II.

OF PROFESSOR JAMIESON'S "APPLIED MECHANICS."

Prefatory Note.—It is very tantalising, as well as a great inconvenience to Students and Engineers, to find so many different symbol letters being used for denoting one and the same thing by various writers on mechanics. It is a pity, that British Civil and Mechanical Engineers have not as yet *standardised* their symbols in the same way that Chemists and Electrical Engineers have done. The Committee on Notation of the Chamber of Delegates to the International Electrical Congress, which met at Chicago in 1893, recommended a set of "Symbols for Physical Quantities and Abbreviations for Units," which have ever since been (almost) universally adopted throughout the world by Electricians.* This at once enables the results of certain new or corroborative investigations and formulæ, which may have been made and printed anywhere, to be clearly understood anywhere else, without having to specially interpret the precise meaning of each symbol letter.

In the following list of symbols, abbreviations and index letters, the *first* letter of the chief noun or most important word has been used to indicate the same. Where it appeared necessary, the *first* letter or letters of the adjectival substantive or qualifying words have been added, either as a following or as a subscript or suffix letter or letters. For certain specific quantities, ratios, coefficients and angles, small Greek letters have been used, and I have added to this list the complete Greek alphabet, since it may be refreshing to the memory of some to again see and read the names of these letters, which were no doubt quite familiar to them when at school.

* These "Symbols for Physical Quantities and Abbreviations for Units" will be found printed *in full* in the form of a table at the commencement of Munro and Jamieson's *Pocket-Book of Electrical Rules and Tables*. If a similar recommendation were authorised by a committee composed of delegates from the chief Engineering Institutions, it would be gladly adopted by "The Profession" in the same way that the present work of "The Engineering Standards Committee" is being accepted.

TABLE OF MECHANICAL ENGINEERING QUANTITIES, SYMBOLS, UNITS AND THEIR ABBREVIATIONS.

(As used in Vols. I. and II. of Prof. Jamieson's "Applied Mechanics.")

Quantities.	Symbols.	Defining Equations.	Practical Units.	Abbreviations of the Practical Units.
FUNDAMENTAL.				
Length, . . .	L, l	...	Yard, . . . Foot, . . . Inch, . . .	yd. ft. in.
Mass, . . .	M, m	...	Pound, . . .	lb.
Time, . . .	T, t	...	Second, . . Minute, . . Hour, . . .	s. m. h.
GEOMETRIC.				
Surface, . . .	S, s	$S = L^2$	Square foot, . . Square inch, . .	sq. ft. sq. in.
Volume, . . .	V	$V = L^3$	Cubic foot, . . Cubic inch, . .	cb. ft. cb. in.
Angle, $\angle$ . .	α, β θ, ϕ	$\alpha = \frac{\text{arc}}{\text{radius}}$	Degree, . . Minute, . . Second, . . Radian $= \frac{180^\circ}{\pi}$.	1° $1'$ $1''$ rn.
MECHANICAL.				
Velocity, . .	v	$v = \frac{L}{T}$	Foot per second, .	$\frac{\text{ft.}}{\text{s.}}$
Angular velocity, .	ω	$\omega = \frac{v}{L} = \frac{\theta}{t}$	Revs. per second, . Revs. per minute, Radians per second,	r.p.s. r.p.m. ω
Acceleration, . .	a, g	$a = \frac{v}{T}$	Foot per sec. per sec.	$\frac{\text{ft.}}{\text{s}^2}$
Force, . . .	F, f	...	Pound weight (gravitational unit),	lb. wt. (or lb.)
	W, w	$F = M a$	Poundal (absolute unit),	pdl.
Pressure (per unit area),	p	$p = \frac{F}{s}$	Pound per sq. inch,	lb. □″
Work, . . .	$(W h)$	$W h = F L$	Foot-pound, . .	ft.-lb.
Potential energy, .	E_P	$E_P = W h$	Foot-pound, . .	ft.-lb.
Kinetic energy, .	E_K	$E_K = \frac{W v^2}{2 g}$	Foot-pound, . .	ft.-lb.
Power or activity, .	HP	$\text{H.P.} = \frac{W h}{T}$	Horse power, . Ft.-lb. per min., . Ft.-lb. per sec., .	H.P. ft.-lb./m. ft.-lb./s.
Moment of inertia, .	I	$I = M k^2$	... ...	lb.-ft.^{2}
Density, . . .	ρ	$\rho = \frac{M}{V}$	Pound per cb. ft., . Pound per cb. in., .	$\frac{\text{lb.}}{\text{ft.}^3}$ $\frac{\text{lb.}}{\text{in}^3}$

OTHER SYMBOLS AND ABBREVIATIONS IN VOLS. I. AND II.

A for Areas.
B, b ,, Breadths.
C, c, k ,, Constants, ratios.
c.g. ,, Centre of gravity.
D, d ,, Diameters depths, deflections.
D_1, D_2, D_3 ,, Drivers in gearing.
E ,, Modulus of elasticity.
e ,, Velocity ratio in wheel gearing.
F_1, F_2, F_3 ,, Followers in gearing.
f_s, f_t ,, Forces of shear and tension.
H, h ,, Heights, heads.
H.P., h.p. ,, Horse-power.
B.H.P. ,, Brake horse-power.
E.H.P. ,, Effective ,,
I.H.P. ,, Indicated ,,
k ,, Radius of gyration, or, Coef. of discharge in hydraulics.
N, n ,, Numbers—*e.g.*, number of revs. per min., number of teeth, &c.
P, Q ,, Push or pull forces.
R_1 R_2 ,, Reactions, resultants, radii, resistances.
s ,, Seconds, space, surface. Displacement, distance.
S F ,, Shearing force.
T M ,, Torsional moment.
T R ,, Torsional resistance.
B M ,, Bending moment.
M R ,, Moment of resistance.
R M ,, Resisting moment.
T_d, T_s ,, Tensions on driving and slack sides of belts or ropes, &c.
W_L, W_T, W_U ,, Lost, total, and useful work.

x, y, z for Unknown quantities.
Z ,, Modulus of section.
Z_t ,, ,, tension.
Z_c ,, ,, compression.

Δ, δ, d for Differential signs which are prefixed to another letter; then the two together represent a very small quantity.
ϵ, e ,, Represents base of Naperian Logs = 2·7182; for example, $\log_e 3 = 1{\cdot}1$.
η ,, Efficiency.
λ ,, Length ratio of ship to model.
μ ,, Coefficient of friction.
π ,, Circumference of a circle ÷ its diameter.
ρ ,, Radius of curvature, radian.

Σ for Symbol for sum total of a number of quantities.
$\int_0^x$,, Sign of integration or summation between limits 0 and x.
$\sim$,, Sign for the difference between two quantities.
□ ,, Sign for square—*e.g.*, 10 □″ = 10 square inches.
— ,, Sign over two letters, $\overline{PQ}$ for a force acting from P to → Q, means that they represent a vector quantity, which has (1) magnitude, (2) direction, (3) sense.
$\geqq$,, Sign for equal to or greater than.
$\leqq$,, Sign for equal to or less than.

GREEK ALPHABET.

Α	α	Alpha.	Ι	ι	Iota.	Ρ	ρ	Rho.
Β	β	Beta.	Κ	κ	Kappa.	Σ	σ or ς	Sigma.
Γ	γ	Gamma.	Λ	λ	Lambda	Τ	τ	Tau.
Δ	δ	Delta.	Μ	μ	Mu	Υ	υ	Upsilon.
Ε	ε	Epsilon.	Ν	ν	Nu	Φ	φ	Phi.
Ζ	ζ	Zeta.	Ξ	ξ	Xi.	Χ	χ	Chi.
Η	η	Eta.	Ο	ο	Omicron.	Ψ	ψ	Psi.
Θ	θ	Theta.	Π	π	Pi.	Ω	ω	Omĕga.

VOLUME I.

Of 540 *Pages;* 300 *Figures and* 540 *Questions.*

APPLIED MECHANICS.

VOLUME II.

PART III.—MOTION AND ENERGY.—PRACTICAL APPLICATIONS TO GOVERNORS, FLYWHEELS, AND CENTRIFUGAL MACHINES.

LECTURE XX.

CONTENTS. — Definitions — Motion — Velocity — Acceleration — Graphical Methods—Velocity Diagrams—Falling Bodies—General Formulæ—Rotation—Angular Velocity—Circular Measure—Angular Acceleration — Composition and Resolution of Velocities — Parallelogram of Velocities—Triangle of Velocities—Polygon of Velocities—Rectangular Resolution—Composition and Resolution of Accelerations—The Hodograph—Hodograph for Motion in a Circle—Examples I., II., III., and IV.—Instantaneous Centre—Varying Velocities—Questions.

DEFINITION.—**A body is said to be in Motion when it is continually changing its position in space, and to be at Rest when it retains a fixed position in space.**

These are the definitions of *absolute* motion and *absolute* rest. We can never know the absolute motion of any body because we know no fixed bodies to which we may refer its positions at different times. We, therefore, can only deal with the *relative motion* of a body.

DEFINITION.—**A body is said to have Relative Motion with respect to another body when it is continually changing its position relatively to that body.**

Thus, take the case of a train moving on a railway. We always consider its motion relatively to some part of the earth's surface. But the train is carried round the earth's axis and also round the sun by the rotation of the earth itself. And this is not all, for we have reason to believe that the sun itself is not fixed in space but is in motion. A passenger in the train might be at rest relative to the train but he would be in motion relatively to the houses, trees, &c., which the train passed on its way.

Motions of Translation and Rotation.—The motion of a body may be either *Translatory* or *Rotary.*

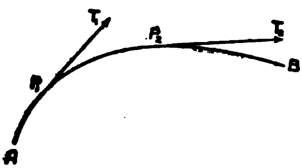
A body is said to have a motion of *simple translation* when all points in the body move with the same velocity and in the same direction at the same instant, so that no line in the body changes its direction. Hence, the motion of the whole body is known when that of any point in it is known.

A body is said to have a motion of *simple rotation* when the various points in the body describe circles about some fixed axis either within or without the body. Hence, the motion of the whole is known when that of any *line* in the body (other than the axis about which the motion takes place) is known.

The motion of a body may be *complex;* being composed or compounded of motions of translation and rotation. Thus, the connecting-rod of an engine has a complex motion. It has a motion of translation in a vertical plane containing the centre line of the engine, and a motion of rotation in the same plane about the crosshead pin.

DEFINITION.—**The Path of a moving point is the line, straight or curved, which passes through all the successive positions of the point.**

Direction of Motion.—The direction of motion of a body is, at any particular instant, the tangent to the path of the body at that instant, or the path itself if the motion is rectilinear.



ILLUSTRATING DIRECTION OF MOTION.

Thus, let A B be the path of a moving body. When the body occupies the position, P_1, its direction of motion is along P_1, T_1, the tangent to the path at that point. Similarly, when the body occupies the position P_2, its direction of motion is along the tangent P_2, T_2.

Hence, when a body moves in a circular path its direction of motion at any instant will be perpendicular to the radius drawn to its position on the circle at that instant.

DEFINITION.—**The Velocity of a body is the rate at which it changes its position.**

A *velocity* is completely specified when we know (1) its *direction*, and (2) its *magnitude*.

Hence, a velocity can be completely represented by a straight line of finite length with a suitably-directed arrow head.

DEFINITION.—**A body is said to be moving with Uniform Velocity when it is moving in a constant direction and passes over equal distances in equal intervals of time, however small these may be.**

The last clause in the above definition is necessary, because a body *might* describe equal distances in equal times, and yet its motion might not be uniform. Thus, a train may describe 20 miles in each of two consecutive hours, and yet its motion may have varied continuously during that time; sometimes its velocity may be 60 miles an hour, and at other times it may be nil.

Uniform Velocity, how Measured.—When uniform, the velocity of a body is measured by its displacement in unit time. Thus:—

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}.$$

DEFINITION.—**A body is said to have Unit Velocity when it describes unit distance in unit time.**

The unit of distance in this country is the *foot*, and the unit of time is usually the *second*, although engineers often take the *minute*, or even the *hour*, as the unit of time. For example, the speed of a railway train is always spoken of as so many *miles per hour*, and that of the piston of an engine as so many *feet per minute*.

Whatever units may be used, we get:—

$$\left.\begin{aligned} v &= \frac{s}{t} \\ \text{Or,}\quad s &= v\,t \end{aligned}\right\} \quad \dots\dots\dots \quad \text{(I)}$$

Where, s = Displacement, or distance described, in time, t.

And, v = Velocity, supposed to be uniform.

* [From the above definition and equation it is evident that v must be the same however small t may be. Thus, let the displacement be very small, say Δs, then the time taken to describe it will be correspondingly small, say Δt, and we get:—

$$v = \frac{\Delta s}{\Delta t}.$$

This being true for the smallest fraction of time, it must also be true in the limit.

$$\left.\begin{aligned} \therefore\quad v &= \frac{d\,s}{d\,t} \\ \text{Or,}\quad d\,s &= v\,d\,t \end{aligned}\right\} \quad \dots\dots \quad \text{(II)}\ ]$$

DEFINITION.—**A body is said to be moving with Variable Velocity when it is either changing its direction of motion or passing over unequal distances in equal intervals of time.**

* Students who have no knowledge of the notation of the *Calculus*, and those merely reading for examination in the Advanced Stage of this subject, may omit for the present the text within the brackets, thus [ ].

From this definition it appears that a body has a *variable velocity* when the direction or magnitude of its velocity is variable. Thus, a point on the rim of the flywheel of an engine has a variable velocity whether the rotary motion of the wheel be uniform or not.

This follows at once from the fact that a velocity is only completely specified when we know its *direction* and *magnitude*, and a change in either the direction or in the magnitude causes a change in the velocity. It is usual, however, in most problems, to speak of the velocity as being uniform or variable, according as the *magnitude* of the velocity is uniform or variable.

Variable Velocity, how Measured.—When variable, the velocity of a body is measured at any particular instant by the displacement which the body would have received if it moved for a unit of time with the same velocity which it had at the instant under consideration.

Thus, we see a train approaching a station and say that its velocity is 10 miles an hour, although we at the same time observe that its velocity is diminishing rapidly, and will soon be zero. By the expression "10 miles an hour" we, therefore, do not mean that it will run 10 miles during the next hour, but simply that *if* the train continued to run for one hour with the same speed that it had at the instant the remark was made, it would travel a distance of 10 miles.

Average Velocity.—When the velocity of a body is variable, and we know its magnitudes for several positions of the body, then its *average* velocity can be found in the same way as we find the average of a series of numbers.

Thus, let v_1, v_2, v_3, ... v_n denote the velocities at n different points in its path; then :—

$$\textbf{Average velocity} = \bar{v} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n}.$$

Or it may be defined as follows :—

DEFINITION.—**When a body moves through a certain distance with a variable velocity, its average velocity is that uniform velocity which it would require to have in order to traverse the same distance in the same time.**

$$\left.\begin{array}{ll}\text{Therefore,} & \bar{v} = \dfrac{s}{t} \\ \text{Or,} & s = \bar{v}\,t\end{array}\right\} \quad \dots\dots\dots \quad (\mathrm{I}_a)$$

If the velocity increase or decrease uniformly, then the mean or average velocity is half the sum of the initial and final velocities.

$$\text{Or,} \qquad \bar{v} = \frac{v_1 + v_2}{2}. \quad \dots\dots \quad (\mathrm{III})$$

Where v_1 and v_2 denote the initial and final velocities respectively. (See end of this Lecture for a Graphic Method.)

DEFINITION.—**The acceleration of a body is its rate of change of velocity.**

Acceleration may be either uniform or variable.

DEFINITION.—**Acceleration is uniform when equal changes of velocity take place in equal intervals of time, however small these may be.**

Otherwise, the acceleration is variable.

Acceleration, how Measured.—Uniform acceleration is measured by the change in the velocity in a unit of time.

Variable acceleration is measured at any particular instant by what would be the change of velocity in a unit of time, on the supposition that during that unit of time the acceleration remained the same as at the instant under consideration.

If the student thoroughly understands the method of measuring a variable velocity, he should have no difficulty in perceiving from the above statement how variable acceleration is measured.

Uniformly Accelerated Motion.—We shall now deduce the ordinary formulæ for the motion of a body uniformly accelerated in its line of motion.

Let v_1 = Velocity of body at end of time t_1,
„ v_2 = „ „ „ t_2,
„ s = Distance described during interval $(t_2 - t_1)$,
„ a = Acceleration per unit time.

Then, *Change of velocity* $= v_2 - v_1$.

$\therefore$ *Rate of change of velocity* $= \dfrac{v_2 - v_1}{t_2 - t_1}$.

But, *Rate of change of velocity* = *acceleration.*

$$\therefore \qquad a = \frac{v_2 - v_1}{t_2 - t_1}.$$

Or, denoting the interval of time $(t_2 - t_1)$ by t, we get :—

$$\left.\begin{aligned} a &= \frac{v_2 - v_1}{t} \\ \text{Or,} \quad v_2 &= v_1 + at \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad \text{(IV)}$$

That is:—**Final Velocity = Initial Velocity + Change of Velocity.**

Again, since the acceleration is uniform, we get:—

$$\textit{Average velocity} = \frac{v_1 + v_2}{2}$$

$$\therefore \qquad s = \frac{v_1 + v_2}{2} \times t.$$

But,
$$v_2 = v_1 + a\,t,$$

$$\therefore \qquad s = \frac{v_1 + (v_1 + a\,t)}{2} \times t,$$

Or,
$$s = v_1 t + \tfrac{1}{2} a\,t^2. \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{V})$$

In many problems the time, t, is not given, and we require to find one of the four quantities, s, a, v_1, v_2, having given the other three. From equations (IV) and (V) the following relation between these quantities can easily be deduced by eliminating t. Thus:—

From equation (IV), $a = \dfrac{v_2 - v_1}{t}$,

From equation (V), $s = \dfrac{v_1 + v_2}{2} \times t.$

Multiplying together the corresponding sides of these equations and equating the products, we get:—

$$a\,s = \frac{v_2^2 - v_1^2}{2},$$

$$\left.\begin{array}{ll} \therefore & v_2^2 - v_1^2 = 2\,a\,s \\ \text{Or,} & v_2^2 = v_1^2 + 2\,a\,s \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{VI})$$

The above formulæ are true for all cases of *uniformly increasing* or *uniformly decreasing* velocity; but in the latter case, the acceleration will be negative, and a must be preceded by the *minus* sign.*

If the body start from rest, that is, if the time, t, be reckoned from the commencement of the motion, then, the initial velocity, $v_1 = 0$, and we get, from the above equations:—

$$v = a\,t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{IV}_a)$$

$$s = \tfrac{1}{2} a\,t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{V}_a)$$

$$v^2 = 2\,a\,s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{VI}_a)$$

Where v = velocity at end of time, t.†

* There is no need for deducing, or even stating, the corresponding formulæ when the acceleration is negative. The fewer formulæ to be committed to memory the better, and the student should learn to distinguish between positive and negative (increasing or decreasing) acceleration as indicated by difference in sign, and to supply the proper sign where necessary.

† The general formulæ (IV), (V), and (VI) should be used in all cases. When the body starts from rest, substitute $v_1 = 0$.

Graphical Methods.—Equations (III) and (IV) can be very easily represented by means of a diagram. We may here remark that diagrams of velocities, accelerations, &c., are very useful in assisting the student to answer many problems on the motion of a body, and in what follows we shall have several instances of their use when dealing with the moving parts of engines. Before explaining the following diagrams, it is necessary to remind the student that a velocity, or an acceleration, can be completely represented by a straight line. We have already seen that a *velocity* may be represented by a finite straight line. But an *acceleration* is a change of velocity per unit time. Hence, an acceleration may also be represented by a finite straight line. In the meantime, we are not concerned with the *direction* of the velocity or acceleration, so that the lines representing these may be drawn in any convenient direction.

Velocity and acceleration diagrams are constructed in a way similar to those representing work, viz., by drawing two axes at right angles, along which the velocities or accelerations and intervals of time may be plotted.

Diagram for Uniform Velocity.—Let v = velocity, supposed to be uniform, and t = time. Draw the line A B, along which intervals of time have to be plotted. Thus, let A B represent t. From A, set up A C at right angles to A B, and let A C represent the velocity, v. Complete the rectangle A B D C. Then, clearly, the area of A B D C represents the displacement during the time, t. Thus:—

$$\text{Displacement} = s = v\,t$$
$$= \text{area A B D C}.$$

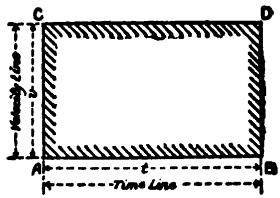


DIAGRAM FOR UNIFORM VELOCITY.

The area, ABDC, represents the displacement in time, t.

$\therefore \quad s = v\,t.$

Diagram for Uniformly Increasing Velocity.—Let a = the acceleration, and v = velocity at the end of time, t; the initial velocity being zero. As before, let A B represent the interval of time, t. At B, the end of interval t, draw B C to represent v, and join A C. Then, as before, the area of triangle A B C represents the displacement during time, t; since,

$$\textit{Displacement} = s = \textit{mean velocity} \times \textit{time} = \tfrac{1}{2}v \times t$$
$$\text{,,} \qquad = \tfrac{1}{2}\,\text{B C} \times \text{A B} = \text{area A B C}.$$

The velocity at any other time can be found by drawing the ordinate from the point on A B representing the given instant.

Thus, suppose A B represents 4 seconds. Then, the velocity at the end of 3 seconds from the beginning of the motion is represented by the ordinate 3 F. Similarly, at the end of the first second, the velocity is represented by the ordinate 1 D. But in this case, the velocity at the end of the first second is a measure of the acceleration; therefore, 1 D represents the acceleration.

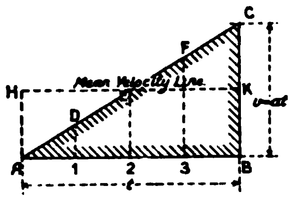


DIAGRAM FOR VELOCITY INCREASING UNIFORMLY FROM 0 TO v.

The area, A B C, represents the displacement in time, t.

$\because \quad s = \frac{1}{2} v t.$

If the acceleration be given instead of the final velocity, v, then the diagram can be set out in the following manner :—

Let A 1 represent a unit of time. Draw 1 D at right angles to A B to represent the acceleration, a. Join A D and produce it. Then A C is the velocity line. From this it will be seen that v = B C = $a\,t$.

$$\therefore \quad s = \tfrac{1}{2} a t^2$$
$$= \tfrac{1}{2} a t \times t$$
$$= \tfrac{1}{2} \text{B C} \times \text{A B}$$
$$= \text{area A B C.}$$

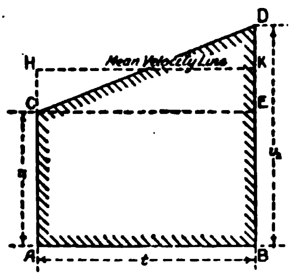


DIAGRAM FOR VELOCITY INCREASING UNIFORMLY FROM v_1 TO v_2.

The area, A B D C, represents the displacement in time, t.

$\because \quad s = v_1 t + \frac{1}{2} a t^2.$

If the body does not start from rest let the initial velocity be v_1, and the final velocity, v_2. Then, at each end of A B, the line representing t, draw the ordinates A C and B D to represent v_1 and v_2 respectively, and join C D.

Here, *Displacement* = $s = \frac{1}{2}(v_1 + v_2) \times t = \frac{1}{2}(\text{A C} + \text{B D}) \times \text{A B}$

" = area A B D C.

Also, E D = *Change of velocity in time*, $t = a\,t$.

And, B D = B E + E D = $v_1 + a\,t$.

$$\therefore \quad s = \tfrac{1}{2}(v_1 + \overline{v_1 + a\,t}) \times t = v_1 t + \tfrac{1}{2} a t^2.$$

We have not drawn the corresponding diagrams for the case when the acceleration is *negative*, but the student should have

no difficulty in doing this for himself. Thus, when a is negative, the last diagram would be drawn with the velocity line sloping in the opposite direction.

Motion Due to Gravity.—The most familiar instance of uniformly accelerated motion is that of a body falling under the influence of gravity. Experiments show that if a body be allowed to fall freely *in vacuo* its motion will be uniformly accelerated, and this acceleration is the same for every body (large or small, heavy or light) at the same locality. The letter g is always used to denote this acceleration. Its value depends on the distance of the falling body from the centre of mass of the earth, and varies inversely as the square of this distance. Hence, g is different at different latitudes, being greatest at the poles and least at the equator. When the units of distance and time are the foot and the second, the value of g at the poles is about 32·255, and 32·091 at the equator. Its value at the sea level in the latitude of London is about 32·19, and is generally taken at 32·2 for any place in the British isles.

Formulæ for the motion of bodies under the action of gravity alone are derived from those previously given for uniformly accelerated motion by substituting g for a. Thus:—

(I) *When let fall without initial velocity.*

$$v = g\,t \qquad \text{(IV}_b\text{)}$$

$$s = \tfrac{1}{2} g\,t^2 \qquad \text{(V}_b\text{)}$$

$$v^2 = 2 g\,s \qquad \text{(VI}_b\text{)}$$

(II) *When let fall with initial velocity*, v_1.

$$v_2 = v_1 + g\,t \qquad \text{(IV}_c\text{)}$$

$$s = v_1 t + \tfrac{1}{2} g\,t^2 \qquad \text{(V}_c\text{)}$$

$$v_2^2 = v_1^2 + 2 g\,s \qquad \text{(VI}_c\text{)}$$

If the body is thrown upwards with an initial velocity, v_1, then the acceleration due to gravity will be in the opposite direction to that of the motion; consequently, we must make *either* v_1 or g *negative*, according as we consider the downward or the upward direction to be *positive*. In such a case it is usual to make g negative. The rule usually observed is to take the acceleration *positive* or *negative* according as the motion is *increased* or *decreased*.

[**General Formulæ for Linear Motion.**—We have already seen that the velocity of a body is expressed generally as:—

$$v = \frac{d\,s}{d\,t}.$$

We can now give similar expressions for the acceleration when this varies according to any law whatever. Thus, at any instant of time let the velocity of the body be v, and at the end of an interval of time, Δt, let it be $v + \Delta v$, then :—

$$\textit{Acceleration} = \frac{\text{Change of velocity}}{\text{Time required}},$$

$$\text{,,} \qquad = \frac{(v + \Delta v) - v}{\Delta t} = \frac{\Delta v}{\Delta t}.$$

This being true, however small Δt may be, it is, therefore, true in the limit, hence :—

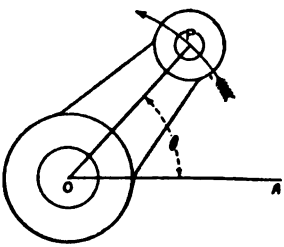
$$a = \frac{d v}{d t}.$$

But,

$$v = \frac{d s}{d t},$$

$$\therefore \qquad a = \frac{d v}{d t} = \frac{d^2 s}{d t^2}. \quad . \quad . \quad . \quad . \quad . \quad \text{(VII)}$$

Body Rotating about an Axis.—Angular Velocity.—We have already said that the motion of a body rotating about an axis is completely known when that of any line in the body, other than the axis of rotation, is known. It is most convenient to take this line passing through the axis, and perpendicular to it. Thus, let O be the intersection of the axis with the plane of the paper, O P a line in the body perpendicular to the axis through O. Then the motion of the body is known when that of the line O P is known. The motion of the line O P is measured by the angle which it describes round the point, O, in unit time. This angle is then spoken of as the **angular velocity** of the body. Hence the following :—



To illustrate Angular Velocity.

Definition.—**The angular velocity of a body about an axis is the rate of the angular displacement of any line in the body perpendicular to that axis.**

Angular velocity, like linear velocity, may be either *uniform* or *variable*, according as equal or unequal angles are described in equal intervals of time.

Uniform Angular Velocity.—Let the centre line, O P, of the crank in the above figure sweep out the angle, A O P = θ, in the interval of time, t; then the angular velocity of the body (usually denoted by the Greek letter ω) is :—

$$\omega = \frac{\theta}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{VIII})$$

The angle, θ, is measured in *circular units*, and not in degrees. The unit angle in circular measure is called the *radian*, and may be defined as *the angle subtended at the centre of a circle by an arc of its circumference, equal in length to the radius of the circle*. Hence, if t is in seconds, the *unit of angular velocity* will be the *radian per second.*

Since the length of the arc subtending a right angle is $\frac{\pi}{2} \times r$, and, therefore, the circular measure of a right angle equal to $\frac{\pi}{2}$ radians, we may easily determine the number of *degrees* in a radian. Thus :—

Degrees in 1 *radian* : *Degrees in* 1 *right angle* = 1 : $\frac{\pi}{2}$.

$$\therefore \qquad \text{Degrees in 1 radian} = \frac{90}{\frac{\pi}{2}} = \frac{180}{3{\cdot}1416} = 57{\cdot}29.$$

In general, if ρ be radians or the circular measure of an angle of θ°, then :—

Since, $\qquad 90^\circ : \theta^\circ :: \frac{\pi}{2} : \rho,$

We get, $\qquad \rho = \frac{\theta \pi}{180}$ radians.

Hence, if the angle described in time, t, by OP, be θ°, we get :—

$$\omega = \frac{\theta^\circ \pi}{180\, t}. \quad . \quad . \quad . \quad . \quad (\text{VIII}_a)$$

When the *linear* velocity of any point, P, in the body, and its distance from the axis are known, the angular velocity of the body can be found. Thus :—

Let $\quad v$ = Component of linear velocity of P perpendicular to O P (see the previous figure).

" $\quad r$ = Radius, O P.

Then, v = Arc described by P in unit time.

$\therefore \quad \frac{v}{r}$ = Circular measure of angle described by O P in *unit time.*

i.e., $$\omega = \frac{v}{r}$$
Or, $$v = \omega r \quad \Bigg\} \quad \ldots\ldots\ldots \quad (IX)^*$$

Variable Angular Velocity.—Angular Acceleration.—When the angular velocity is variable, it is measured in a way similar to that of variable linear velocity.

[Let $\Delta\,\theta$ = small angle described by O P, in small interval of time, $\Delta\,t$; then we have :—

$$\omega = \frac{\Delta\,\theta}{\Delta\,t},$$

which, in the limit, becomes :—

$$\omega = \frac{d\,\theta}{d\,t}. \quad \ldots\ldots\ldots \quad (X)]$$

DEFINITION.—**The angular acceleration of a rotating body is the rate of change of its angular velocity.**

Angular acceleration may be either *uniform* or *variable* according as equal changes of angular velocity take place in equal or unequal intervals of time. When uniform, angular acceleration is measured by the increase or decrease of angular velocity per unit time.

Let ω_1, ω_2 = Angular velocities at the beginning and end of interval of time, t.

" θ_1, θ_2 = Angular displacements at the beginning and end of interval of time, t.

" α = Angular acceleration.

Then, $$\alpha = \frac{\omega_2 - \omega_1}{t}$$
Or, $$\omega_2 = \omega_1 + \alpha\,t \quad \Bigg\} \quad \ldots\ldots\ldots \quad (XI)$$

From these equations and those previously deduced for uniformly accelerated linear motion, the student will notice the similarity of the relations between the terms s, v, and a, and θ, ω, and α respectively.

Hence, we get the remaining and corresponding equations for rotary motion, viz. :—

$$\theta = \omega_1\,t + \tfrac{1}{2}\,\alpha\,t^2 \quad \ldots\ldots\ldots \quad (XII)$$

$$\omega_2^2 = \omega_1^2 + 2\,\alpha\,\theta. \quad \ldots\ldots\ldots \quad (XIII)$$

* It is sometimes convenient to speak about the angular velocity of a *point*, such as P in the foregoing figure. Such a phrase is not strictly correct, and when used, it should be understood to mean the angle described in unit time by the radius drawn through the point, P.

[Generally, we have :—

$$\alpha = \frac{d\,\omega}{d\,t} = \frac{d^2\,\theta}{d\,t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XIV)}]$$

Composition and Resolution of Velocities.—A moving body may have at any instant two or more velocities in different directions, and it then becomes an important problem to be able to determine the resultant velocity, both in magnitude and in direction. Thus, the magnitude and direction of the motion of a man who walks across the deck of a moving ship is different from that of the ship and also from that of his motion relative to the deck. Similarly, the motion of a point on the rim of a carriage wheel in motion is, in general, different in magnitude and direction from its circular motion about the axle, and also from the onward motion of the wheel as a whole.

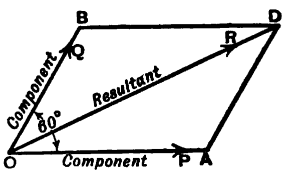
The process of finding a single velocity equivalent in effect to two or more velocities is called the *Composition of Velocities.*

The process of finding two or more velocities equivalent in effect to a single velocity is called the *Resolution of Velocities.*

DEFINITIONS.—The single velocity which is equivalent to two or more velocities is called their Resultant, and these two or more velocities are called the Components.

Parallelogram of Velocities.—If two component velocities be represented, in magnitude and direction, by two adjacent sides, O A, O B, of a parallelogram, their resultant velocity will be represented by the diagonal, O D, through their intersection.

Thus, if a moving point, O, possess simultaneously two velocities, P and Q, in directions O A and O B respectively, and, if O A and O B represent the magnitudes of these velocities, their resultant velocity, R, will be represented both in magnitude and in direction by the diagonal, O D, of the parallelogram constructed on O A, and O B, as adjacent sides.



PARALLELOGRAM LAW.

Let θ = angle between the directions of the velocities, P and Q.

„ $\alpha = \angle$ A O D, and $\beta = \angle$ B O D, the angles between the direction of the resultant, R, and the components P and Q respectively.

Then the student may easily prove from Euclid II., 13 and 14, or by trigonometry, that:—

$$R^2 = P^2 + Q^2 + 2\,P\,Q \cos\theta. \quad . \quad . \quad . \quad \text{(XV)}$$

$$\left.\begin{aligned} \text{And,} \quad \tan\alpha &= \frac{Q \sin\theta}{P + Q\cos\theta} \\ \text{Or,} \quad \tan\beta &= \frac{P \sin\theta}{Q + P\cos\theta} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XVI)}$$

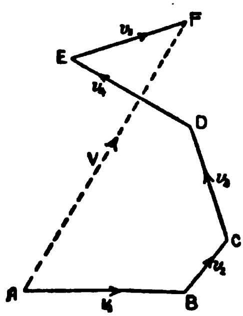
From these equations the magnitude and direction of the resultant velocity can be calculated.

It is not necessary to complete the parallelogram as explained above, it being quite sufficient to draw but one-half of the figure. Thus, A D is equal and parallel to O B; hence, as much can be determined from the triangle, O A D, as from the complete parallelogram, O A D B.

Triangle of Velocities.—If two component velocities be represented in magnitude and direction by two sides of a triangle taken in order, their resultant will be represented in magnitude and direction by the third side taken in the reverse direction.

Hence, if there be simultaneously impressed on a point three velocities represented in magnitude and direction by the sides of a triangle taken in order, then the point will remain at rest.*

Polygon of Velocities.—If several component velocities be represented by all but one of the sides of a polygon, A B C D E F, taken in order —the resultant velocity will be represented in magnitude and direction by the remaining side, A F, taken in the opposite direction.



POLYGON OF VELOCITIES.

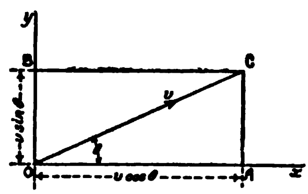
Thus, if a moving point have simultaneously impressed upon it velocities, $v_1, v_2, \ldots v_5$, and these are represented in magnitude and direction by the sides A B, B C, . . . E F of a polygon, A B C D E F, then the resultant velocity will be represented in magnitude and direction by the side, A F, required to complete the polygon.

* In setting out the *Parallelogram*, or *Triangle of Velocities*, it is not necessary to draw the sides parallel to the velocities represented. The sides may be drawn in directions *perpendicular* to the respective velocities, or, indeed, at any other angle, so long as the angle is the same for all the sides. In such cases the line representing the resultant will be equally inclined to its true direction. These several proofs are applicable, in the same way, to the *Parallelogram*, *Triangle*, and *Polygon of Statical Forces*, as explained in my *Elementary Manual on Applied Mechanics.*

If the figure whose sides represent the component velocities be closed or completed when the last velocity has been represented, then there is no resultant velocity, and the point will remain at rest.

It is equally important to be able to *resolve* a given velocity into two or more component velocities. Thus, the velocity, R (see the figure for *Parallelogram of Velocities*), can be resolved into two components, P and Q, in the directions O A, O B respectively. Or, the velocity, V (in the last figure), may be resolved into a number of components, v_1, v_2, . . ., in directions A B, B C, . . . Further, the directions of the component velocities may be anything we like. Thus, in resolving a given velocity, R, into two components, we can do so in an infinite number of ways, since an infinite number of parallelograms, such as O A D B, can be found having O D for one of their diagonals. When, however, the *directions* of the components are fixed, their magnitudes will be definite and easily determined. Referring to the figure for the *Parallelogram of Velocities*, let O D represent a velocity, R, which has to be resolved into two components in the directions O A and O B. From D draw D A parallel to B O and D B parallel to A O, meeting the lines O A and O B in the points A and B respectively. Then O A and O B represent the component velocities P and Q to the same scale that O D represents the velocity R.

The most important case of resolution is that wherein the given velocity has to be resolved into components whose directions are at right angles to each other. Thus, let it be required to resolve the velocity, v, whose direction is O C, into its **Rectangular Components** along O x and O y.



RECTANGULAR RESOLUTION.

From C drop the perpendiculars C A, C B on the axes O x and O y. Then, O A, O B are the components in the required directions.

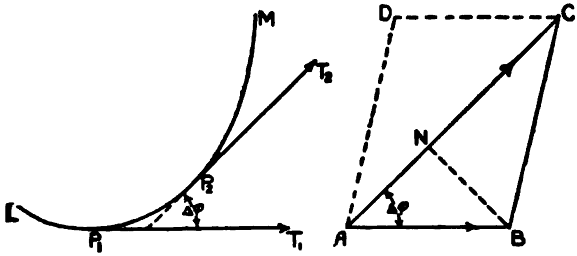
Let v_x, v_y = Components of v in directions O x, O y respectively.
" θ = Angle between the directions of v and v_x.

$$\left.\begin{aligned}\text{Then,}\qquad v_x &= v\cos\theta\\ \text{And,}\qquad v_y &= v\sin\theta\end{aligned}\right\}\quad \ldots\ldots \quad \text{(XVII)}$$

Composition and Resolution of Accelerations.—Since an *acceleration* is a *rate of change of velocity*, whether in *magnitude* or in *direction*, it follows that accelerations may be compounded or resolved according to the same rules as velocities.

If the direction of motion of a body be constant, then change of velocity can only take place in that direction. Thus, if a body is constrained to move in a rectilinear path its only acceleration is one of magnitude, and takes place along the straight line in which the body moves.

Again, the velocity of a body may be constant in *magnitude*, but variable in *direction*, as in the case of a body moving with uniform speed in a circle. Or, it may vary both in magnitude and in direction, as in the case of the bob of a pendulum swinging to and fro. The **Total Acceleration**, in any case, may be found in the following manner:—



TOTAL ACCELERATION OF A MOVING BODY.

Let L M be the path of a moving body, and P_1, P_2 its positions at the beginning and end of an interval of time, t.

At P_1, its velocity is in the direction of the tangent, $P_1\,T_1$, and at P_2, its velocity is in the direction, $P_2\,T_2$.

From A draw A B and A C to represent in magnitude and direction the velocities of the body at the points P_1 and P_2 respectively. Join B C, and complete the parallelogram, A B C D. Then A C represents the resultant velocity whose components are A B and A D or B C. But, if the velocity of the body had remained constant in magnitude and in direction during the time, t, its velocity at the end of that interval of time would have been represented by A B. Hence, in the above case, A D, or B C, represents, in magnitude and direction, the *change of velocity* during the time, t.

$$\therefore \qquad \textbf{Total acceleration} = \frac{BC}{t}.$$

[Suppose the arc $P_1\,P_2$ to be very small; and

Let v = Velocity of body at point, P_1.

„ $v + \Delta v$ = Velocity of body at point, P_2.

„ Δt = Small interval of time required to traverse the small arc, $P_1\,P_2$.

„ $\Delta \varphi$ = Angle between tangents to curve at P_1 and P_2.

From B draw B N perpendicular to A C. Then B N and N C represent respectively the components of the total acceleration, B C, along lines normal and tangential to the curve at a point near to P_1.

Hence, *Normal Acceleration* $= \text{limit of } \frac{BN}{\Delta t}$

And, *Tangential Acceleration* $= \text{limit of } \frac{NC}{\Delta t}$.

Now, $BN = AB \sin \Delta\phi = v \sin \Delta\phi$.

$\therefore$ $\text{Limit of } \frac{BN}{\Delta t} = v\frac{d\phi}{dt}$.

In the limit, let ds denote the infinitesimally small arc, $P_1 P_2$, and let ρ denote the radius of curvature at the point, P_1 or P_2.

Then, $$v\frac{d\phi}{dt} = v\frac{ds}{dt} \cdot \frac{d\phi}{ds} = v^2\frac{d\phi}{ds}.$$

But, from the properties of plane curves, we know that :—

$$\frac{d\phi}{ds} = \frac{1}{\rho}.*$$

$$\therefore \quad \text{Normal Acceleration} = \frac{v^2}{\rho} \quad . \quad . \quad . \quad . \quad \text{(XVIII)}$$

Again, $\text{Limit of } \frac{NC}{\Delta t} = \text{limit of } \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$.

$$\therefore \quad \text{Tangential Acceleration} = \frac{dv}{dt} \quad . \quad . \quad . \quad . \quad \text{(XIX)}$$

The result expressed in equation (XIX) agrees with the corresponding general equation previously deduced.

In the case of a body moving in a circle with uniform motion, we get $\rho = r =$ radius of circle, and v is constant. Then the tangential acceleration is *nil*, and the

$$\text{Normal or Radial Acceleration} = \frac{v^2}{r}. \quad \text{(XVIII}_a\text{)}$$

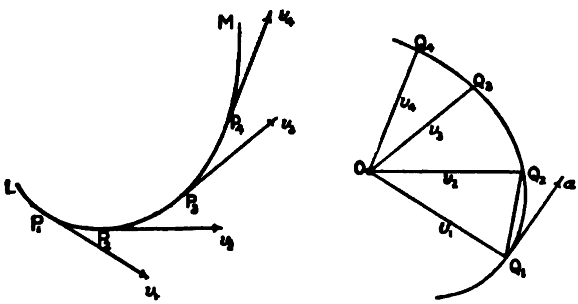
This is usually spoken of as the *Centripetal Acceleration.*]

The Hodograph—Uniform Motion in a Circle.—We shall now extend the foregoing principles to the determination of the acceleration of a body which moves with uniform velocity in a circle. In the first place we shall briefly describe the properties of the *Hodograph.*

* See Todhunter's *Diff. Calculus*, p. 343.

Definition.—If a point, P, be moving in any manner in a straight or curved path, and if from a fixed point, straight lines be drawn representing in magnitude and direction the velocities of P at different points of its path, the locus of the extremities of those lines will be a curve which is the Hodograph of P's motion.

Thus, let L M be the path of a moving point, P. Let the velocities at the points P_1, P_2, P_3 . . . be v_1, v_2, v_3 . . . From any point, O, draw O Q_1, O Q_2, O Q_3 . . . respectively parallel to v_1, v_2, v_3 . . . and of lengths representing these velocities. Then the curve, Q_1, Q_2, Q_3, Q_4, which is the *locus* of



The Hodograph.

the point Q, is the hodograph of P's motion in the path, L M. Hence, to every point on the curve, L M, there will be a corresponding point on the hodograph, so that while the body describes the curve, L M, we may imagine a point to describe the hodograph.

We shall now prove the following property of the hodograph:—

The acceleration of the body at any point on the curve, L M, is represented in magnitude and direction by the velocity of the corresponding point on the hodograph.

Let v = Average velocity between P_1 and P_2.

„ Δt = Indefinitely small time required to describe arc $P_1 P_2$.

Then,
$$v = \frac{P_1 P_2}{\Delta t}.$$

But O Q_1, O Q_2, represent the velocities of the body at the beginning and end of the interval of time, Δt. Therefore, chord $Q_1 Q_2$ represents the *change of velocity* of the body, during that interval of time.

That is, $\left.\begin{matrix}\textit{Acceleration of body between} \\ P_1 \textit{ and } P_2\end{matrix}\right\} = \frac{Q_1 Q_2}{\Delta t}.$

But, in the limit, when P_2 approaches indefinitely near to P_1, and, therefore, also Q_2 approaches indefinitely near to Q_1, we get:—

$$\text{Chord } Q_1 Q_2 = \text{Arc } Q_1 Q_2.$$

But, $$\frac{\text{Arc } Q_1 Q_2}{\Delta t} = \textit{Velocity of } Q \textit{ in hodograph.}$$

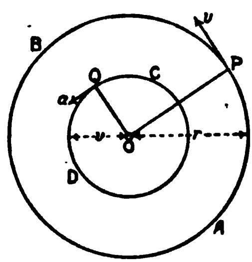
$$\therefore \left.\begin{array}{l}\textbf{Acceleration of}\\ \textbf{body in curve}\\ \textbf{L M}\end{array}\right\} = \textbf{Velocity of Q in hodograph.}$$

Again, since the direction of motion of a point on a curve is along the tangent to the curve at that point, so the direction of motion of Q on the hodograph at any point is along the tangent to the hodograph at that point. Hence, the *direction* of the acceleration of the moving body at any point on the curve, L M, is represented by the tangent at the corresponding point on the hodograph.

Thus, let P_1 and Q_1 be corresponding points on the path and hodograph respectively. Then, O Q_1 represents the velocity of the body at P_1, and the tangent to the hodograph at Q_1 represents the *direction* of the acceleration at the same point.

When a body describes a circle with uniform velocity, it is evident that there can be no tangential acceleration.

Let A P B represent the circular path of a body moving with uniform velocity, v. Then, it is clear that the hodograph of the moving body will also be a circle whose radius is v. With centre, O, and radius representing v, describe a circle, C Q D. Then, circle C Q D is the hodograph. Let P be the position of the body at any instant. Draw the radius, O Q, of the hodograph parallel to the tangent at P; or, what is the same thing, draw O Q perpendicular to O P. Since the radius, O P, describes equal angles in equal times, it follows at once that the radius, O Q, of the hodograph will also describe equal angles in equal times. In other words, the velocity of Q in the hodograph is uniform. Now, the magnitude and direction of the *velocity* of Q represent the magnitude and direction of the *acceleration* of P. Therefore, the direction of the acceleration of P is that of the tangent to the hodograph at the point, Q; that is, it



HODOGRAPH FOR UNIFORM MOTION IN A CIRCLE.

is along the radius, P O. As to the magnitude of this acceleration, we observe that:—

$$\textit{Acceleration of } P = a = \textit{Velocity of } Q.$$

Since Q describes the circle, C Q D, in the same time, t, as P describes the circle, A P B, we get:—

$$\text{For hodograph,} \quad t = \frac{\textit{Circumference of } C\,Q\,D}{\textit{Velocity of } Q} = \frac{2\,\pi \times v}{a} \quad . \quad (1)$$

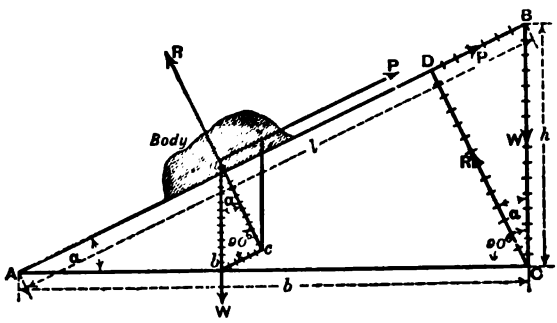
$$\text{For Path of P,} \quad t = \frac{\textit{Circumference of } A\,P\,B}{\textit{Velocity of } P} = \frac{2\,\pi \times r}{v} \quad . \quad (2)$$

Equating (1) and (2) we get:—

$$\frac{2\,\pi\,v}{a} = \frac{2\,\pi\,r}{v}.$$

$$\therefore \quad \textbf{Acceleration of P} = a = \frac{v^2}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (XX)$$

EXAMPLE I.—A body slides down a smooth inclined plane, determine its velocity at the foot of the plane. If the plane has



MOTION ON SMOOTH INCLINED PLANE.

a rise of 25 per cent., what distance would a body, descending along it from a state of rest, describe in five seconds? Find also the time occupied in sliding down the first 50 feet of the length of the plane.

ANSWER.—Let a = Acceleration of the body along the plane.
„ α = Inclination of plane to the horizon.

(1) If the body were free to move vertically downwards its acceleration in that direction would be g. But since it is con-

strained to move in the direction B A, its acceleration in this direction will be less, being, in fact, the component of g, along B A.

Hence, resolve g into its rectangular components in directions along and at right angles to B A. Then :—

$$\textit{Acceleration along } \mathrm{B\,A} = g \sin \alpha.$$

$$\textit{Acceleration perpendicular to } \mathrm{B\,A} = g \cos \alpha.$$

The latter component has no effect on the motion of the body. Hence :—

$$\text{Acceleration down the plane} = a = g \sin \alpha. \quad . \quad . \quad (1)$$

Let t = Time required to slide along a length, s.

„ v = Velocity at the end of time, t.

Then, from equation (IV_a) $v = a\,t$.

$$\therefore \qquad v = g\,t \sin \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From equation (V_a) $s = \frac{1}{2} a\,t^2$.

$$\therefore \qquad s = \tfrac{1}{2} g\,t^2 \sin \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

And from equation (VI_a) $v^2 = 2\,a\,s$.

$$\therefore \qquad v^2 = 2\,g\,s \sin \alpha.$$

But, $s \sin \alpha$ = Height of plane of length, s,

„ $= h$, say.

$$\text{Then,} \qquad v^2 = 2\,g\,h. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

That is,—*The velocity acquired by a body in sliding down a smooth inclined plane is the same as that acquired by a body falling freely through a distance equal to the height of the plane.*

From the given data, we get :—

$$\sin \alpha = \frac{25}{100} = \cdot 25,$$

$$t = 5 \text{ seconds.}$$

$\therefore$ From equation (3), we get :—

$$s = \tfrac{1}{2} g\,t^2 \sin \alpha = \tfrac{1}{2} \times 32{\cdot}2 \times 5 \times 5 \times \cdot 25$$

$$„ = 100{\cdot}625 \text{ feet.}$$

(2) Here, s = 50 feet, and we require t.

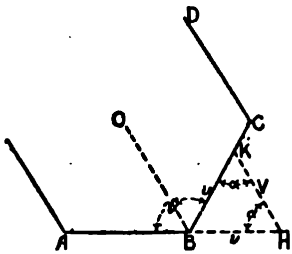
From equation (3), we get :—

$$s = \tfrac{1}{2} g\,t^2 \sin \alpha.$$

$$\therefore \qquad t = \sqrt{\frac{2\,s}{g \sin \alpha}} = \sqrt{\frac{2 \times 50}{32{\cdot}2 \times \cdot 25}} = 3{\cdot}52 \text{ seconds.}$$

Example II.—State the rule for the composition of two velocities. If a particle describes the perimeter of a regular polygon with a constant velocity, v, show that there must be impressed on it, at each angular point a velocity equal to $2\,v\cos\alpha$, directed towards the centre of the circumscribing circle, where α denotes half an angle of the polygon. (S. & A. Hons. Theor. Mechs. Exam., 1885.)

Answer.—For answer to first part, see *Parallelogram of Velocities.* Let A B C D . . represent the sides of a regular polygon, whose centre is O. When the particle arrives at B, its direction of motion is suddenly changed from A B to B C, while the magnitude of the velocity remains unaltered. To find the magnitude and direction of the velocity which must have been imparted to the particle at the point, B, we may proceed as follows:—



To Illustrate Example IV.

Produce A B, and set off B H, to represent the velocity, v, of the particle along A B, and B K along B C, to represent the velocity in that direction. Then H K represents in magnitude and direction the change of velocity which must have been imparted to the particle at the point, B. The magnitude of this velocity can be found from the triangle, B H K, or equation (XV).

$$\text{For, } V^2 = v^2 + v^2 - 2\,v^2\cos(180^\circ - 2\,\alpha) = 2\,v^2(1 + \cos 2\,\alpha)$$

$$,, \;= 4\,v^2\cos^2\alpha. \quad [\text{Since } 1 + \cos 2\,\alpha = 2\cos^2\alpha.]$$

$$\therefore \quad \mathbf{V = 2\,v\cos\alpha.}$$

Join B to O, the centre of the polygon, and we get:—
In triangle B H K;

$$\text{Exterior} \angle ABC = \angle BHK + \angle BKH.$$

$$\text{But,} \quad \angle BHK = \angle BKH,$$

$$\therefore \quad \angle BHK \text{ or } \angle BKH = \tfrac{1}{2}\angle ABC = \alpha.$$

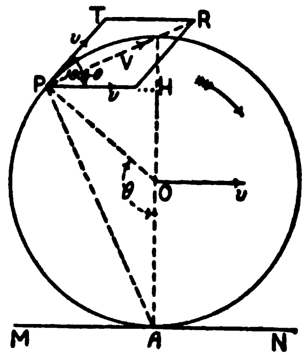
$\therefore$ H K is parallel to B O, since B O bisects $\angle$ A B C.

Therefore, the velocity impressed on the particle at B is directed along B O, towards the centre.

Example III.—Find, at any instant, the magnitude and direction of the velocity of a point on the rim of a wheel which rolls along a road with a constant speed, v.

ANSWER.—Take any point, P, on the rim of the wheel, and let the radius, drawn through P, make an angle, θ, with the vertical radius, O A.

Then, since the centre of the wheel is moving with velocity, v, it follows that the tangential velocity of any point on the rim is also v. This is, however, but one of the components of the actual velocity of P. The actual velocity of P is the resultant of two velocities—viz., v, along the tangent at P, and v, horizontally, since the point, P, in addition to moving round O, as a centre, is also being carried in a horizontal direction along with the wheel as a whole. By completing the parallelogram of velocities, as shown, the resultant velocity, V, can be found. The angle between the component velocities is $180° - \theta$. Hence, from equation (XV) :—



VELOCITY OF A POINT ON A ROLLING WHEEL.

$$V^2 = v^2 + v^2 + 2\,v^2 \cos(180° - \theta).$$

$$\therefore \qquad ,, \; = 2\,v^2\,\{1 + \cos(180° - \theta)\} = 4\,v^2 \sin^2\frac{\theta}{2}.$$

$$\therefore \qquad V = 2\,v \sin\frac{\theta}{2}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

Next, as to the direction of the resultant velocity, V.
Since P R bisects the angle between P T and P H,

$\therefore$ $\qquad \angle RPH = 90° - \frac{\theta}{2}.$

But, $\qquad \angle OAP = \frac{1}{2} \angle POH = 90° - \frac{\theta}{2}.$

$\therefore$ $\qquad \angle RPH = \angle OAP.$

In the $\triangle$ A P H, $\angle OAP$ = complement of $\angle APH$.

$\therefore$ $\qquad \angle RPH$ = ,, ,,

$\therefore$ $\qquad \angle RPA$ is a right angle.

Hence, the direction of motion of P is perpendicular to the line joining P with A, the point of the wheel which is in contact with the ground.

The point, A, is called the **Instantaneous Centre** of motion for all points on the rim of the wheel; because any point, such as P, is moving at any instant on the circumference of a circle having A for its centre and A P as its radius.

The direction of the actual motion of any point, P, is, at any instant, inclined to the horizontal at an angle equal to $90° - \frac{\theta}{2}$.

EXAMPLE IV.—In the previous example find the magnitude and direction of the actual velocity of the point, P, when the radius, O P, makes angles of 0°, 90°, 180°, and 270° with the vertical radius, O A. Also, find the position of P when the resultant velocity, V, is equal to v.

ANSWER.—(1) *When* $\theta = 0°$. From equation (1), Example III., we get :—

$$V = 2v \sin\frac{\theta}{2} = 0, \text{ since } \sin 0° = 0.$$

i.e., the point is at rest when it is in contact with the ground A.

(2) *When* $\theta = 90°$. Here $\sin\frac{\theta}{2} = \sin 45° = \frac{\sqrt{2}}{2}$.

$$\therefore \qquad V = 2v \sin\frac{\theta}{2} = 2v \times \frac{\sqrt{2}}{2} = v\sqrt{2}.$$

Also, $\left.\begin{array}{l}\text{Direction which}\\ \text{V makes with}\\ \text{the horizon}\end{array}\right\} = 90° - \frac{\theta}{2} = 45°.$

(3) *When* $\theta = 180°$. $\sin\frac{\theta}{2} = \sin 90° = 1$.

$$\therefore \qquad V = 2v \sin\frac{\theta}{2} = 2v \times 1 = 2v.$$

And, $\left.\begin{array}{l}\text{Direction which}\\ \text{V makes with}\\ \text{the horizon}\end{array}\right\} = 90° - \frac{\theta}{2} = 0°.$

That is, when P is vertically over O, it is moving horizontally with a velocity equal to twice the speed of the wheel.

(4) *When* $\theta = 270°$. $\sin\frac{\theta}{2} = \sin 135° = \frac{\sqrt{2}}{2}$.

$$\therefore \qquad V = 2v \sin\frac{\theta}{2} = 2v \times \frac{\sqrt{2}}{2} = v\sqrt{2}.$$

And, $\left.\begin{array}{l}\text{The inclination}\\ \text{of V to the}\\ \text{horizon}\end{array}\right\} = 90° - 135° = -45°.$

(5) *To find θ when* $V = v$.

Here, $$V = 2\,v \sin\frac{\theta}{2}.$$

$$\therefore \qquad \sin\frac{\theta}{2} = \frac{V}{2\,v} = \tfrac{1}{2}.$$

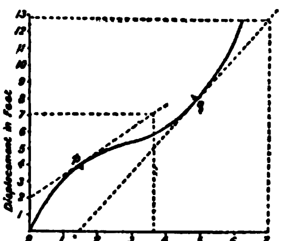
$$\therefore \qquad \frac{\theta}{2} = 30^\circ, \text{ or } 150^\circ,$$

$$\therefore \qquad \theta = 60^\circ, \text{ or } 300^\circ.$$

These agree with the two positions of P when the chord, A P, is equal to the radius, O A.

Varying Velocities.—When a body is caused to move through different distances in equal times, its speed or velocity is said to be variable. The velocity of the body at a particular instant, or during a very short interval of time, such as the fraction of a second, will be the distance it travels during that time.

The instantaneous velocity of a body moving with a variable speed may be graphically represented and estimated in the following manner:—Plot an abscissa or horizontal line to represent seconds and fractions of a second, and an ordinate or vertical line to represent displacement in feet. Then, suppose that the curved line represents to scale the varying velocity of a body, and we desire to know its instantaneous values at the positions p and q.



VARIABLE VELOCITIES.

(1) Draw a tangent to the curve at the point p. Then, from the slope of this tangential dotted line we see, that if the velocity of the body had been uniform whilst passing through a point represented by p, a distance of $(7 - 2) = 5$ feet, would have been passed over in 3·75 seconds. Consequently, we estimate that its *instantaneous volocity* at p was $(5 \div 3{\cdot}75) = 1{\cdot}\dot{3}$ feet per second. In the same way, we find that had the velocity of the body remained uniform whilst passing through a point represented by q on the curve, then 13 feet would have been passed over in $(7 - 1{\cdot}5)$ or 5·5 seconds. Therefore its *instantaneous velocity* at $q = (13 \div 5{\cdot}5) = 2{\cdot}36$ feet per second.

In this way the velocity of the body may be ascertained at any other instant.

LECTURE XX.—QUESTIONS.

1. Define the terms velocity and acceleration, and explain how these are measured—(1) when uniform; (2) when variable. Give examples of bodies having accelerations—(*a*) constant in magnitude and direction; (*b*) constant in magnitude but not in direction; (*c*) constant in direction but not in magnitude; (*d*) variable both in magnitude and direction.

2. When the velocity of a particle is uniformly accelerated, show that $s = \frac{1}{2} a t^2$. A particle moves from a state of rest under the action of a force which increases its velocity by 20 feet per second in every second of its motion. After four seconds the force ceases to act on it. What distance does it describe in the first six seconds of its motion? (S. & A. Adv. Theor. Mechs. Exam., 1880.) *Ans.* 320 feet.

3. Establish the formulæ for uniform acceleration in the direction of motion:—$v_2 = v_1 + a t$; $s = v_1 t + \frac{1}{2} a t^2$, and from these results deduce the formula—$v_2^2 = v_1^2 + 2 a s$. Find an expression for the distance described in the nth second.

4. A cage is ascending the shaft of a mine at a uniform rate of 10 feet per second. When it is 50 feet from the top the speed is diminished, so that it now moves with a uniformly retarded velocity, and finally comes to rest at the top. Find the retardation. *Ans.* 1 foot per second.

5. State the rule for the composition of two velocities. Draw two lines, A B, A C, containing an acute angle. A particle is at A moving with a given velocity, V, from A towards B. Give a construction for determining the velocity that must be impressed on it, to make it move with a velocity, 2 V, from A towards C.

6. A particle describes the perimeter of a regular hexagon with a constant velocity of 100 feet a second. Find the magnitude and direction of the velocity that must be communicated to it, at the instant it reaches an angular point. *Ans.* 100 feet per second towards centre of hexagon.

7. Two bodies start together from rest, and move in directions at right angles to each other. One moves with a uniform velocity of 3 feet per second, while the motion of the other is uniformly accelerated. At the end of four seconds the bodies are found to be 20 feet apart. Determine the acceleration of the latter body. *Ans.* 2 feet per second.

8. Two bodies, P and Q, move with different velocities along the same line. What is the relative velocity of Q to P? If Q is allowed to fall freely, and two seconds after P is allowed to fall freely from the same point, find the relative velocity of Q to P at any subsequent time.
Ans. 64·4 feet per second.

9. Define angular velocity. P is a point of a body turning uniformly round a fixed axis, and P N is a line drawn from P at right angles to the axis. If P N describes an angle of 375° in three seconds, what is the angular velocity of the body? and if P N is 6 feet long, what is the linear velocity of P? *Ans.* (1) 0·7 π radians per second; (2) 4·2 π feet per second.

10. A point is describing a circle of radius 21 feet, with a uniform velocity of 12 feet per second. Find the change in its velocity after it has described one-sixth of a whole circumference. *Ans.* 12 feet per second, at 120° with first direction.

11. A wheel, whose diameter is 5 feet, turns forty times a minute; find its angular velocity and the linear velocity of a point on its circumference. If the centre of the wheel moves in a straight line with a velocity of 20 miles an hour; what are the velocities, relative to a very distant fixed point in the straight line, of the ends of the diameter which is at any instant vertical? *Ans.* (1) $\omega = \frac{4\pi}{3}$ radians per second; (2) 10·5 feet per second; (3) upper end = 40 miles per hour; lower end = zero.

12. What is the numerical value of the angular velocity of a body which turns uniformly round a fixed axis twenty-five times a minute? A B C is a triangle right angled at C. It is turning with a given angular velocity, ω, round an axis through A, at right angles to its plane. Find the magnitude and direction of the velocities of B and C; and also the relative velocity of B to C. *Ans.* $\frac{5}{6}\pi$ radians per second.

13. A train descending a gradient increases its speed from 40 to 49 miles per hour in four and a-half minutes. Find the mean acceleration. Taking the acceleration due to gravity at 32 in feet and seconds, determine the gradient. *Ans.* (1) 0·049 foot per second per second, or 120 miles per hour per hour; (2) 1 in 654.

14. Given the base, b, of a smooth inclined plane, find its height, h, so that the horizontal component of the velocity of a body at the foot of the plane shall be a maximum. *Ans.* $h = b$.

15. Define the hodograph, and prove that the acceleration of a point's motion is equal to the velocity with which the hodograph is traced out. Determine, by means of the hodograph, the acceleration of a body which moves with uniform velocity in a circle.

16. Define the angular velocity of a moving point with respect to a fixed point. Under what circumstances will the angular velocity of the moving point be equal to its linear velocity divided by its distance? Draw an equilateral triangle A B C, having each side 12 feet long; a point moves along B C with a velocity of 10 feet a second; when it is at C, what is its angular velocity with respect to A? (S. & A. Adv. Theor. Mechs. Exam., 1896.)

17. Two circles touch each other externally, and the point of contact (A) is in the same vertical line as the centres; from any point (P) of the upper circumference draw a straight line P A Q to meet the lower circumference in Q; if a particle is allowed to fall from P along P Q, show that the time it takes to reach Q is constant for all positions of P. Also compare the times in which P A and A Q are described. (S. & A. Adv. Theor. Mechs. Exam., 1896.)

18. A body weighing 322 lbs. is lifted by a force of F lbs. which alters. When the body has risen through the distance x feet, the force in lbs. for the several values of x is as follows (or would be if the body rose as far):—

x	0	1	2	3	4	5·5	7	9	11	12·5	14	17	20
F	540	540	540	530	500	460	310	220	190	190	190	190	190

Using squared paper, find the velocity in each position and the time taken by the body to get to each position counting from $x = 0$, the velocity then being 5 feet per second. (S. and A. H., Part I., 1899.)

19. A body weighing 3,220 lbs. is lifted vertically by a rope, there being a damped spring balance to indicate the pulling force F pounds of the rope. When the body has been lifted x feet from its position of rest the pulling force is automatically recorded as follows :—

x	0	18	43	60	74	95	111	130
F	7,700	7,680	7,430	7,130	6,770	5,960	5,160	3,970

Using squared paper, find the velocity, v, for values of x of 20, 50, 80, 120, and draw a curve showing the probable values of v for all values of x In what time does the body get from $x = 75$ to $x = 85$? *Ans.* 34·6; 53·6; 65; 69·5 feet per second; 0·154 second. *Ans.* 740,600 ft.-lbs.; 370,300 ft.-lbs.; 370,300 ft.-lbs. (B. of E. Adv. & H., Part I., 1900.)

Lecture XX.—A.M.Inst.C.E. Exam. Questions.

1. Having given a diagram setting out the relation between distances and times in the movement of a body, show how you might use it to find the velocity at any time. What form does such a diagram take in the case of a body starting from rest with a uniform acceleration? (I.C.E., *Oct.*, 1897.)

2. In plane motion of a rigid body, given the acceleration of a point A and the path of a point B, show how we find a diagram for the acceleration of any point in the body. Prove your construction to be correct. (I.C.E., *Oct.*, 1898.)

3. Prove that if the link and orce polygons are both closed, a system of forces in a plane is in equilibrium. (I.C.E., *Oct.*, 1898.)

4. Two bodies, A and B, are moving along any two straight lines; state what is meant by the velocity of A relatively to B, and explain how you would determine this having given the velocities of A and B. A train is travelling at the rate of 20 miles an hour, and a man, sitting in a corner of a compartment with both windows down, observes a stone pass in a straight line at right angles to the length of the train through both windows. If it appears to the man to have a velocity of 20 feet per second, with what horizontal velocity must the stone have been thrown? (I.C.E., *Feb.*, 1899.)

5. Draw a rectangle A B C D having A B = 4 inches and B C = 2 inches. A body is acted on by a force P of 14·2 tons weight in the direction from C to B. You are required to find, graphically, three forces which will balance P, their lines of action being given as follows:—i. A line through D and a point E which is on A B, and 1 inch from A. ii. A line through B at right angles to D E. iii. The line C D. Show clearly, by means of arrows, the direction of each force. (I.C.E., *Feb.*, 1899.)

6. What is meant by the instantaneous centre of a piece moving in a fixed plane? A rigid body has a plane motion. Three points on the body A, B, and C, in the same plane, are such that A B = 3 feet, B C = 2 feet, and A C = 2·6 feet. At a certain instant it is known that the point A has a velocity of 4 feet per second in the direction from A to C, and that the point B is moving in the direction from C to B. Show how the velocity of any other point on the body may be obtained, graphically or otherwise, and determine the values for the velocities of B and the point midway between A and B. (I.C.E., *Feb.*, 1899.)

7. A body of 250 lbs. is acted on by a resultant force, F, which varies in amount. When the body was passed through the distance x feet, the force in lbs. weight is as follows:—

x	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
F	25	26	26	24	21·5	20	17	12·5	7	3	0

Find, either by a graphical or arithmetical method—(1) The average force acting on the body during the first foot. (2) The work done on the body during the first half of a foot. (3) If the velocity is 4 feet per second when $x=0$, find what it is when $x=0·5$ foot. (I.C.E., *Feb.*, 1899.)

8. Show that the parallelogram of force applies also to couples, and that the parallelogram of velocities is applicable to rotations.
(I.C.E., *Oct.*, 1899.)

9. Draw a curve of velocities referred to time which might apply to the starting of a train from rest, and show how to deduce the curve of acceleration from it. (I.C.E., *Oct.*, 1899.)

10. A body moving at a velocity of 10 feet per second to start with is acted upon by forces which produce an acceleration varying uniformly from 10 foot-seconds per second to 20 foot seconds per second during the 10 seconds they act; find the distance the body travels during the time.
(I.C.E., *Oct.*, 1899.)

11. What do you understand by the instantaneous centre of motion? Four links are pivoted together in a plane; find the instantaneous centres of motion of the opposite links with respect to each other, with proof of same. (I.C.E., *Oct.*, 1899.)

12. A body is rotating about an axis with an angular velocity of 4 radians per second, and about an axis intersecting the former at an angle of 60° with an angular velocity of 9 radians per second; find the axis about which the resultant rotation is taking place and its amount.
(I.C.E., *Feb.*, 1900.)

13. Define acceleration. Find an expression for the acceleration of a weight, W, rotating at a uniform angular velocity round an axis at the end of an arm whose weight may be neglected. (I.C.E., *Feb.*, 1900.)

14. A vehicle is proceeding in a westerly direction at the rate of 10 miles per hour. The wind is blowing from the south-west at the rate of 5 miles per hour. Find the velocity of the wind relatively to the vehicle in amount and direction. If the vehicle returns at the same rate, what is the velocity and direction of the wind relatively to it? (I.C.E., *Feb.*, 1900.)

15. What do you mean by an instantaneous centre of rotation? Give an example. What points on the rim of a carriage wheel are moving at the same rate as the middle of the axle? (I.C.E., *Feb.*, 1901.)

16. Explain the usual British and Continental units of mass, force, energy, power, and heat. Given that 1 kilogram equals 2·2 lbs, compare the velocities generated in 1 second by (1) the weight of 1 oz acting on a kilogram, and (2) the weight of a gram acting on 1 lb. (I.C.E., *Feb.*, 1901.)

17. Give a proof of the formulæ for falling bodies, (*a*) $v = g\,t$, (*b*) $s = \frac{1}{2}\,g\,t^2$, (*c*) $v^2 = 2\,g\,s$, where g is the acceleration due to gravity. Find the distance traversed by a falling body starting from rest, during the 8th and 10th seconds of its motion. (I.C.E., *Oct.*, 1901.)

18. State the proposition of the parallelogram of forces and describe an experiment to verify it. Find by calculation the magnitude of the resultant of two forces of 5 lbs. and 10 lbs. weight acting at an angle of 60°. (I.C.E., *Oct.*, 1901.)

19. A cam moves a roller up and down between vertical guides, the displacement of the roller being noted for each twelfth of a revolution of the cam as follows:—

Twelfths of revolution of cam.	1	2	3	4	5	6	7	8	9	10	11	12
Displacement of roller in inches.	0·19	0·97	2·15	4	5·41	6	5·4	4·01	2·16	0·97	0·24	0·00

Plot on squared paper to a suitable scale the displacement or space curve of the roller, and from it determine points in and draw the curves representing the velocity and acceleration of the roller for a complete revolution of the cam. Assuming the angular velocity of the cam shaft to

be uniform, and the time of one revolution to be ½ second, find from the curves the numerical value of the maximum velocity and acceleration of the cam roller. (I.C.E., *Oct.*, 1901.) (*Refer also to Lecture XIX., Vol. I.*)

20. Prove that the velocity acquired by a body in sliding down a smooth inclined plane is the same as that acquired in falling freely through a distance equal to the height of the plane. (I.C.E., *Feb.*, 1902.)

21. State the principle of the "parallelogram of velocities." If a man rows a boat at the rate of 3 miles an hour in a direction 60° east of north, and in a current which flows due south at the rate of 1½ miles an hour, find the direction the boat will move and the rate at which it will move.
(I.C.E., *Feb.*, 1902.)

22. Explain how to determine graphically the relative velocity of two points, the magnitudes and directions of whose velocities are known. Find the true course and velocity of a steamer steering due north by compass at 12 knots through a 4-knot current setting south-west, and determine the alteration of direction by compass in order that the steamer should make a true northerly course. (I.C.E., *Oct.*, 1902.)

23. Explain what is meant by the instantaneous axis of a link. A horizontal engine has a connecting-rod five cranks in length. Find the velocity ratio of slide-block and crank-pin at one quarter stroke. What is the mean velocity ratio of crank-pin and slide-block? (I.C.E., *Feb.*, 1903.)

24. Explain how to determine the relative velocity of two bodies. A is travelling due north at constant speed. When B is due west of A and at a distance of 21 miles from it, B starts travelling north east with the same constant speed as A. Determine graphically, or otherwise, the least distance which B attains from A. (I.C.E., *Feb.*, 1903.)

LECTURE XXI.

CONTENTS.—Quantity of Motion—Definition of Momentum—Example I.—Newton's Laws of Motion—Examples II. and III.—Motion on a Double Inclined Plane—Examples IV. and V.—Energy—Definition of Energy—Definitions of Potential and Kinetic Energy—Expression for Kinetic Energy—Energy Equations—Examples VI., VII., and VIII.—Questions.

Quantity of Motion.—In the preceding Lecture we have confined our attention chiefly to cases of pure motion—that is, motion considered apart from mass and force. In this Lecture we shall treat of the motion of bodies as produced by the action of external forces, and establish the relations between the *quantity of motion* thus produced and the magnitude of the forces producing it. **Quantity of motion** is measured by the product of the *mass* and its *velocity*. The term **Momentum** is used instead of *Quantity of motion*, and hence we get the following:—

DEFINITION.—**The momentum of a moving body is the product of its mass and velocity.**

Thus, let m be the mass, and v the velocity of a body:—

Then, $\quad$ **Momentum** $= m\,v$.

EXAMPLE I.—Of two steam hammers, one weighs 5 tons and the other 10 tons. The former has a drop of 10 feet and the latter 6 feet. Compare their momenta at the end of their respective strokes.

ANSWER.—In order to find their velocities at the moment of impact, we may employ formula (VI_b) of Lecture XX.:—

$$v^2 = 2\,g\,s,$$

$\therefore$ for the first hammer, $v_1 = \sqrt{2 \times 32 \times 10} = 8\sqrt{10}$ ft. per sec.

And, for the second hammer, $v_2 = \sqrt{2 \times 32 \times 6} = 8\sqrt{6}$ „

$$\therefore \frac{\text{Momentum of first hammer}}{\text{Momentum of second hammer}} = \frac{m_1 v_1}{m_2 v_2} = \frac{5 \times 8\sqrt{10}}{10 \times 8\sqrt{6}} = \frac{\sqrt{5}}{2\sqrt{3}} = \frac{1}{1{\cdot}549}.$$

Newton's Laws of Motion.—The three fundamental laws of Dynamics, called *Laws of Motion*, were first clearly set forth by Newton, and may be stated as follows:—

LAW I. (*Law of Inertia*).—**Every body continues in a state of rest, or of uniform motion in a straight line, except in so far as it may be compelled to change that state by external force acting on it.**

LAW II. (*Law of Force and Motion*).—**Rate of change of momentum is proportional to the force which causes it, and takes place in the direction of the force.**

LAW III. (*Law of Stress*).—**When two bodies mutually act upon each other, the momenta developed in the same time are equal but opposite in direction.**

Or, To every action there is an equal and opposite reaction.

Law I.—This Law asserts that matter is *indifferent* to motion, *i.e.*, has no *innate* tendency to start into motion when at rest, nor to change its motion, either in magnitude or in direction, when once it is made to move. Hence, a body at rest or in motion, and unacted upon by force, will continue to remain at rest, or to move on in a straight line with uniform motion. Should any change take place in its motion, then we immediately infer that the body has been acted upon by some external force. This tendency of matter to resist change in its state of rest or of uniform motion in a straight line is called **Inertia**, and the first Law is often spoken of as the *Law of Inertia*.

Law II.—The first Law asserts that change of momentum is caused by the action of force, and the second Law gives us a means of measuring this force, viz., that the force is proportional to the *rate of change of momentum*.

Let F = Force producing change of momentum.
,, m = Mass of body.
,, v_1, v_2 = Initial and final velocities of body.
,, t = Time during which F acts.

Then, *Change of momentum* $= m(v_2 - v_1)$.

And, *Rate of change of momentum* $= \dfrac{m(v_2 - v_1)}{t}$.

∴ By the *Second Law of Motion*, we get :—

$$F \propto \frac{m(v_2 - v_1)}{t}.$$

Or, $$F = \frac{m(v_2 - v_1)}{t} \times \text{constant.}$$

It now remains to establish the exact relation between those quantities. If we accept the definition of *Unit Force* given on page 2, Lecture I., Vol. I., as being *that force which, acting for unit time on unit mass, produces unit change of velocity*, we find the numerical value of the *constant* in the above equation to be *unity*.

i.e., $$F = \frac{m(v_2 - v_1)}{t}.$$

But we have shown in Lecture XX. that :—

$$a = \frac{v_2 - v_1}{t},$$

where a denotes the acceleration produced when the motion is uniformly accelerated.

∴ $$F = m\,a.$$

The above definition is that of the *Absolute Unit* of Force; and, therefore, the force, F, as given by these equations, is expressed in absolute units. Engineers, however, prefer measuring their forces by the weights which they are capable of supporting, and the above equations may be modified to suit these units. Let P, P_1, be the statical measures of the forces required to produce accelerations, a, a_1, on a given mass, m; then by Law II., we get :—

$$P : P_1 = a : a_1.$$

If one of these forces be that due to gravity, viz., the weight, w, of a body, then the aceleration is g, and we get :—

$$P : w = a : g.$$

Or, $$P = \frac{w\,a}{g} \quad \ldots\ldots\ldots \quad \text{(I)}$$

This equation expresses the force, P, in the same units as w, and if w be stated in *pounds weight* that will be in what we have previously called *gravitation* units.

Law III.—This Law asserts that when two bodies mutually act upon each other, the momenta generated in each are equal, but in opposite directions. Thus, when a shot is fired from a gun, the force of the explosion produces momentum in the gun equal in amount to that of the shot, and causes the recoil. We shall, however, see later on that the other effects produced in the gun and the shot are not numerically equal. In the case of mutual action between two bodies incapable of relative motion, the Law asserts that they act and react on each other with equal forces. Thus, a weight lies on a table, and presses on it with a certain force; then the table reacts on the weight with an equal and opposite force, so that every action is accompanied by an equal and opposite reaction.

The truth of this Law has been assumed throughout the whole of the preceding parts of this treatise—viz., that the effort exerted between two bodies is always equal to the resistance overcome. The two equal and opposite forces caused by the mutual action between two bodies are together spoken of as a Stress, and for this reason the above Law is sometimes called the *Law of Stress.* The subject of *internal stress* will be discussed in another part of this work.

We shall now apply the preceding results to some examples.

EXAMPLE II.—A 40-lb. shot is fired from a 5-ton gun with an initial velocity of 1,500 feet per second. Find the velocity of the gun's recoil, and the mean force of the explosion, supposing the gun to be 10 feet long.

ANSWER.—Let W, w = Weight of gun and shot respectively.

„ V, v = Velocity „ „

(1) By the *Third Law*:—

Momentum of gun = momentum of shot.

$$\therefore \quad W\,V = w\,v$$

$$i.e., \quad 5 \times 2240 \times V = 40 \times 1500,$$

$$\therefore \quad V = \frac{40 \times 1500}{5 \times 2240} = 5{\cdot}36 \text{ ft. per sec.}$$

(2) In order to find the mean effort exerted during the explosion of the powder, we must first determine the acceleration of the shot along the muzzle of the gun. Since the gun is 10 feet long, and the velocity of the shot as it leaves the

gun is 1,500 ft. per second, we get, from the formula (VI_a), Lecture XX. :—

$$v^2 = 2\,a\,s,$$

$$\therefore \quad 1500^2 = 2 \times a \times 10,$$

$$\therefore \quad a = \frac{1500^2}{20} = 112{,}500 \text{ ft. per second per second.}$$

But, $P = \dfrac{w}{g} \times a,$

$$\therefore \quad P = \frac{40}{32} \times 112{,}500 = \mathbf{140{,}625 \text{ lbs.}}$$

EXAMPLE III.—A railway train, exclusive of engine, weighs 200 tons, and moves on a level line. In 10 minutes its speed is increased from 10 miles per hour to 40 miles per hour. Determine the mean pull between the engine and train, the frictional resistances being taken at 10 lbs. per ton.

ANSWER.—The pull between the engine and train consists of two parts; (1) the force required to accelerate the train, and (2) the force required to overcome the frictional resistances.

(1) Let P_1 = Force required to accelerate the train,

Then, $P_1 = \dfrac{w\,a}{g} = \dfrac{w\,(v_2 - v_1)}{g\,t}.$

But, $v_1 = 10$ miles per hour $= \dfrac{44}{3}$ ft. per second.

$v_2 = 40$ " $= \dfrac{176}{3}$ "

And, $t = 10$ minutes $= 600$ seconds.

$$\therefore \quad P_1 = \frac{200 \times 2240 \times \left(\frac{176}{3} - \frac{44}{3}\right)}{32 \times 600} = \mathbf{1026 \cdot 6 \text{ lbs.}}$$

(2) The resistance of friction being 10 lbs. per ton,

The *total frictional resistance* $= P_2 = 200 \times 10 = \mathbf{2000 \text{ lbs.}}$

$$\therefore \quad \left.\begin{array}{c}\text{Mean pull between}\\ \text{engine and train}\end{array}\right\} = P_1 + P_2 = 1026\cdot\dot{6} + 2000 = \mathbf{3026\cdot\dot{6} \text{ lbs.}}$$

Motion on a Double Inclined Plane.—Let A B C, D B C, be the two planes placed back to back, and let W_1, be the ascending, and W_2, the descending load, these loads being connected by a weightless rope passing over a frictionless and weightless pulley at B. We require to determine the motion—*i.e.*, the *acceleration* of the bodies, and the tension in the connecting rope.

Let α_1, α_2 = Inclinations of planes A B C, D B C respectively.

" μ_1, μ_2 = Coefficients of friction between W_1, W_2, and their respective planes.

" F_1, F_2 = Frictional resistances in the two cases.

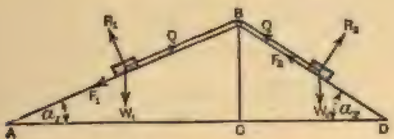
" P_1, P_2 = Effective forces acting on W_1, W_2 respectively in causing motion.

" Q = Tension in connecting rope.

" a = Acceleration due to effective forces P_1, P_2.

Then, the effective force causing the *upward* motion of W_1, is :—

$$P_1 = Q - W_1 \sin \alpha_1 - F_1.$$



DOUBLE INCLINED PLANE.

Similarly, the effective force in causing the *downward* motion of W_2, is :—

$$P_2 = W_2 \sin \alpha_2 - Q - F_2.$$

But, $F_1 = \mu_1 W_1 \cos \alpha_1.$

And, $F_2 = \mu_2 W_2 \cos \alpha_2.$

$\therefore$ $P_1 = Q - W_1 (\sin \alpha_1 + \mu_1 \cos \alpha_1), \quad . \quad . \quad . \quad . \quad (1)$

And, $P_2 = W_2 (\sin \alpha_2 - \mu_2 \cos \alpha_2) - Q. \quad . \quad . \quad . \quad . \quad (2)$

Again, $P_1 = \dfrac{W_1}{g} a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$

And, $P_2 = \dfrac{W_2}{g} a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$

To determine the acceleration, a :—

From, (1) + (2),

$$P_1 + P_2 = W_2(\sin\alpha_2 - \mu_2\cos\alpha_2) - W_1(\sin\alpha_1 + \mu_1\cos\alpha_1)$$

From, (3) + (4),

$$P_1 + P_2 = \frac{W_1 + W_2}{g} \times a.$$

$$\frac{W_1 + W_2}{g} \times a = W_2(\sin\alpha_2 - \mu_2\cos\alpha_2) - W_1(\sin\alpha_1 + \mu_1\cos\alpha_1),$$

$$\therefore \quad a = \frac{W_2(\sin\alpha_2 - \mu_2\cos\alpha_2) - W_1(\sin\alpha_1 + \mu_1\cos\alpha_1)}{W_1 + W_2} \times g. \quad \text{(II)}$$

To determine the tension in the rope :—

Equation (1) ÷ (2), $\dfrac{P_1}{P_2} = \dfrac{Q - W_1(\sin\alpha_1 + \mu_1\cos\alpha_1)}{W_2(\sin\alpha_2 - \mu_2\cos\alpha_2) - Q}$,

,, (3) ÷ (4), $\dfrac{P_1}{P_2} = \dfrac{W_1}{W_2}$.

$$\therefore \quad \frac{Q - W_1(\sin\alpha_1 + \mu_1\cos\alpha_1)}{W_2(\sin\alpha_2 - \mu_2\cos\alpha_2) - Q} = \frac{W_1}{W_2},$$

$$\therefore \ (W_1 + W_2)Q = W_1 W_2(\sin\alpha_1 + \mu_1\cos\alpha_1 + \sin\alpha_2 - \mu_2\cos\alpha_2)$$

$$\therefore \quad Q = \frac{W_1 W_2(\sin\alpha_1 + \sin\alpha_2 + \mu_1\cos\alpha_1 - \mu_2\cos\alpha_2)}{W_1 + W_2} \quad \text{(III)}$$

We shall show how these formulæ are modified to suit some particular cases, but the student should try to prove each particular case independently of the general case just demonstrated.

Case I.—Suppose the planes to be equally inclined to the horizon, and equally rough, so that $\alpha_1 = \alpha_2 = \alpha$, and $\mu_1 = \mu_2 = \mu$; then, from equation (II) :—

$$a = \frac{W_2(\sin\alpha - \mu\cos\alpha) - W_1(\sin\alpha + \mu\cos\alpha)}{W_1 + W_2} g,$$

$$\therefore \ a = \frac{(W_2 - W_1)\sin\alpha - \mu(W_2 + W_1)\cos\alpha}{W_1 + W_2} g \quad . \quad . \quad \text{(II}_a\text{)}$$

From equation (III),

$$Q = \frac{2 W_1 W_2 \sin\alpha}{W_1 + W_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III}_a\text{)}$$

CASE II.—Let the planes be equally inclined, and smooth; so that $\alpha_1 = \alpha_2 = \alpha$, and $\mu_1 = \mu_2 = 0$; then, from equation (II):—

$$a = \frac{W_2 \sin \alpha - W_1 \sin \alpha}{W_1 + W_2} g,$$

$$\therefore \qquad a = \frac{(W_2 - W_1) \sin \alpha}{W_1 + W_2} g \quad . \; . \; . \; . \; . \; . \quad (II_b)$$

And from equation (III),

$$Q = \frac{2 W_1 W_2 \sin \alpha}{W_1 + W_2} \quad . \; . \; . \; . \; . \; . \; . \quad (III_b)$$

Equations (III_a) and (III_b) show that the degree of roughness of the planes does not affect the tension in the rope, when the planes are equally inclined to the horizon.

CASE III.—Let the plane, AB, be horizontal, and $\mu_1 = \mu_2 = \mu$, and suppose W_2 by falling vertically to drag W_1 along AB. In this case $\alpha_1 = 0$, and $\alpha_2 = 90°$; then, from equation (II):—

$$a = \frac{W_2 (\sin 90° - \mu \cos 90°) - W_1 (\sin 0 + \mu \cos 0)}{W_1 + W_2} g,$$

$$\therefore \; a = \frac{W_2 - \mu W_1}{W_1 + W_2} g \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (II_c)$$

And from equation (III),

$$Q = \frac{W_1 W_2 (1 + \mu)}{W_1 + W_2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (III_c)$$

CASE IV.—In the previous case, let the horizontal plane be smooth, so that, $\mu = 0$:—

Then, $$a = \frac{W_2}{W_1 + W_2} g \quad . \; . \; . \; . \; . \; . \quad (II_d)$$

And, $$Q = \frac{W_1 W_2}{W_1 + W_2} \quad . \; . \; . \; . \; . \; . \; . \quad (III_d)$$

CASE V.—Suppose the weights to be suspended over the frictionless and weightless pulley B, and the parts of the rope to hang vertically.

In this case, $\alpha_1 = \alpha_2 \, 90°$; $\mu_1 = \mu_2 = 0$; then:—

From equation (II),

$$a = \frac{W_2 - W_1}{W_1 + W_2} g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II_e)$$

From equation (III),

$$Q = \frac{2 W_1 W_2}{W_1 + W_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (III_e)$$

These last equations are of great importance to the student of *Theoretical Mechanics*, because they enable him, by means of an Atwood's machine, to determine the value of g, at the place where the experiment is conducted.

EXAMPLE IV.—A cage weighing 1 ton is being raised from a mine with an acceleration of 10 feet per second. Find the tension in the rope. If a miner, whose weight is 150 lbs., is raised with the cage, find the pressure between him and the cage. Again, if the cage be lowered with the same acceleration, what would then be the tension in the rope, and the pressure between the man and cage?

ANSWER.—(1) *To find tension in rope during ascent of cage.*

Let W = Weight of cage = 1 ton = 2,240 lbs.
„ w = Weight of man = 150 lbs.
„ Q = Tension of rope in lbs.
„ a = Acceleration of cage = 10 ft. per sec. per sec.

Then, neglecting the weight of the rope, and in the meantime that of the man, we get:—

Effective pull causing motion = $P = Q - W$.

But, by the *Second Law of Motion*, $P = \frac{W}{g} a$.

$$\therefore \qquad Q - W = \frac{W}{g} a.$$

Hence,

$$Q = W\left(1 + \frac{a}{g}\right)$$

$$\therefore \qquad Q = 2240 \times \left(1 + \frac{10}{32}\right) = \mathbf{2{,}940\ lbs.}$$

That is, the tension in the rope is *greater* than the weight raised by 700 lbs.

If the weight of the miner be taken into account, we must increase W, by 150, and then we get :—

$$Q = 3136{\cdot}9 \text{ lbs.}$$

(2) *To find the pressure between man and cage.*

$$\left.\begin{array}{l}\textit{Pressure between man}\\ \quad\textit{and cage}\end{array}\right\} = \left\{\begin{array}{l}\textit{Weight of man + Force required}\\ \quad\textit{to accelerate his upward motion}\end{array}\right.$$

$$\text{,,} \qquad \text{,,} \qquad = w + \frac{w}{g}a,$$

$$\therefore \qquad w_1 = 150 + \frac{150}{32} \times 10 = 196{\cdot}9 \text{ lbs.}$$

Under these circumstances he will feel heavier by 46·9 lbs.

(3) *To find tension in rope during descent of cage.*

In this case, we get :—

$$\left.\begin{array}{l}\textit{Effective pull causing}\\ \quad\textit{motion}\end{array}\right\} = P = W - Q,$$

$$\text{And,} \qquad P = \frac{W}{g} \times a,$$

$$\therefore \qquad W - Q = \frac{W}{g} \times a,$$

$$\therefore \qquad Q = W\left(1 - \frac{a}{g}\right)$$

$$Q = 2{,}240 \times \left(1 - \frac{10}{32}\right) = 1{,}540 \text{ lbs.}$$

That is, the tension in the rope is *less* than the weight of the cage by 700 lbs.

Similarly, it can be shown that the pressure between the man and the floor of the cage during descent, is 103·1 lbs.; or, 46·9 lbs. less than his real weight.

Example V.—In a double inclined plane, having a rise of 1 in 20, the loaded and empty trucks run on parallel lines of rails, the connection being made by means of two ropes passing round drums at the summit of the plane. Five loaded trucks

when descending pull up an equal number of empty ones. Each empty truck weighs 5 cwts., and when loaded carries 20 cwts. of material. The diameter of the drums at the top of the incline is 8 feet, and on the same shaft is fitted a brake pulley 6 feet in diameter. The length of the inclined plane is 1 mile. Taking the coefficient of friction between the trucks and the rails at 20 lbs. per ton, but neglecting other frictional resistances; determine (1) the acceleration of the trucks and their speed at the end of one minute after starting; (2) the tension in the ropes during the free motion of the whole; and (3) the constant frictional resistance which must be exerted at the rim of the brake pulley, during the last three-eighths of the run, in order to just bring the whole to rest at the end of the journey.

ANSWER.—Using the same letters as in the text.

Let W_1 = Total weight of five empty trucks = 25 cwts.
„ W_2 = „ „ loaded „ = 125 „
„ P_1 = Effective force causing motion of W_1.
„ P_2 = „ „ „ W_2.
„ Q = Tension in ropes.
„ α = Inclination of the plane.
„ μ = Coefficient of friction = 20 lbs. per ton = $\frac{1}{112}$.

Then, $\sin \alpha = \frac{1}{20}$; and since α is small we may assume $\cos \alpha = 1$.

(1) *To find the acceleration of the trucks.*

The effective pull causing the motion of the empty trucks, is:—

$$P_1 = Q - W_1 (\sin \alpha + \mu \cos \alpha),$$

$$\therefore \quad P_1 = Q - 25\left(\frac{1}{20} + \frac{1}{112} \times 1\right) = Q - \frac{165}{112} \text{ cwt.} \quad . \quad . \quad (1)$$

The effective pull causing the motion of the loaded trucks, is:—

$$P_2 = W_2 (\sin \alpha - \mu \cos \alpha) - Q,$$

$$,, \quad = 125\left(\frac{1}{20} - \frac{1}{112} \times 1\right) - Q = \frac{575}{112} - Q \text{ cwt.} \quad . \quad . \quad (2)$$

Again, by *Second Law of Motion*, we get :—

$$P_1 = \frac{W_1}{g} \times a = \frac{25}{32} \times a \text{ cwt.} \quad . \quad . \quad . \quad . \quad (3)$$

$$P_2 = \frac{W_2}{g} \times a = \frac{125}{32} \times a \text{ cwt.} \quad . \quad . \quad . \quad (4)$$

$$(1) + (2), \quad P_1 + P_2 = \frac{575}{112} - \frac{165}{112} = \frac{205}{56} \text{ cwt.}$$

$$(3) + (4), \quad P_1 + P_2 = \left(\frac{25}{32} + \frac{125}{32}\right) \times a = \frac{150}{32} a \text{ cwt.}$$

$$\therefore \quad \frac{150}{32} \times a = \frac{205}{56}$$

$$\therefore \quad a = 0{\cdot}78 \text{ ft. per sec. per sec.}$$

That is, the trucks move with an acceleration of 0·78 foot per second per second.

At the end of one minute from starting the speed would be :—

$$v = a\,t = {\cdot}78 \times 60 = 46{\cdot}8 \text{ ft. per sec.}$$

Or, at the end of one minute they would be moving with a speed somewhat greater than 30 miles per hour.

(2) *To find tension in the ropes.*

Since we have assumed that the machinery at the top of the incline offers no resistance to the motion, it is evident that the tension in each rope will be the same. Hence :—

$$(1) \div (2), \quad \frac{P_1}{P_2} = \frac{Q - \frac{165}{112}}{\frac{575}{112} - Q} = \frac{112\,Q - 165}{575 - 112\,Q}.$$

$$(3) \div (4), \quad \frac{P_1}{P_2} = \frac{W_1}{W_2} = \frac{25}{125} = \frac{1}{5}.$$

$$\therefore \quad \frac{1}{5} = \frac{112\,Q - 165}{575 - 112\,Q}.$$

$$\therefore \quad Q = \frac{1400}{672} = 2{\cdot}08 \text{ cwt.}$$

(3) *To find the frictional resistance at the rim of the brake pulley in order to bring the trucks to rest at the end of the run.*

Here we have first to obtain the speed of the trucks at the instant when the brake is applied, and then find the retardation or negative acceleration necessary to bring the trucks to rest at the desired place.

The velocity v, of the trucks at the instant when the brake is applied is given by the formula :—

$$v^2 - v_1^2 = 2\,a\,s.$$

Where v_1 = Initial velocity = 0 in this case.

" a = Acceleration just found = 0·78 ft. per sec. per sec.

" s = Distance traversed = $\frac{5}{8}$ mile.

The acceleration during the application of the brake may be found by the same formula. In this case, however, the initial velocity is v, and the final velocity is zero.

Let a_1 = Acceleration of the trucks during the application of the brake.

" s_1 = Distance traversed = $\frac{3}{8}$ mile.

Then, before the brakes are applied :—

$$v^2 - 0^2 = 2\,a\,s$$

Or, $$v^2 = 2\,a\,s.$$

And after the brakes have been applied :—

$$0^2 - v^2 = 2\,a_1\,s_1,$$

Or, $$v_2 = -2\,a_1\,s_1.$$

$$\therefore \quad a_1 = -\frac{a\,s}{s_1} = -\frac{\cdot 78 \times \frac{5}{8}}{\frac{3}{8}} = -1\cdot 3 \text{ ft. per sec. per sec.}$$

(4) *To determine the tensions in the two ropes.*

These will not now be equal as when the motion was free. The tension in the rope coming on to the drum will be much less than before, whilst that on the other rope will be greater.

Let Q_1, Q_2 = Tensions in the ropes attached to the empty and loaded trucks respectively.

Then the effective pull P_1, causing the motion of the ascending trucks is as before :—

$$P_1 = Q_1 - W_1 (\sin \alpha + \mu \cos \alpha)$$

But, $$P_1 = \frac{W_1 a_1}{g}$$

$\therefore$ $$Q_1 = W_1 \left\{ \sin \alpha + \mu \cos \alpha + \frac{a_1}{g} \right\}$$

$$,, \quad = 25 \left\{ \frac{1}{20} + \frac{1}{112} \times 1 - \frac{1 \cdot 3}{32} \right\} \text{cwt.},$$

$$,, \quad = \frac{25 \times 20 \cdot 5}{1120} \text{ cwt.} = 51 \cdot 25 \text{ lbs.}$$

Similarly, the tension Q_2, in the rope attached to the loaded trucks is,

$$Q_2 = W_2 \left\{ \sin \alpha - \mu \cos \alpha - \frac{a_1}{g} \right\}$$

$$,, \quad = 125 \left\{ \frac{1}{20} - \frac{1}{112} \times 1 + \frac{1 \cdot 3}{32} \right\} \text{cwt.}$$

$$,, \quad = \frac{125 \times 91 \cdot 5}{1120} \text{ cwt.} = 1143 \cdot 75 \text{ lbs.}$$

The difference in the tensions in the two ropes is caused by the resistance offered by the brake. Hence, the resultant couple due to this difference in the tension must be balanced by the couple at the brake wheel.

Let F = Frictional resistance at the rim of the brake wheel.
,, R = Radius of the drums = 4 ft.
,, r = Radius of brake wheel = 3 ft.

Then, $$F \times r = (Q_2 - Q_1) \times R,$$

$\therefore$ $$F \times 3 = (1143 \cdot 75 - 51 \cdot 25) \times 4,$$

$\therefore$ $$F = 1456 \cdot 7 \text{ lbs.}$$

Energy.—If we raise a body of W lbs. weight through a vertical height of h feet from some given datum level, we confer upon that body the capability of doing work equal to $W h$ ft.-lbs. For, in raising the body we expend $W h$ ft.-lbs. of work, and if it be allowed to return to its original level it will give out an equal amount of work.

Again, we have seen that if a body be in motion and its speed reduced, some force must have acted upon it in bringing about this change of state. Further, this resisting force must have been overcome through some distance, and, therefore, work is expended. Thus, a body in motion is capable of doing work, the measure of which is the work done against a resisting force or forces in bringing the body to rest.

This capability of doing work which a body possesses in virtue of its position or condition is called **Energy**. Hence, we have the following:—

DEFINITION.—**The energy of a body is its capability of doing work in virtue of its Position, Condition, or Motion.**

It is usual to distinguish between that form of energy due to the position or state of a body, and that due to its motion. To the former the term **Potential** is applied, and to the latter **Kinetic**. This distinction may be stated in the form of a definition.

DEFINITIONS.—**Potential Energy is that form of energy which a body possesses in virtue of its Position or Condition.**

Kinetic Energy is that form of energy which a body possesses in virtue of its Motion.

Thus, a raised weight, such as the weight of a clock, or the monkey of a pile-driving engine, has *potential* energy in virtue of its *position*. In the first case the slowly falling weight gives up its stored energy to the mechanism of the clock in overcoming frictional resistances, and thus keeps the clock going, the pendulum being simply a regulator or governor. In the second case, the monkey is allowed to fall freely and its energy is employed in forcing the pile into the ground at the instant of the blow. Similarly, the water in a mill dam possesses potential energy due to its *position* relatively to the water wheel. Again, a stretched helical spring, or coiled spiral spring such as is used in watches and clocks, possesses potential energy due to its *stretched condition*. A lump of coal, or gunpowder, has potential energy in virtue of its *chemical condition;* a magnet has potential energy in virtue of its *magnetic condition;* and the steam in a boiler has potential energy in virtue of its *heat condition*, and so on.

When the monkey of the pile driver is at the top of its stroke its energy is entirely in the potential form. When it is descending it is evident that its potential energy is rapidly decreasing whilst its kinetic energy is increasing.

Neglecting frictional and other resistances, the **Principle of the Conservation of Energy** (see Lecture IV., Vol. I.) asserts that—

The Loss of Potential Energy = The Gain of Kinetic Energy.

Consequently, at the instant when the monkey strikes the head of the pile, the energy of the monkey is wholly Kinetic. The work done in driving the pile into the ground is immediately derived from the kinetic energy of the falling weight, but the whole of this energy is not thus employed, for the faces of the pile and monkey have been heated by the blow. This shows that part of the energy stored in the falling weight has been transformed into heat energy. Further, at the instant of striking, a loud noise is heard, which shows that there is also a transformation into sound energy. Thus, energy appears under many different forms, such as mechanical, electrical, chemical, heat, light, sound, &c., and can, by suitable arrangements, be changed from one kind into any of the others. In nature all is change or transformation, but there is no annihilation; so what appears as a loss to the engineer simply means change into some other form which he does not desire, but which he has no power to entirely prevent.

Expression for Kinetic Energy.—We have already seen, that the expression for mechanical potential energy is:—

$$\textbf{Potential Energy} = E_P = W\,h$$

Where, W = Weight of body,

And, h = Height of body above zero level.

It now remains to determine the expression for kinetic energy.

First, take the case of the raised weight whose potential energy in its highest position is $W\,h$, and suppose it to fall freely. Its kinetic energy at the instant when it strikes the ground is:—

$$E_K = W\,h.$$

But, if v = velocity at that instant, we have:—

$$v^2 = 2\,g\,h,$$

Or, $$h = \frac{v^2}{2\,g}.$$

$$\therefore \quad E_K = \frac{W\,v^2}{2\,g} \quad \ldots\ldots \quad \text{(IV)}$$

Thus, if the monkey of the pile driver weigh 10 cwts., and reaches the pile with a velocity of 40 feet per second, it has kinetic energy :—

$$E_K = \frac{10 \times 112 \times 40^2}{2 \times 32} = 28{,}000 \text{ ft.-lbs.}$$

If the pile be driven 3 inches into the ground at each blow, what is the *mean* resistance offered to its motion, supposing there are no losses from heating, &c.?

Let R_m = Mean resistance of ground in lbs.
,, s = Distance *in feet* through which the pile is driven.

Then, $\left.\begin{array}{l}\textit{Work done in driv-}\\ \textit{ing the pile}\end{array}\right\} = \left\{\textit{Work given out by monkey.}\right.$

But, $\left.\begin{array}{l}\textit{Work given out by}\\ \textit{monkey}\end{array}\right\} = \left\{\begin{array}{l}\textit{Kinetic energy at instant of}\\ \textit{striking the pile + Work}\\ \textit{done in falling through 3''.}\end{array}\right.$

$\therefore$ $$R_m \times s = \frac{W v^2}{2g} + W \times s,$$

i.e., $$R_m \times \frac{3}{12} = 28{,}000 + 10 \times 112 \times \frac{3}{12},$$

$\therefore$ $$R_m = 113{,}120 \text{ lbs.} = 50{\cdot}5 \text{ tons.}$$

Energy Equations.—The expression for the kinetic energy given in equation (IV) is perfectly general, and therefore independent of the manner in which the velocity, v, is acquired. That is to say, if a body of weight, W, be moving with a velocity, v, in any direction whatever, its kinetic energy is still given by the equation :—

$$E_K = \frac{W v^2}{2g}.$$

For, manifestly, the direction of motion cannot in any way affect its energy state, other things being equal. Nevertheless, we can deduce the expression from more general considerations as follows :—

Let a body of weight, W, have its velocity changed in magnitude from v_1 to v_2 by a constant force P, acting through a distance s. Then—

$\left.\begin{array}{l}\textit{Change of body's kinetic}\\ \textit{energy}\end{array}\right\} = \left\{\begin{array}{l}\textit{Work done by or against}\\ \textit{the force}\end{array}\right.$

And, *Work done by or on* P $=$ P $\times$ s.

But, by eqation (I), $\quad P = \dfrac{W a}{g}$,

And, by equation (VI), Lecture XX.,

$$s = \frac{v_2^2 - v_1^2}{2a},$$

$$\therefore \qquad P \times s = \frac{W(v_2^2 - v_1^2)}{2g} = \text{change of } E_K . \text{(V)}$$

If the body start from rest, and have a final velocity, v, we get :—

$$\text{Change of } E_K = \frac{W v^2}{2g} = \text{Total } E_K \text{ in body,}$$

which is just the same result as that given by equation (IV).

Next, take the case of a body moving with uniform velocity, v, against some constant resistance, F, which resistance may be frictional or otherwise. To maintain this constant speed a force equal to F must act on the body in opposition to the resistance; but no part of this force is employed in changing the kinetic energy of the body, since, by supposition, no change occurs in its velocity. The kinetic energy of the body is constantly $= \dfrac{W v^2}{2g}$, and the *Work done against resistances* $=$ F s.

If, now, some other force, P, acts on the body in the direction of motion, the velocity will change, and, therefore, the energy of the body will also change.

Let Q = Resultant force acting on body = $P \sim F$.
" v_1, v_2 = Velocities of body before and after action of P.
" s = Distance through which body moves under P.

Then, $$Q \times s = F \times s + \frac{W(v_2^2 - v_1^2)}{2g} \quad . \quad . \quad . \quad \text{(VI)}$$

This is a very general equation of energy, and is sometimes stated thus :—

Energy exerted = Work done + Change in Kinetic Energy.

Example VI.—The height and length of an inclined plane are 20 feet and 100 feet respectively: a body weighing 100 lbs. is placed at the top of the plane and allowed to slide along its whole length; the coefficient of friction between the plane and

the body is 0·15; how many units of work (foot-pounds) are accumulated in the body, and what is its velocity when it reaches the foot of the plane? (You may assume the pressure on the plane equal to the weight of the body). (S. & A. Adv. Exam.)

ANSWER.—Let F = Resultant force urging body down the plane.

Then, $F = W \sin\alpha - \mu R = W \sin\alpha - \mu W$, very approximately,

$$,, = 100\left(\frac{20}{100} - 0{\cdot}15\right) = 5 \text{ lbs.}$$

When body reaches the foot of the plane, we have :—

$$E_K = F \times l = 5 \times 100 = 500 \text{ ft.-lbs.}$$

Let v = Velocity at foot of plane.

Then, $$\frac{W v^2}{2g} = E_K.$$

i.e., $$\frac{100 \times v^2}{2 \times 32} = 500,$$

$\therefore$ $$v = 17{\cdot}9 \text{ ft. per sec.}$$

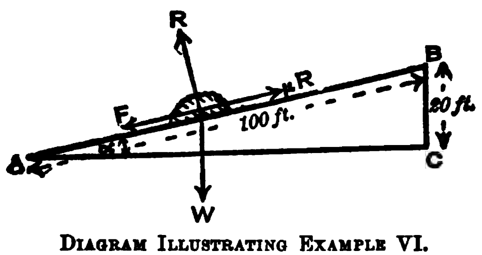


DIAGRAM ILLUSTRATING EXAMPLE VI.

The kinetic energy at the foot of the plane could be obtained immediately from equation (VI), thus :—

$$\textit{Energy exerted} = W \sin\alpha \times l = 100 \times \frac{20}{100} \times 100 = 2000 \text{ ft.-lbs.}$$

$$\textit{Work done} = \mu W l = 0{\cdot}15 \times 100 \times 100 = 1500 \text{ ft.-lbs.}$$

$$\textit{Change in } E_K = \textit{Energy at foot of plane} = \frac{W v^2}{2g}.$$

$\therefore$ **Energy exerted = Work done + Change in E_K.**

$$\therefore \qquad 2000 = 1500 + \frac{W v^2}{2 g}.$$

$$\therefore \quad \left.\begin{array}{l}E_K \text{ at foot of} \\ \text{plane}\end{array}\right\} = \frac{W v^2}{2 g} = 500 \text{ ft.-lbs.}$$

$$\therefore \text{ also,} \qquad v = \sqrt{\frac{500 \times 2 \times 32}{100}} = 17{\cdot}9 \text{ ft. per sec.}$$

EXAMPLE VII.—Show by a diagram the amount of work done in *slowly compressing* a spiral spring through 6 inches, supposing the spring to shorten 1 inch for every 100 lbs. pressure. If a weight of 100 lbs. *falls* from a height of 4 feet on the top of the spring, how much will it be compressed? (S. & A. Adv. Exam., 1892.)

ANSWER.—As explained in Lecture II., Vol. I., the diagram of work done in slowly compressing a spiral spring, is a right angled triangle whose base represents the compression produced, and its perpendicular side the force required to produce that compression.

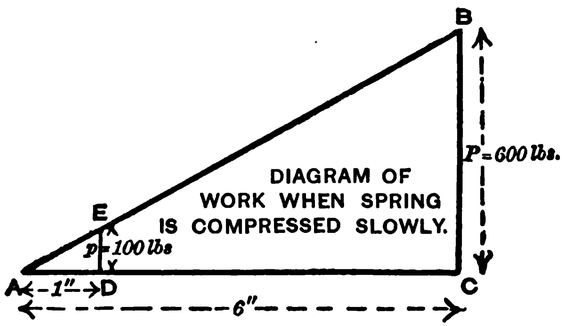


DIAGRAM ILLUSTRATING EXAMPLE VII.

Let A B C represent the diagram of work done in slowly compressing the spring.

Where, A C = Compression produced = 6″.

And, C B = Force required = P.

Let, D E = Force required to compress spring 1″, = p = 100 lbs.

Then, $\quad P : p = CB : DE = AC : AD.$

That is, $\quad P : 100 = 6'' : 1''.$

$\therefore \quad P = 600$ lbs.

$\therefore \quad$ Work done $= \frac{1}{2} P \times L = \frac{1}{2} \times 600 \times 6'' = 1800$ in.-lbs.

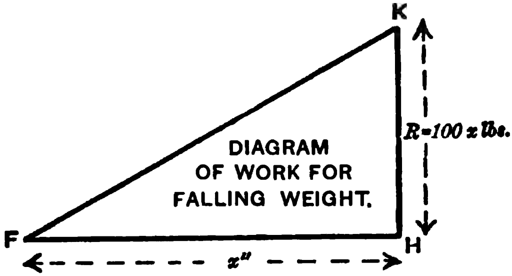


Diagram Illustrating Example VII.

Next, suppose the spring to be compressed by a weight which falls from a height of 4 feet.

Let x = Number of *inches* by which spring is compressed by falling weight.

Then, $\quad 48 + x$ = Number of inches through which weight actually falls.

Since a force of 100 lbs. is required to compress the spring 1 inch, a force $R = 100\,x$ lbs. will be required to compress it x inches.

But,

Work done in compressing spring = *Work done by falling weight.*

$$\therefore \quad \tfrac{1}{2} R \times x = W(48 + x).$$

$$\therefore \quad \tfrac{1}{2} \times 100\,x \times x = 100\,(48 + x).$$

That is, $\quad x^2 = 2\,x + 96.$

Or, $\quad x = 10{\cdot}85$ inches.

Example VIII.—A blowing-fan 30 inches in diameter revolves at a speed of 1,000 revolutions per minute, and propels the wind with a velocity equal to $\frac{7}{8}$ of the velocity of the tips of the vanes; the wind is driven through a pipe having a sectional area

of 200 square inches. Neglecting the power that is required to overcome friction, will you show the amount of power which is required to force the above quantity of air? Work it out in arithmetic, either by the law of falling bodies or in any better way that may suggest itself to you. (S. & A. Hons. Exam.)

ANSWER.—This is simply a question on work and energy.

Let W = Weight of air expelled from fan *per second.*
„ A = Sectional area of delivery pipe of fan = 200 sq. ins.
„ v = Velocity of air as it leaves tips of vanes.

Then, $v = \frac{7}{8} \times$ *Velocity of tips of vanes*

$$,, = \frac{7}{8} \times \pi d n$$

$$,, = \frac{7}{8} \times \frac{22}{7} \times \frac{30}{12} \times \frac{1000}{60} = \textbf{114·6 ft. per sec.}$$

$$\left.\begin{array}{l}\textit{Volume of air expelled} \\ \textit{per second}\end{array}\right\} = \mathrm{A}\,v$$

$$,, \quad ,, \quad = \frac{200}{144} \times 114·6 = \textbf{159·16 cub. ft. per sec.}$$

By calculation from the density of air, it can be shown that *13 cubic feet of air at atmospheric pressure weigh 1 lb. very nearly,* and assuming this, we get :—

$$\textit{Weight of } 159·16 \textit{ cub. ft.} = \mathrm{W} = \frac{159·16}{13} = \textbf{12·24 lbs. nearly.}$$

$$\text{Hence, } \textit{Work done per sec.} = \textit{Energy exerted} = \frac{\mathrm{W}\,v^2}{2\,g}$$

$$,, \quad ,, \quad = \frac{12·24 \times 114·6^2}{2 \times 32} \text{ ft.-lbs. per sec.}$$

$$,, \quad ,, \quad = \frac{12·24 \times 114·6^2 \times 60}{2 \times 32} \text{ ft.-lbs. per min.}$$

$$\therefore \quad \text{H.P. exerted} = \frac{12·24 \times 114·6^2 \times 60}{2 \times 32 \times 33{,}000} = \textbf{4·57 nearly.}$$

Lecture XXI.—Questions.

1. Define momentum, and state how it is measured. State and explain, by aid of examples, Newton's three laws of motion. A shot weighing half a ton is fired from a 100-ton gun with a velocity of 2,000 feet per second. Neglecting the weight of the powder, find the velocity of the gun's recoil. *Ans.* 10 feet per second.

2. A man weighing 140 lbs. descends in a lift with an acceleration equal to $\frac{1}{8}g$. What pressure does he exert on the floor of the lift? How would your answer differ if the lift were ascending instead of descending? *Ans.* 122·5 lbs.; 157·5 lbs.

3. A railway train, exclusive of engine, weighs 150 tons, and in starting along a level line from rest it attains a speed of 30 miles an hour in 5 minutes. What has been the mean pull between the engine and train, the resistance being taken at 10 lbs. per ton? *Ans.* 3,040 lbs.

4. A locomotive and its train weigh 220 tons, and the frictional resistance at all speeds may be taken at 2,000 lbs. If the tractive force of the engine is constantly 3,500 lbs., find in what time from starting the train can attain a speed of 40 miles per hour (1) on a level line, and (2) going down an incline of 1 in 220. Find, also, the distance travelled in both cases in attaining the above speed. *Ans.* (1) 10 minutes 2 seconds, 3·34 miles; (2) 4 minutes 1 second, 1·34 miles.

5. In a steam engine, the piston, which is 40 inches diameter and weighs 2,000 lbs., comes off the rod just as it is commencing its inward stroke. The mean steam pressure is 50 lbs. per square inch. Find the velocity with which the piston will strike the cover at the opposite end of the cylinder, the stroke being 4 feet. *Ans.* 89·7 feet per second.

6. One end of a string is fixed; it then passes over a movable pulley to which a weight, W, is attached. The string then passes over a fixed pulley, and a smaller weight, w, is attached to its other end, all three sections of the string being vertical. Show that, neglecting the weights of the pulleys, the acceleration with which W descends is $\left(\frac{W - 2w}{W + 4w}\right)g$. Verify this result (1) when w is small compared with W, and (2) when W is small compared with w. (Wool. Roy. Milly. Acad. Exams.)

7. It is very evident that a railway train requires a considerable amount of force to set it in motion, but there is a popular notion existing that a less amount of power or force is required to bring the same train to a state of rest. Will you explain clearly the natural principles upon which the whole case depends, and compare the force necessary both for giving motion to the train and in producing the opposite condition? (S. & A. Hons. Exam.)

8. Distinguish between work and energy, and between potential and kinetic energy. Give examples of both forms of energy. State the principle of the conservation of energy, and show its connection with the axiom that "perpetual motion" is impossible. A simple pendulum is pulled aside till its heavy bob is raised h inches vertically, and then let go. Find its velocity when it passes its lowest point. *Ans.* $\sqrt{\frac{gh}{6}}$.

9. Prove the formula which gives the number of units of work stored up in a given weight when moving with a given velocity. A weight of

100 lbs. is moving with a velocity of 64 feet per second, how many foot-pounds of work have been expended in producing this result? (S. & A. Adv. Exam.) *Ans.* 6,400 foot-pounds.

10. A hammer head weighs 5 tons and reaches the anvil with a velocity of 10 feet per second; what amount of energy measured in foot-pounds, is stored up in the hammer at the instant of the blow? *Ans.* 17,500 foot-pounds.

11. The head of a steam hammer weighs 10 cwts., and has a fall of 8 feet. If it indent the iron on which it falls by 1 inch, find the mean force exerted on the iron during compression. *Ans.* 970 cwts.

12. Of two steam hammers, one weighs 5 tons and reaches the anvil with a velocity of 10 feet per second, and the other weighs 10 tons and reaches the anvil at a velocity of 5 feet per second; will you compare and distinctly characterise the conditions of the blow of each of the two hammers? (S. & A. Hons. Exam.)

13. Referring to a steam hammer, in which steam is admitted above the piston to assist gravitation, will you describe the combination of forces at work in producing the blow, and, as far as you may be able, the nature of the blow as depending on velocity and mass or weight of the hammer at the moment of impact? (S. & A. Hons. Exam.)

14. What do you understand by energy, and how is it measured? The head of a steam hammer weighs 50 cwts., steam is admitted on the under side for lifting only, and there is a drop of 5 feet. What will be the average compressive force exerted during a blow from this hammer, on the supposition that the duration of the blow—that is, the time during which the hammer is compressing the iron under operation—is $\frac{1}{10}$ second? *Ans.* 2,236 cwts.

15. The monkey of a pile driver weighs 15 cwts., and the drop is 6 feet. The blow causes the pile to go down through 4 inches; what is the frictional resistance of the earth? *Ans.* 285 cwts.

16. Compare the force expended in pile driving by a ram or monkey of 1 ton falling 20 feet, with that of a weight of 2 tons falling 10 feet. If one blow of the former moves the pile 9 inches, what is the average resistance that is opposed to its motion? (S. & A. Adv. Exam., 1896.) *Ans.* (1) 2 : 1, (2) 27·7 tons.

17. Two bodies, weighing 5 lbs. and 3 lbs. respectively, are connected by a perfectly flexible weightless string which passes over a smooth pulley. The heavier body draws up the lighter. When it has fallen through 5 feet, what is the kinetic energy of the bodies and the velocity? ($g = 32.$) Determine also the acceleration of the system, and the tension in the string. *Ans.* (1) 10 foot-pounds; (2) 8·94 feet per second; (3) 8 feet per second per second; (4) 3·75 lbs.

18. State Newton's third law of motion, and give his illustrations of it. Weights of 5 and 11 lbs. are connected by a weightless thread. The latter is placed on a smooth horizontal table, while the former hangs over the edge. If the bodies are then allowed to move under the action of gravity, what is the tension of the thread? Find, also, the acceleration produced, and the kinetic energy of the system at the end of 4 seconds. *Ans.* (1) 3·44 lbs., (2) 10 feet per second per second, (3) 400 foot-pounds.

19. A train of locomotive and carriages weighs 60 tons. If it be supposed to run down an incline of 1 in 265 for $7\frac{1}{2}$ miles, starting with zero velocity, unopposed by anything but its own inertia, and unaccelerated by anything but its own weight; what would be its velocity, its momentum, and its kinetic energy at the foot of the incline? *Ans.* 97·84 feet per second, 5,870 foot-tons per second, 8,925 foot-tons.

20. What meaning do you attach to the phrase *horse-power?* A fire-engine pump is provided with a nozzle, the sectional area of which is 1 square inch, and the water is projected through the nozzle with a velocity of 130 feet per second; find the horse-power of the engine required to drive the pump, irrespective of the loss by resistance of the working parts. The weight of a cubic foot of water is $62\frac{1}{4}$ lbs. (S. & A. Hons. Exam.) *Ans.* 27·1 H.P.

21. State Newton's second law of motion. Explain briefly how the measure of force is derived from this law. In the equation $P = mf$, in what units is P, when the units of mass, distance, and time are a pound, a foot, and a second? (S. & A. Adv. Theor. Mechs. Exam., 1896.)

22. A steam engine is employed to raise coals, and it is calculated that in order to set in motion the winding drums, flywheel, cages, ropes, &c., which are concerned in the motion, it has to do the work of imparting a linear velocity of 36 feet per second, to 60 tons of material in *half-a-minute* at each lift. What effective horse-power is consumed in overcoming the inertia of the aggregate weight of 60 tons, and in setting up the velocity, estimated as above stated, in the time assigned? (S. & A. Hons. Exam.) *Ans.* 165 H.P.

23. In lifting water into a tender by a scoop running along a trough while the train is going at rapid speed, the height of the lift is $7\frac{1}{2}$ feet. What speed in miles per hour will just cause the water to be lifted through that height? (S. & A. Hons. Exam.) *Ans.* 15 miles per hour nearly.

24. A vertical pipe, carried by the tender of a locomotive engine, and terminating in a scoop with a flat mouth, picks up water from a trough laid on a railway. If the speed of the engine and tender be 22 miles per hour, find the height to which the water will rise in the pipe. Upon what theory do you proceed? *Ans.* 16·3 feet.

25. A hammer head of $2\frac{1}{2}$ lbs., moving with a velocity of 50 feet per second, is stopped in 0·001 second. What is the average force of the blow? What do you mean by this average? What is the difference between a time average and a space average? When are they the same? (S. & A. Adv. Exam., 1897.)

26 A body of 4 lbs. moving with a speed of 20 feet per second overtakes one of 200 lbs. moving at 2 feet per second in the same direction. When the collision events are finished (friction stilling the relative motions) and both bodies go on together, what is their common velocity? What mechanical energy has been lost? (S. & A. Hons. Exam., Part I., 1898.)

27. A body of 200 lbs. is acted on by a force F, which alters. No other force acts on the body. When the body has passed through the distance x feet, the force in pounds is as follows:—

x	0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1·0
F	20	21	21	20	19	18·5	18·0	13·5	9	4·5	0

Using either a graphical or arithmetical method, find—

(*a*) The average force acting on the body through this total distance of 1 foot.

(*b*) The work done upon the body from $x = 0$ to $x = ·4$.

(*c*) The answer to (*b*) being the kinetic energy added to the body; if the velocity was 0 when x is 0, what is the velocity when $x = ·4$? (S. & A. Adv. Exam., 1898.)

28. Prove the formula for the kinetic energy of a moving body. A ball weighing 1 lb. moving at 1,200 feet per second passes through a plate of iron in 0·002 second, and its velocity is reduced to 200 feet per second. Find the work done in passing through the plate, and the average force during the time of its passage. (S. and A. Adv., 1899.)

29. A machine is found to have 300,000 foot-pounds stored in it as kinetic energy when its main shaft makes 100 revolutions per minute. If the speed changes to 98 revolutions per minute, how much kinetic energy has it lost? A similar machine (that is, made to the same drawings but on a different scale) is made of the same material but with all its dimensions 20 per cent. greater. What will be its store of energy at 70 revolutions per minute? What energy will it store in changing from 70 to 71 revolutions per minute? (B. of E. Adv. & H., Part I., 1901.)

30. A body weighing 1,610 lbs. is lifted vertically by a rope, there being a damped spring balance to indicate the pulling force, F lbs., of the rope. When the body has been lifted x feet from its position of rest, the pulling force was automatically recorded as follows:—

x	0	11	20	34	45	55	66	76
F	4,010	3,915	3,763	3,532	3,366	3,208	3,100	3,007

(using squared paper). Find approximately the work done on the body when it has risen 70 feet. How much of this is stored as potential energy, and how much as kinetic energy? What is then the velocity of the body? Find the velocity, v, feet per second for values of x of 10, 30, 50, 70, and draw a curve showing the probable values of v for all values of x up to 80. In what time does the body get from $x = 45$ to $x = 55$?

(B. of E. Adv. & H., Part I., 1901.)

31. An electric tramcar, loaded with 52 passengers, weighs altogether 10 tons. On a level road it is travelling at a certain speed. For the purpose of finding the tractive force, the electricity is suddenly turned off, and an instrument shows that there is a retardation in speed: how much will this be if the tractive force was 315 lbs.? If the tractive force is found on several trials to be on the average—

342 lbs. when the speed is 12 miles per hour,
315 ,, ,, ,, 10 ,, ,,
294 ,, ,, ,, 8 ,, ,,

what is the probable tractive force at 9 miles per hour?

(B. of E. Adv. & H , Part I., 1901.)

32. A car weighs 10 tons, what is its mass in engineers' units? It is drawn by the pull, P lbs., varying in the following way, t being seconds from the time of starting:—

P	1,020	980	882	720	702	650	713	722	805
t	0	2	5	8	10	13	16	19	22

The retarding force of friction is constant, and equal to 410 lbs. Plot

$P - 410$ and the time, t, and find the *time average* of this excess force. What does this represent when it is multiplied by 22 seconds? What is the speed of the car at the time 22 seconds from rest? Tabulate values of speed and time, and draw a curve showing speed and time.

(B. of E. Adv. & H., Part I., 1902.)

LECTURE XXI.—A.M.INST.C.E. EXAM. QUESTIONS.

1. What is meant by the conservation of momentum? A fireman holds a round nozzle from which a jet of water $\frac{1}{2}$ inch in diameter is projected with a velocity sufficient to carry it to a height of 100 feet. Find the force in lbs. which he has to exert in holding the nozzle. (I.C.E., *Oct.* 1897.)

2. If a man coasting on a bicycle down a uniform slope of one in fifty attains a limiting speed of 8 miles per hour, what horse-power must he exert to drive his machine up the same hill at the same speed, there being no wind in either case? The weight of man and bicycle together is 200 lbs. (I.C.E., *Oct.*, 1897.)

3. Apply the principle of the conservation of energy to find the velocity of a thin hollow circular cylinder after rolling a distance of 12 feet down a plane inclined at a slope of 1 vertical in 5 horizontal. (I.C.E., *Feb.*, 1898.)

4. Assuming that a train may be accelerated by the application of a force equal to one-fortieth of its gross weight and be braked with a force equal to one-tenth of its gross weight, find the least time in which it may be run from one to another of two stopping stations 5,000 feet apart. What is its greatest speed during the run? (I.C.E., *Feb.*, 1898.)

5. A hammer-head of $2\frac{3}{4}$ lbs., moving with a velocity of 75 feet per second, is stopped in 0·0007 second. What is the average force of the blow? (I.C.E., *Oct.*, 1898.)

6. Define velocity, acceleration, momentum, mass or inertia, force, impulse, rate of change of momentum per second, kinetic energy. In every case, when you make a statement relating to linear or translational motion, make the analogous statement about angular motion.
(I.C.E., *Oct.*, 1898.)

7. A body of 30 lbs. moves towards the south at 30 feet per second, in 2 minutes it moves towards the south-west at 40 feet per second—what is the added velocity? Find the average acceleration. What constant force would produce the change? (I.C.E., *Oct.*, 1898.)

8 The pull, F, in lbs. on a tramcar was registered when the car was at the following distances x from a certain point. Use squared paper to find

x	0	12	20	34	50	62
F	510	480	460	450	470	490

the average pull and the work done. If the distance is passed over in 21 seconds, find the average horse-power. (I.C.E., *Oct.*, 1898.)

9. Taking the resistance, F lb., of a bicycle on a level road to be given by $F=W\left\{0·002 + 0·00012\,(v + w)^2\right\}$, where W is the weight, in lbs., of rider and machine, and v is the speed in miles per hour, w being the speed of an opposing wind; calculate what H.P. is expended in going up an incline of 1 in 80 at a speed of 10 miles an hour, the weight of the rider and machine being 180 lbs., the helping wind being at 5 miles an hour. What would this amount to going down the same slope at double the speed, the wind now opposing the rider? (I.C.E., *Feb.*, 1899.)

10. In a steam engine the piston at the beginning of its stroke is exposed to a total pressure of 2,000 lbs., but the inertia is such that the thrust of the piston-rod at the crosshead is only 1,600 lbs. The speed of the engine

is now raised until it becomes half as great again as before, while the steam pressure is unchanged, what is the thrust of the piston-rod? (I.C.E., *Feb.*, 1899.)

11. State Newton's second law of motion in its most useful form. Suppose that a Maxim gun delivers 250 1-oz. bullets per minute with a speed of 1,500 feet per second, what average force in lbs. weight must be provided to hold the gun still? (I.C.E., *Feb.*, 1899.)

12. A vehicle weighing 4 tons is proceeding at a rate of 10 miles an hour along a level road; the pull on it is suddenly stopped; supposing the whole resistance equivalent to 500 lbs. applied to the rim of one of the wheels 4 feet diameter, calculate how far the vehicle will run before stopping. (I.C.E., *Oct*, 1899.)

13. An engine of 10 B.H.P. drives the axle of a vehicle at one-fifth the angular velocity of its own shaft; the driving wheels are 4 feet diameter, and the speed of the vehicle is 8 miles per hour, find the equivalent resultant pull on the driving axle. (I.C.E., *Oct.*, 1899.)

14. Give Newton's second law of motion. Calculate the force necessary to increase the velocity of a body weighing 100 lbs. by 10 feet per second in 4 seconds, supposing the acceleration uniform. (I.C.E., *Feb.*, 1900.)

15. A mass whose weight is 50 lbs., moving at the rate of 15 feet per second, is acted upon for 20 seconds by a force of 20 lbs.; find the distance moved during the time. (I.C.E., *Feb.*, 1900.)

16. A bicycle and rider, weighing together 180 lbs., are travelling at the rate of 10 miles per hour on the level. Supposing a brake is applied to the top of the front wheel 30 inches in diameter, and that this is the only resistance acting, how far will the bicycle travel before stopping if the pressure of the brake is 20 lbs., and the coefficient of friction 0·5? In what respect is such a bicycle brake more efficient than a brake on a vehicle with springs? (I.C.E., *Feb.*, 1900.)

17. Explain the chief units adopted in the measurement of acceleration, force, energy, and power. If a bicyclist always works at $\frac{1}{10}$ H.P. and goes 12 miles per hour on the level, find the resistance of the road, and show that, if the mass of the machine and the rider together be 12 stone, the speed on an incline of 1 in 50 will be reduced to about 5·8 miles per hour. (I.C.E., *Oct.*, 1900.)

18. A man pushes an 8-ton railway truck from rest with a uniform force of $\frac{1}{2}$ cwt. on a smooth horizontal line of rails. Neglecting all resistances, find what speed he will get up in 22 seconds. At what H.P. will he be working at the end of that time? (I.C.E., *Feb.*, 1901.)

19. A 1-oz. bullet fired horizontally, with velocity 1,000 feet, into a 1-lb. block of wood, resting on a smooth table, penetrates 2 inches and remains imbedded. With what velocity does the block move off (without rotation)? Why would the bullet have penetrated more if the block had been fixed? (I.C.E., *Feb.*, 1901.)

20. A man weighing 140 lbs. stands on the floor of a lift. Find the pressure he exerts on the floor (*a*) when the lift ascends and descends with uniform velocity, (*b*) when it ascends with a velocity which decreases by the acceleration of $\frac{1}{10}g$, (*c*) when it descends with a velocity which increases at the rate of 8 feet per second per second. Under what conditions can the pressure be (i.) zero, (ii.) greater than the weight of the man? (I.C.E., *Oct.*, 1901.)

21. A cannon weighs 35 tons and the shot 1,200 lbs. The velocity of the shot on leaving the muzzle is 1,200 feet per second, find the velocity of the recoil of the cannon. If the velocity of recoil is to be destroyed while the gun moves through 3 feet, find the average resistance to be applied. (I.C.E., *Oct.*, 1901.)

22. Distinguish between kinetic and potential energy and show how they are measured in the case of a body falling from rest. Show that if a body be let fall from rest its energy remains constant till it reaches the ground. (I.C.E., *Oct.*, 1901.)

23. Define "impulse" and "momentum." A force acting on a mass of 10 lbs. increases its velocity in every second by 7 feet a second. Another force acting on a body of mass 25 lbs. increases its velocity in 2½ minutes from 500 feet per second to 1,850 feet per second. Compare the forces. (I.C.E., *Feb.*, 1902.)

24. State exactly what is meant by "a pound mass," "a pound weight," "a poundal," and by the terms "work" and "energy." Show that if a body of mass m has a velocity v imparted to it by the action of a constant force F acting through a distance S, then the work done by the force is $\frac{1}{2} m v^2$. (I.C.E., *Feb.*, 1902.)

25. Two bodies, P and Q, of unequal mass, are connected by a fine string passing over a frictionless pulley. Find expressions for the acceleration of the bodies, and for the tension in the string. (I.C.E., *Feb.*, 1902.)

26. The resistance of a passenger train on the level road is 17·3 lbs. per ton, the speed being 48 miles per hour. If the total weight of engine and train is 190 tons, find the horse-power of the engine. If the train is brought to a standstill by the application of the brakes in 14½ seconds, find the average resistance of the brakes. (I.C.E., *Feb.*, 1902.)

27. A train increases its speed from 40 miles to 49 miles an hour while descending an incline in 4½ minutes. Find its average acceleration. Find also the slope of the incline, taking the acceleration due to gravity to be 32 foot-second units. (I.C.E., *Feb.*, 1902.)

28. At what horse-power must a bicyclist work when riding at 20 miles per hour on a track, the resistance of which is 1 per cent. of the total weight (180 lbs.)? If he goes 60 yards from rest before getting speed up, find the mean moment he must exert during that time. Diameter of wheel 28 inches, and geared so as to make 9 revolutions for 4 turns of the crank. (I.C.E., *Oct.*, 1902.)

29. Explain the term "kinetic energy," and show that if the motion of a body be retarded by a resistance, the decrease of kinetic energy is equal to the work done against that resistance. At what distance from a given point must a carriage be detached from a train going at 20 miles an hour in order that it may come to rest at the given point, the brake being applied and the coefficient of friction being ⅛? (I.C.E., *Oct.*, 1902.)

30. Distinguish between the measurements of force and impulse. The head of a steam hammer weighs 50 cwts.; steam is admitted on the under side for lifting only, and there is a drop of 5 feet. What will be the velocity and momentum of the head the instant before the blow is given, if there is no resistance to the fall. If the time during which the compression of the iron takes place be $\frac{1}{50}$ second, find the average force of the blow. (I.C.E., *Oct.*, 1902.)

31. Explain the principle of conservation of energy. Two cylindrical tanks, A and B, of, respectively, 4 square yards and 2 square yards horizontal cross section, stand on the same floor and are connected near the bottom by a narrow pipe. A at first contains 8 cubic yards of water, and B is empty. The water flows slowly into B; find the amount of heat which will have been generated when the water has ceased to flow and it has all come to rest. (I.C.E., *Feb.*, 1903.)

32. Two men put a railway wagon weighing 5 tons into motion by exerting on it a force of 80 lbs. The resistance of the wagon is 10 lbs. per ton, or altogether 50 lbs.; how far will the wagon have moved in 1 minute?

Calculate at what fraction of a horse-power the men are working at 60 seconds after starting. (I.C.E., *Feb.*, 1903.)

33. State and explain fully Newton's Third Law of Motion. A 100-lb. shot leaves a gun horizontally with a muzzle velocity of 2,000 feet per second. The gun and attachments, which recoil, weigh 4 t ns. Find what the resistance must be that the recoil may be taken up in 4 feet, and compare the energy of recoil with the energy of translation of the shot.

(I.C.E., *Feb.*, 1903.)

LECTURE XXII.

CONTENTS.—Energy of a Rotating Body—Moment of Inertia of a Body about an Axis—Definitions of Moment of Inertia and Radius of Gyration—Propositions I., II., and III.—Methods of Calculating Moments of Inertia—Examples I., II., and III.—Tables of Radii of Gyration of Solids and Sections—Equation of Energy for a Rotating Body—Examples IV., V., VI., and VII.—Determination of Energy of Flywheels—Centripetal and Centrifugal Forces—Definitions of Centripetal and Centrifugal Forces—Example VIII.—Straining Actions due to Centrifugal Forces—Example IX.—Questions.

Energy of a Rotating Body.—The deduction of the equation for energy of rotation is complicated by the fact that particles of the body at different distances from the axis of rotation possess different energies, due to their different linear velocities. To obtain the energy of the whole body, we must, therefore, take the sum of the energies of the various particles composing it. In general, this process must be performed by the aid of higher mathematics; and even then, only in those cases in which the bodies are of regular geometrical form.

Moment of Inertia of a Body about an Axis.—Before deducing the expression for the kinetic energy of a rotating body, it may be as well to explain certain terms and quantities which we shall have occasion to make use of.

DEFINITION.—**If the mass of every particle of a body be multiplied by the square of its distance from a given axis, the sum of the products is called the Moment of Inertia of the body about that axis.**

Let I = Moment of inertia of the body about a given axis.
" m = Mass of any particle or element of body.
" r = Distance of m from the given axis.

Then, $$I = \Sigma m r^2. \quad \text{. . . . . (I)}$$

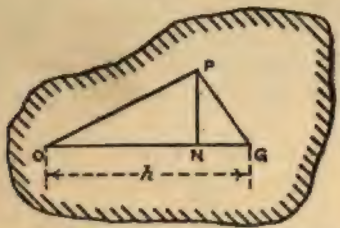
DEFINITION.—**If M be the mass of a body, and k be such a quantity that M k^2 is its Moment of Inertia about a given axis, then k is called the Radius of Gyration of the body about that axis.**

Thus, $$\left.\begin{aligned} \mathrm{M}k^2 &= \mathrm{I} \\ k^2 &= \frac{\mathrm{I}}{\mathrm{M}} \end{aligned}\right\} \quad \text{. . . . . . . (II)}$$
Or,

The following PROPOSITIONS relating to moments of inertia and radii of gyration are so important that we here give their proofs in full:—

PROPOSITION I.—**If M be the mass of any body, I_G the moment of inertia about any axis through its centre of gravity, G, and I_O, that about a parallel axis through any other point, O, at a distance, h, from G, then:—**

$$I_O = I_G + M h^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (III)$$



MOMENTS OF INERTIA ABOUT PARALLEL AXES.

Let G, and O, be the points of intersection of the axes with the plane of the paper, which is at right angles to them. Let P be any particle of mass, m. Draw P N perpendicular to O G.

Then, in triangle O P G, we get (Euc. II., 12):—

$$O P^2 = P G^2 + O G^2 - 2 O G \cdot G N.$$

Multiplying both sides by m the mass of particle at P, we get:—

$$m \cdot O P^2 = m \cdot P G^2 + m \cdot O G^2 - 2 m \cdot O G \cdot G N.$$

Repeating this process for every other particle of the body, and adding the results, we have:—

$$\Sigma m \cdot O P^2 = \Sigma m \cdot P G^2 + \Sigma m \cdot O G^2 - 2 \Sigma m \cdot O G \cdot G N.$$

But, clearly, $\quad \Sigma m \cdot O P^2 = I_O$, and $\Sigma m \cdot P G = I_G$.

And, since $\quad O G = h = \text{constant},$

$\therefore \quad \Sigma m \cdot O G^2 = O G^2 \Sigma m = h^2 M$, or $M h^2$.

Also, $\quad 2\Sigma m \cdot O G \cdot G N = 2 O G \cdot \Sigma m \cdot G N = 2 h \Sigma m \cdot G N.$

But the quantity, $\Sigma m \cdot G N$, is the sum of the moments of the various particles about their centre of gravity, G, and is therefore zero, from the definition of the centre of gravity.

$$\therefore \qquad 2\Sigma m \cdot \mathrm{OG} \cdot \mathrm{GN} = 2h\Sigma m \cdot \mathrm{GN} = 0.$$

$$\therefore \qquad \mathrm{I_O} = \mathrm{I_G} + \mathrm{M}h^2.$$

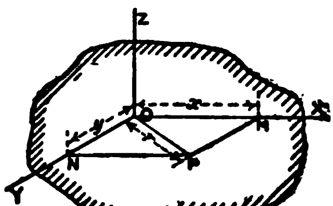
COROLLARY.—Denoting the radii of gyration of the body about the axes, O, and G, by k_O, and k_G, respectively, we get:—

$$\mathrm{M}k_O^2 = \mathrm{M}k_G^2 + \mathrm{M}h^2.$$

$$\therefore \qquad k_O^2 = k_G^2 + h^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IV)$$

PROPOSITION II.—If I_x and I_y respectively denote the moments of inertia of a lamina, or plane area, about two axes 0 X, 0 Y, at right angles, lying in the plane of the lamina or area, and I_z, the moment of inertia about an axis, 0 Z, through 0, perpendicular to the plane of the lamina or area; then I_z is equal to the sum of I_x, and I_y.

i.e.,
$$I_z = I_x + I_y. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (V)$$



MOMENT OF INERTIA OF LAMINA ABOUT RECTANGULAR AXES.

Take any particle, P, of mass, m, and draw P M, and P N, perpendicular to O X and O Y respectively.

Let x and y denote the co-ordinates of P, with respect to the axes, O X, O Y, so that O M $= x$, O N $= y$, and O P $= r$. Then:—

Moment of inertia of P about O X $= m \cdot \mathrm{PM}^2 = m \cdot y^2.$

" " O Y $= m \cdot \mathrm{PN}^2 = m \cdot x^2.$

" " O Z $= m \cdot \mathrm{OP}^2 = m \cdot r^2.$

But, $$r^2 = y^2 + x^2,$$

$$\therefore \qquad m \cdot r^2 = m \cdot y^2 + m \cdot x^2.$$

Hence, the moment of inertia of P, about the axis, O Z, is equal to the sum of the moments of inertia of the same particle about the axes, O X, and O Y. But this is equally true for every other particle.

$$\therefore \qquad \Sigma m r^2 = \Sigma m y^2 + \Sigma m x^2.$$

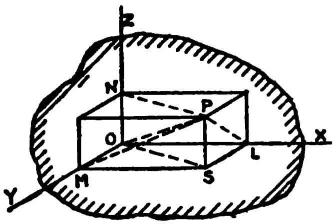
$$i.e., \qquad I_z = I_x + I_y.$$

Corollary.—Denoting the radii of gyration of the lamina, about the axes by the letters k_x, k_y, k_z, we get :—

$$k_z^2 = k_x^2 + k_y^2. \quad \ldots \ldots \quad \text{(VI)}$$

Proposition III.—If I_x, I_y, I_z respectively denote the moments of inertia of any body about three rectangular axes drawn from any point, O, in the body, then the sum, I_x, + I_y, + I_z, is equal to twice the moment of inertia, I_o, of the body about the point, O.

$$i.e., \qquad I_x + I_y + I_z = 2\, I_o \quad \ldots \ldots \quad \text{(VII)}$$



Moment of Inertia of Body about Rectangular Axes.

Let O X, O Y, O Z be the three rectangular axes drawn from any point, O; P, any particle of mass m, whose co-ordinates are x, y, z, so that O L = x, O M = y, O N = z, and O P = r. Then :—

Moment of inertia of P about $\Big\}$ $O X = m \cdot P L^2 = m (y^2 + z^2).$

„ „ $O Y = m \cdot P M^2 = m (z^2 + x^2).$

„ „ $O Z = m \cdot P N^2 = m (x^2 + y^2).$

$$\therefore \qquad I_x = \Sigma m (y^2 + z^2) = \Sigma m y^2 + \Sigma m z^2,$$

$$I_y = \Sigma m (z^2 + x^2) = \Sigma m z^2 + \Sigma m x^2,$$

$$\text{And,} \qquad I_z = \Sigma m (x^2 + y^2) = \Sigma m x^2 + \Sigma m y^2.$$

$$\therefore \qquad I_x + I_y + I_z = 2 \left\{\Sigma m x^2 + \Sigma m y^2 + \Sigma m z^2\right\}$$

$$\text{„} \quad \text{„} \quad = 2 \Sigma m (x^2 + y^2 + z^2),$$

$$i.e., \qquad I_x + I_y + I_z = 2 \Sigma m r^2 = 2\, I_o.$$

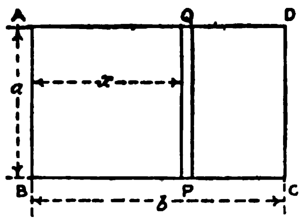
Corollary.—Denoting the radii of gyration of the body about the three axes and the point, O, by the letters k_x, k_y, k_z and k_o respectively, we get :—

$$k_x^2 + k_y^2 + k_z^2 = 2\,k_o^2 . \quad . \quad . \quad . \quad . \qquad \text{(VIII)}$$

Proposition II. is a particular case of this general one. There O P and O S will always coincide, and, therefore, I_z and I_o are identical. By putting $I_o = I_z$ in equation (VII), we at once get equation (V).

Methods of Calculating Moments of Inertia.—We shall show by working out a few examples how the moments of inertia or radii of gyration can be calculated in certain cases, wherein the density is uniform.

Example I.—Determine the moment of inertia and radius of gyration of a rectangular lamina (1) about its shorter edge, (2) about an axis in its plane through its *c.g.* and parallel to a short edge, and (3) about an axis through its *c.g.* perpendicular to its plane.



Moment of Inertia of Rectangle about AB.

Answer.—Let A B C D be the rectangular lamina, and let the edge A B = a, and B C = b.

(1) *About the shorter edge* A B.

Divide the rectangle into n, equal and narrow strips, P Q, parallel to the axis A B.

Let M = Mass of whole figure, A B C D.
 „ m = Mass of elementary rectangle, P Q.
 „ x = Distance of P Q from axis A B.
 „ h = Breadth of elementary strip P Q $= \dfrac{b}{n}$.

The whole of the strip P Q is at the same distance from A B,

$$\therefore \quad \left.\begin{array}{c}\textit{Mom. of inertia of}\\ \textit{element}\ \mathrm{P\,Q}\end{array}\right\} = m\,x^2.$$

$$\therefore \quad \left.\begin{array}{c}\textit{Mom. of inertia of}\\ \textit{whole figure}\end{array}\right\} = \Sigma\, m\,x^2.$$

But $\qquad m : \mathrm{M} = h : b.$

$$\therefore \quad m = \frac{\mathrm{M}\,h}{b}.$$

$$\therefore \quad \mathrm{I} = \frac{\mathrm{M}}{b}\Sigma\, h\,x^2. \quad . \quad . \quad . \quad . \qquad (1)$$

Beginning at edge A B, the distances of the various strips from this edge will be $x_0 = 0$, $x_1 = h$, $x_2 = 2h, \ldots x_n = nh$.

$$\therefore \quad \Sigma h x^2 = h(0^2 + h^2 + 2^2 h^2 + 3^2 h^2 + \ldots + n^2 h^2),$$

$$\text{,,} \quad = h^3 (1^2 + 2^2 + 3^2 + \ldots + n^2),$$

$$\text{,,} \quad = h^3 \frac{n(n+1)(2n+1)}{6}, \text{ [See Treatises on Algebra]}$$

$$\text{,,} \quad = \frac{(nh)^3}{6}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right).$$

When the number of strips, n, is infinitely large, the reciprocal, $\frac{1}{n}$, will be infinitely small, and may, therefore, be neglected.

Also, $\quad (nh)^3 = b^3.$

$$\therefore \quad \Sigma h x^2 = \frac{b^3}{6} \times 2 = \tfrac{1}{3} b^3,$$

$\therefore$ From eqn. (1), $\quad \mathrm{I} = \tfrac{1}{3} \mathrm{M} b^2.$

Let k = Radius of gyration, then :—

$$\mathrm{M} k^2 = \mathrm{I},$$

$$\therefore \quad k^2 = \tfrac{1}{3} b^2,$$

Or, $$k = \frac{b}{\sqrt{3}}.$$

[The above method of finding the moment of inertia is precisely the same as that employed in higher mathematics. For those who understand the calculus we here repeat the above calculation, using its notation.

Let dx = Breadth of elementary strip, P Q.

Then, $\quad m = \frac{\mathrm{M}}{b} dx.$

$$\therefore \quad d\mathrm{I} = \frac{\mathrm{M}}{b} x^2 dx.$$

$$\therefore \quad \mathrm{I} = \frac{\mathrm{M}}{b} \int_0^b x^2 dx = \frac{\mathrm{M}}{b}\left[\frac{x^3}{3}\right]_0^b = \frac{\mathrm{M}}{b} \cdot \frac{b^3}{3} = \tfrac{1}{3} \mathrm{M} b^2.]$$

(2) *About an axis through c.g. parallel to edge* A B.—We may obtain the moment of inertia in this case by proceeding in

exactly the same way as before, but there is no need for this repetition, as we can very easily get the result from the relation given in PROPOSITION I.

Let I_G = Moment of inertia of the rectangle about an axis through its *c.g.* parallel to the edge A B.
„ I = Moment of inertia about the edge A B = $\frac{1}{3} M b^2$.
„ h = Distance between these axes = $\frac{1}{2} b$.

Then, from equation (III) :—

$$I = I_G + M h^2.$$

$$\therefore \quad I_G = I - M h^2 = \tfrac{1}{3} M b^2 - \tfrac{1}{4} M b^2 = \tfrac{1}{12} M b^2.$$

Also, $$k_G^2 = \tfrac{1}{12} b^2.$$

(3) *About an axis through c.g. perpendicular to plane.*

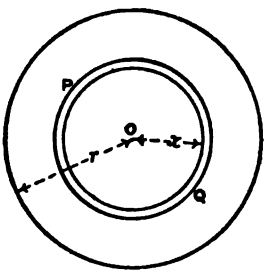
$$\left.\begin{array}{l}\textit{Moment of inertia about an axis}\\ \quad\textit{through c.g. parallel to } \mathrm{A\,B}\end{array}\right\} = I_x = \tfrac{1}{12} M b^2.$$

$$\left.\begin{array}{l}\textit{Moment of inertia about an axis}\\ \quad\textit{through c.g. parallel to } \mathrm{B\,C}\end{array}\right\} = I_y = \tfrac{1}{12} M a^2$$

But, from equation (V), $$I_z = I_x + I_y.$$

$$\therefore \left.\begin{array}{l}\text{Moment of inertia about an axis}\\ \quad\text{through } c.g. \text{ perpendicular to}\\ \quad\text{the plane}\end{array}\right\} = I_z = \tfrac{1}{12} M (a^2 + b^2).$$

EXAMPLE II.—Determine the radius of gyration of a circular disc about an axis through its centre perpendicular to its plane.



RADIUS OF GYRATION OF A CIRCULAR DISC.

ANSWER.—Divide the disc into n, equal, narrow rings of breadth, h. Taking one of these rings, P Q, let x be its distance from the centre, O.

Let M = Mass of the disc.
„ m = Mass of the elementary ring, P Q.
„ r = Radius of the disc.

Then, $\quad m : M$ = *area of ring* : *area of disc.*

i.e., $\quad m : M = 2 \pi x h : \pi r^2.$

$$\therefore \quad m = \frac{2 M}{r^2} x h.$$

The whole of the elementary ring is at the same distance from the centre :—

$$\therefore \quad \left.\begin{array}{l}\textit{Moment of inertia of elementary} \\ \textit{ring about the axis through O}\end{array}\right\} = m\,x^2 = \frac{2\,M}{r^2}\,x^3\,h.$$

$$\therefore \quad \left.\begin{array}{l}\textit{Moment of inertia of the whole} \\ \textit{disc}\end{array}\right\} = I = \frac{2\,M}{r^2}\,\Sigma\,x^3\,h. \qquad (1)$$

Beginning at the centre, O, the distances of the various rings will be $x_o = 0,\ x_1 = h,\ x_2 = 2h, \ldots x_n = nh.$

$$\therefore \qquad \Sigma\,x^3\,h = (0^3 + 1^3\,h^3 + 2^3\,h^3 + \ldots + n^3\,h^3)\,h$$

$$,, \qquad = (1^3 + 2^3 + 3^3 + \ldots + n^3)\,h^4$$

$$,, \qquad = \tfrac{1}{4}\,n^2\,(n+1)^2\,h^4 \qquad \text{[See Treatises on Algebra]}$$

$$,, \qquad = \frac{(n\,h)^4}{4} \cdot \left(1 + \frac{1}{n}\right)^2.$$

When n is infinitely great, the reciprocal, $\frac{1}{n}$, will be infinitely small, and may be neglected.

Also, $$(n\,h)^4 = r^4.$$

$$\therefore \quad \text{From equation (1),} \quad I = \frac{2\,M}{r^2} \times \frac{r^4}{4} = \tfrac{1}{2}\,M\,r^2.$$

$$\therefore \qquad k^2 = \tfrac{1}{2}\,r^2. \quad \ldots \ldots \ldots \quad (2)$$

[These results may also be obtained by the aid of the calculus, thus :—

Let $d\,x$ = Breadth of elementary ring.

Then, $$m = \frac{2\,M}{r^2}\,x\,d\,x.$$

$$\therefore \qquad d\,I = \frac{2\,M}{r^2}\,x^3\,d\,x.$$

$$\therefore \qquad I = \frac{2\,M}{r^2}\int_0^r x^3\,d\,x = \frac{2\,M}{r^2} \cdot \frac{r^4}{4} = \tfrac{1}{2}\,M\,r^2.$$

If, however, the disc be annular, the outside and inside radii being R and r respectively, we get :—

$$m = \frac{2\,M}{R^2 - r^2} \cdot x\,d\,x.$$

And, $\quad \mathrm{I} = \dfrac{2\,\mathrm{M}}{\mathrm{R}^2 - r^2}\displaystyle\int_r^{\mathrm{R}} x^3\,dx$

$$,, = \frac{2\,\mathrm{M}}{\mathrm{R}^2 - r^2} \times \frac{\mathrm{R}^4 - r^4}{4} = \tfrac{1}{2}\,\mathrm{M}\,(\mathrm{R}^2 + r^2).$$

$\therefore \quad k^2 = \frac{1}{2}(\mathrm{R}^2 + r^2).]$

These results also express the moments of inertia and radii of gyration of a solid and of a hollow cylinder about their axes. For a cylinder can be conceived as made up of a great number of circular discs threaded together on the same axis, and the moment of inertia will just be the sum of the moments of inertia of all the discs, since the radius of gyration of each disc is independent of the thickness of the disc, it follows that the radius of gyration of the whole cylinder will be the same as that of one of the discs.

Having found the radius of gyration of a circular disc about an axis through its centre at right angles to its plane, we can very easily find its radius of gyration about a diameter.

Let k_x, k_y = Radii of gyration of disc about two diameters at right angles to each other.

,, $\quad k_z$ = Radius of gyration about axis through centre and perpendicular to plane.

Then, $\quad k_x = k_y = k$

And, from (2) $\quad k_z^2 = \frac{1}{2}r^2.$

But, from equation (VI), PROPOSITION II., we get:—

$$k_x^2 + k_y^2 = k_z^2.$$

$$\therefore \quad 2\,k^2 = k_z^2 = \tfrac{1}{2}r^2.$$

$$\therefore \quad k^2 = \tfrac{1}{4}r^2, \text{ or } k = \frac{r}{2}.$$

If the disc be annular and of radii R and r, then the radius of gyration about any diameter, is given by the equation :—

$$k^2 = \tfrac{1}{4}(\mathrm{R}^2 + r^2).$$

EXAMPLE III.—Determine the radius of gyration of a sphere about a diameter.

ANSWER.—The results of PROPOSITION III. tell us that if three mutually perpendicular axes be drawn from any point

in a body, the sum of the moments of inertia of the body about these axes is equal to twice the moment of inertia of the body about that point. Suppose, then, that the point selected be the centre of the sphere, the axes will then be three mutually perpendicular diameters. But the moments of inertia about all diameters must be the same. Therefore, if I denote the moment of inertia of the sphere about any diameter and I_0 that about the centre, O, we get, from equation (VII):—

$$3\,I = 2\,I_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It only remains now to find the value of I_0 or $\Sigma\, m\, x^2$.

Suppose the sphere divided into a large number n, of concentric shells, the thickness of each shell being h.

Let x = Distance of any one shell from centre of sphere.
„ r = Radius of sphere.
„ m = Mass of shell.
„ M = Mass of sphere.

Then, $\quad m : M =$ vol. of shell : vol. of sphere,

i.e., $\quad m : M = 4\,\pi\, x^2 h : \frac{4}{3}\pi\, r^3,$

$$\therefore \quad m = \frac{3\,M}{r^3}\, x^2\, h,$$

And, $\quad I_0 = \Sigma\, m\, x^2 = \frac{3\,M}{r^3}\,\Sigma\, x^4\, h.$

Beginning at the centre of the sphere and putting successively, $x_0 = 0,\ x_1 = h,\ x_2 = 2\,h, \ldots x_n = n\,h$, we get:—

$$\Sigma\, x^4\, h = (1^4 + 2^4 + 3^4 + \ldots n^4)h^5,$$

$$\text{„} \quad = \left\{\frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}\right\} h^5,$$

$$\text{„} \quad = \left\{\frac{1}{5} + \frac{1}{2\,n} + \frac{1}{3\,n^2} - \frac{1}{30\,n^4}\right\} n^5\, h^5.$$

When n is infinitely great, the quantity inside the brackets reduces to $\frac{1}{5}$, and

$$n^5\, h^5 = r^5.$$

$$\therefore \quad \Sigma\, x^4\, h = \tfrac{1}{5}\, r^5.$$

$$\therefore \quad I_0 = \frac{3\,M}{r^3} \times \tfrac{1}{5}\, r^5 = \tfrac{3}{5}\, M\, r^2.$$

$\therefore$ From equation (I) $\mathrm{I} = \frac{2}{5}\mathrm{I}_0 = \frac{2}{5}\mathrm{M}r^2$.

Hence, $k^2 = \frac{2}{5}r^2$.

[The same result can be easily arrived at by aid of the Calculus. With the usual notation, we get :—

$$m = \frac{3\,\mathrm{M}}{r^3}x^2\,dx,$$

$$\therefore \quad \mathrm{I}_0 = \Sigma\, m\, x^2 = \frac{3\,\mathrm{M}}{r^3}\int_0^r x^4\,dx = \frac{3\,\mathrm{M}}{r^3}\times\frac{r^5}{5} = \frac{3}{5}\mathrm{M}\,r^2,$$

$$\therefore \quad \mathrm{I} = \tfrac{2}{3}\mathrm{I}_0 = \tfrac{2}{5}\mathrm{M}r^2.]$$

If the sphere be hollow, the inside radius being r, and the outside radius R, it can easily be proved that :—

$$\mathrm{I} = \tfrac{2}{5}\mathrm{M}\frac{\mathrm{R}^5 - r^5}{\mathrm{R}^3 - r^3}.$$

The term "moment of inertia" has been defined above with respect to a solid body only, but it is easy to see that by a slight alteration in the wording of the definition it may be made to apply equally to an area or a *section* of a solid. Accordingly, we find the terms "moment of inertia" and "radius of gyration" applied to areas as well as to solids. Thus, we speak about the moment of inertia and radius of gyration of a circle about a diameter, a triangle about its base, and so on.

We may here remark that the moment of inertia of a solid, or section of a solid, about a given axis, is always proportional to the mass of the solid, or to the area of the section as the case may be.

The following rule has been stated by Routh and will be found useful for finding the moments of inertia about an axis of symmetry :—

Moment of Inertia = *Mass* × (*sum of the squares of the perpendicular semi-axes*) ÷ (3, 4, *or* 5, *according as the body is rectangular, elliptical, or ellipsoidal*).

For the sake of reference, we here give tables of the squares of the radii of gyration for some of the more important cases of both solids and sections.

In every case the axis is taken as passing through the centre of mass of the solid or centre of area of the section, so that if the moment of inertia or radius of gyration be required about any other axis, this can easily be computed from the results given in Propositions I., II., and III.

TABLE I.—SQUARES OF RADII OF GYRATION OF SOLIDS.

	Name of Solid, and Dimensions.	Position of Axis through c.g.	Square of Radius of Gyration. $k^2 = \frac{I}{M}$
I.	Circular hoop of thin wire —Radius, r	Perp. to plane of circle	r^2
II.	Circular hoop of thin wire —Radius, r	About a diameter	$\frac{1}{2} r^2$
III.	Uniform circular rod—Length, l; radius, r	Perp. to length	$\frac{1}{12} l^2 + \frac{1}{4} r^2$
IV.	Solid circular cylinder—Radius, r	About its own axis	$\frac{1}{2} r^2$
V.	Hollow circular cylinder or ring—Radii, R, r	About its own axis	$\frac{1}{2} (R^2 + r^2)$
VI.	Thin cylindrical shell—Radius, r	About its own axis	r^2
VII.	Solid sphere—Radius, r	About a diameter	$\frac{2}{5} r^2$
VIII.	Hollow sphere—Radii, R, r	About a diameter	$\frac{2}{5} \frac{R^5 - r^5}{R^3 - r^3}$
IX.	Thin spherical shell—Radius, r	About a diameter	$\frac{2}{3} r^2$
X.	Solid cone—Radius of base, r	About its own axis	$\frac{3}{10} r^2$

TABLE II.—SQUARES OF RADII OF GYRATION OF LAMINA AND SURFACES OR SECTIONS.

	Form of Lamina, Surface, or Section.	Position of Axis through c.g.	Square of Radius of Gyration. $k^2 = \frac{I}{A}$
I.	Rectangle—Sides, a, b	Parallel to side, b	$\frac{1}{12} a^2$
II.	Rectangle—Sides, a, b	Perp. to plane of figure	$\frac{1}{12} (a^2 + b^2)$
III.	Hollow rectangle—Sides, A, B, and a, b	Parallel to sides, B, b	$\frac{1}{12} \frac{A^3B - a^3b}{AB - ab}$
IV.	Triangle — Altitude, a; base, b	Parallel to base, b	$\frac{1}{18} a^2$
V.	Circular section—Radius, r	Perp. to plane of figure	$\frac{1}{2} r^2$
VI.	Circular section—Radius, r	About a diameter	$\frac{1}{4} r^2$
VII.	Hollow circular section—Radii, R, r	Perp. to plane of figure	$\frac{1}{2} (R^2 + r^2)$
VIII.	Hollow circular section—Radii, R, r	About a diameter	$\frac{1}{4} (R^2 + r^2)$
IX.	Elliptical section—Axes, a, b	About axis, b	$\frac{1}{16} a^2$
X.	Elliptical section—Axes, a, b	Perp. to plane of figure	$\frac{1}{16} (a^2 + b^2)$
XI.	Hollow elliptical section—Axes, A, B, and a, b	About axis, B, b	$\frac{1}{16} \frac{A^3B - a^3b}{AB - ab}$

Equation of Energy for a Rotating Body.—We shall now determine the energy possessed by a rotating body.

Let W = Weight of body, and M its mass.
" w = Weight of any particle at a distance, r, from the axis of rotation, and m its mass.
" ω = Angular velocity of body about given axis.
" v = Linear velocity of the particle = ωr.
" k = Radius of gyration about the given axis.

Then, the kinetic energy of the particle $= \frac{w v^2}{2 g} = \frac{w \omega^2 r^2}{2 g}$.

Repeating this process for every particle composing the body, and adding the results together, we get:—

$$\left.\begin{array}{l}\text{The kinetic energy of}\\ \text{the whole body, } E_K\end{array}\right\} = \Sigma \frac{w \omega^2 r^2}{2 g} = \frac{\omega^2}{2 g} \Sigma w r^2 = \frac{\omega^2}{2} \Sigma m r^2,$$

since $w = m g$, and ω is the same for every particle.

But, $\Sigma m r^2 = I = M k^2 = \frac{W k^2}{g}$, about the given axis,

$$\therefore \quad E_K = \tfrac{1}{2} I \omega^2 = \frac{W \omega^2 k^2}{2 g} \quad . \quad . \quad . \quad . \quad (IX)^*$$

Thus, the equation for the energy of a body rotating about a fixed axis is similar in form to that for a body moving without rotation.

Engineers usually measure the angular velocity, ω, of a rotating body by the number of revolutions made in unit time.

Then, if n be the number of revolutions per unit time,

$$\omega = 2 \pi n$$

$$\therefore \quad E_K = \frac{W \times 4 \pi^2 n^2 k^2}{2 g} = \frac{2 \pi^2 n^2 W k^2}{g} \quad . \quad . \quad . \quad . \quad (X)$$

We may also show, as in the previous Lecture, that, if the angular velocity changes from ω_1 to ω_2, or from n_1 to n_2 revolutions per second, then:—

* If W be expressed in absolute units or poundals, the kinetic energy will also be given in absolute units or foot-poundals; but if W be in pounds weight or in gravitation units, then the kinetic energy will be in foot-pounds.

The student should note that the *pound* is the absolute unit of mass, and, therefore, those of the above equations which contain M instead of $\frac{W}{g}$ *always* give the kinetic energy in absolute units or foot-poundals.

$$\left.\begin{array}{ll}\text{The change of kinetic energy} & = \dfrac{W(\omega_2^2 - \omega_1^2)k^2}{2g} \\ \text{Or, ,, ,,} & = \dfrac{2\pi^2 W(n_2^2 - n_1^2)k^2}{g}\end{array}\right\} \quad . \quad . \quad \text{(XI)}$$

Again, if the centre of gravity of the body be moving with a linear velocity, v, and if at the same time the body be rotating about an axis through its centre of gravity with an angular velocity, ω, then the total kinetic energy possessed by the body is :—

$$E_K = \frac{W v^2}{2g} + \frac{W \omega^2 k^2}{2g} = \frac{W}{2g}\{v^2 + \omega^2 k^2\}. \quad . \quad \text{(XII)}$$

Or, if the linear velocity changes from v_1 to v_2, while the angular velocity changes from ω_1 to ω_2, then the total change in the kinetic energy of the body during that period is :—

$$\frac{W(v_2^2 - v_1^2)}{2g} + \frac{W(\omega_2^2 - \omega_1^2)k^2}{2g} = \frac{W}{2g}\{(v_2^2 - v_1^2) + (\omega_2^2 - \omega_1^2)k^2\}. \quad \text{(XIII)}$$

EXAMPLE IV.—Sketch and describe the action of a fly-press as used for punching holes in metal plates. The balls weigh 60 lbs. each, and are fixed at a radius of 30 inches from the axis of the screw. The screw is double threaded, and of 1 inch pitch. Find what diameter of hole can be punched in a wrought iron plate $\frac{3}{8}$ inch thick, if the strength of the plate in shear be taken at 22 tons per square inch, the resistance to shearing be overcome in the first $\frac{1}{16}$ inch, and if the balls at the instant when the punch touches the plate are moving at the rate of 60 revolutions per minute.

ANSWER.—For a sketch and description of a fly-press, the student may refer to Lecture XXI., of the Author's *Elementary Manual on Applied Mechanics*.

Let W = Weight of *each* ball = 60 lbs.

,, k = Radius of gyration of the system = $2\frac{1}{2}$ feet.

,, n = Number of revolutions *per second* = 1.

,, R = Resistance, in lbs., offered by the metal to the punch.

,, s = Distance through which R is overcome = $\frac{1}{16}$ inch $= \dfrac{1}{12 \times 16}$ feet.

,, t = Thickness of plate punched = $\frac{3}{8}$ inch.

,, d = Diameter, in inches, of the hole.

,, f = Resistance of metal to shearing = 22×2240 lbs. per square inch.

Then, $\quad$ *Area sheared* $= \left\{ \begin{array}{l} \textit{Area of cylindrical surface of} \\ \textit{hole} = \pi d t. \end{array} \right.$

$\therefore$ $\left. \begin{array}{l} \textit{Mean resistance offered} \\ \textit{to shearing} \end{array} \right\} = \mathrm{R} = \pi d t f.$

$\therefore$ $\quad$ *Work done against* $\mathrm{R} = \mathrm{R} \times s = \pi d t f \times s.$

But, $\quad$ *Work done against* $\mathrm{R} = \left\{ \begin{array}{l} \textit{Energy of moving balls at the} \\ \textit{instant when punch strikes} \\ \textit{metal} \end{array} \right.$

$$" \quad " \quad = \frac{2 \mathrm{W} v^2}{2 g} = \frac{\mathrm{W} \times 4 \pi^2 n^2 k^2}{g}.$$

$$\therefore \qquad \pi d t f \times s = \frac{\mathrm{W} \times 4 \pi^2 n^2 k^2}{g}.$$

This is the general equation connecting together the given and the required quantities. By substituting the given data, and cancelling π from both sides of the equation, we get :—

$$d \times \tfrac{3}{8} \times 22 \times 2240 \times \frac{1}{12 \times 16} = \frac{60 \times 4 \times \frac{22}{7} \times 1 \times 1 \times 2\frac{1}{2} \times 2\frac{1}{2}}{32}$$

$$\therefore \qquad d = 1{\cdot}53 \text{ inch.}$$

Example V.—A flywheel weighing 4 tons is keyed to a shaft of 9 inches diameter at the journals. The radius of gyration of the wheel is $5\frac{1}{4}$ feet. At a given instant the wheel is found to be making 80 revolutions per minute, and is not acted on by any other retarding forces than the friction at its journals. Find (1) the reduction in speed after the wheel has made 100 turns. (2) The number of turns it will make before it stops if the coeff. of friction between the journals and their bearings = 0·07.

Answer.—(1) To find the reduction in speed after the wheel has made 100 turns, we must equate the work done against friction in 100 turns to the change of kinetic energy of the wheel during that time.

Let W = Weight of wheel = 4 tons = 4 × 2,240 lbs.
" k = Radius of gyration of wheel = $5\frac{1}{4}$ ft.
" n_1, n_2 = Initial and final revolutions per second.
" d = Diameter of journals = $\frac{3}{4}$ ft.
" μ = Coefficient of friction = 0·07.

Using equation (XI), we have :—

$$\textit{Change of } \mathrm{E_K} \textit{ of wheel} = \frac{2 \pi^2 (n_1^2 - n_2^2) \mathrm{W} k^2}{g}.$$

And, from equation (II_b), Lecture VII., Vol. I. :—

Work lost in friction in one turn of journals $= \pi d \mu R$.

Where R is the resultant pressure on the bearings, and therefore = W in this case.

$$\therefore \left.\begin{array}{l}\textit{Work done against fric-}\\ \quad\textit{tion in } 100 \textit{ turns}\end{array}\right\} = 100 \pi d \mu W.$$

$$\therefore \qquad \frac{2\pi^2 (n_1^2 - n_2^2) W k^2}{g} = 100 \pi d \mu W.$$

$$\therefore \qquad n_1^2 - n_2^2 = \frac{100 d \mu g}{2 \pi k^2}.$$

Hence,
$$n_2^2 = n_1^2 - \frac{50 d \mu g}{\pi k^2}$$

$$\therefore \qquad ,, = \left(\frac{80}{60}\right)^2 - \frac{50 \times \cdot 75 \times \cdot 07 \times 32}{\frac{22}{7} \times 5\cdot 25 \times 5\cdot 25}$$

$$,, = 1\cdot 78 - \cdot 97 = \cdot 81.$$

Or,
$$n_2 = \sqrt{\cdot 81} = \cdot 9 \text{ rev. per sec., or } 54 \text{ revs. per min.}$$

$\therefore$ **Reduction in speed** $= n_1 - n_2 = 80 - 54 = 26$ **revs. per min.**

(2) Let n = number of turns made before stopping.

Then, in this case, the whole energy of the wheel when making 80 turns per minute is absorbed in friction at the journals.

$$\therefore \frac{2\pi^2 n_1^2 W k^2}{g} = n \pi d \mu W.$$

$$\therefore \qquad n = \frac{2 \pi n_1^2 k^2}{\mu d g} = \frac{2 \times \frac{22}{7} \times \frac{80}{60} \times \frac{80}{60} \times 5\cdot 25 \times 5\cdot 25}{\cdot 07 \times \cdot 75 \times 32}$$

$$\therefore \qquad n = 183\tfrac{1}{3} \text{ turns.}$$

Example VI.—A right cylinder of radius r, rolls, without slipping, down an inclined plane of height h. Find its velocity at the foot of the plane, and compare this with that which it

would have had by merely sliding. Neglect frictional resistances in both cases.

ANSWER.—Let v = Velocity of *c.g.* of cylinder at foot of plane.
„ ω = Angular velocity „ „
„ W = Weight of cylinder.
„ k = Radius of gyration about its own axis

$$= \frac{r}{\sqrt{2}}.$$

Then, $\left.\begin{array}{c}\textit{Total kinetic energy}\\ \textit{at foot of plane}\end{array}\right\} = \left\{\begin{array}{l}\textit{Energy of Translation}\\ + \textit{Energy of Rotation.}\end{array}\right.$

But, $\left.\begin{array}{c}\textit{Total energy at foot}\\ \textit{of plane}\end{array}\right\} = \mathrm{W}\, h.$

Also, $\textit{Energy of Translation} = \dfrac{\mathrm{W}\, v^2}{2\, g}.$

And, $\textit{Energy of Rotation} = \dfrac{\mathrm{W}\, \omega^2 k^2}{2\, g}.$

$$\therefore \quad \mathrm{W}\, h = \frac{\mathrm{W}\, v^2}{2\, g} + \frac{\mathrm{W}\, \omega^2 k^2}{2\, g}. \quad \therefore \quad 2\, g\, h = v^2 + \omega^2 k^2.$$

But, $\omega = \dfrac{v}{r}$, and $k = \dfrac{r}{\sqrt{2}}$. $\therefore \quad \omega^2 k^2 = \dfrac{v^2}{2}.$

$$\therefore \quad 2\, g\, h = v^2 + \frac{v^2}{2}. \quad \therefore \quad v = \sqrt{\frac{4\, g\, h}{3}}.$$

Had the cylinder been allowed to *slide* down the plane *without rolling*, the velocity at foot of plane would have been :—

$$v = \sqrt{2\, g\, h}.$$

$$\therefore \left.\begin{array}{l}\textbf{Vel. with rolling : Vel.}\\ \quad\textbf{without rolling}\end{array}\right\} = \sqrt{\frac{4\, g\, h}{3}} : \sqrt{2\, g\, h} = \sqrt{2} : \sqrt{3}.$$

Of course, the kinetic energy of the body in both cases is the same, but in the second case the whole energy is translational, hence the reason for the greater speed in this case.

EXAMPLE VII.—A weight, Q, draws up another weight, W, by means of an ordinary wheel and axle. The force ratio (Q : W) is 1 to 6, and the velocity ratio (vel. of Q : vel. of W) is 8 to 1. The diameter of the axle is 6 inches, and the radius

of gyration of the wheel and its axle may be taken at 10 inches. Neglecting frictional resistances and the inertia of the ropes, determine the revolutions per minute of the machine after 10 turns have been made from a state of rest. Take the weight of the wheel and its axle = 2 W.

ANSWER.—We shall first answer this question in a general way.

Let W_1 = Weight of wheel and axle.
„ V = Velocity of effort, Q, in ft. per sec., after N turns.
„ v = Velocity of weight, W, „ „
„ R = Radius of wheel in feet.
„ r = Radius of axle „
„ k = Radius of gyration of wheel and axle in feet.
„ n = Revolutions per sec. of machine, after N turns.

Then, by the *Principle of Energy*, we get :—

Energy exerted = Work done + Change of kinetic energy.

But, $\left.\begin{array}{r}\textit{Energy}\\ \textit{exerted}\end{array}\right\} = Q \times$ *Distance fallen in* N *turns of machine.*

$$„ \quad = Q \times 2\pi R N.$$

Work done = $\left\{\begin{array}{l}W \times \textit{Distance raised in N turns of}\\ \quad \textit{machine} = W \times 2\pi r N.\end{array}\right.$

$\left.\begin{array}{l}\textit{Change of kinetic}\\ \textit{energy}\end{array}\right\} = \left\{\begin{array}{l}\textit{Translational energy of Q and W + Rota-}\\ \quad\textit{tional energy of wheel and axle.}\end{array}\right.$

$$„ \quad „ \quad = \frac{4\pi^2 n^2 Q \times R^2}{2g} + \frac{4\pi^2 n^2 W \times r^2}{2g} + \frac{4\pi^2 n^2 W_1 \times k^2}{2g}$$

$$„ \quad „ \quad = \frac{2\pi^2 n^2}{g}\left\{Q R^2 + W r^2 + W_1 k^2\right\}$$

$$\text{Hence,} \quad Q \times 2\pi RN = W \times 2\pi r N + \frac{2\pi^2 n^2}{g}\left\{QR^2 + Wr^2 + W_1 k^2\right\}$$

$$\text{Or,} \quad (QR - Wr)N = \frac{\pi n^2}{g}\left\{Q R^2 + W r^2 + W_1 k^2\right\}$$

This is the general expression from which n can be found when the other quantities are given.

From the question, we get :—$W = 6\,Q$; $W_1 = 2\,W = 12\,Q$; $N = 10$; $V = 8v$; $r = 3$ inches $= \frac{1}{4}$ foot; $R = \frac{V}{v}r = 8 \times \frac{1}{4}$ $= 2$ feet; $k = \frac{10}{12} = \frac{5}{6}$ foot.

Hence, $\left.\begin{matrix}\textit{Energy}\\ \textit{exerted}\end{matrix}\right\} = Q \times 2\pi RN = Q \times 2\pi \times 2 \times 10 = 40\pi Q$ *ft.-lbs.*

$$\textit{Work done} = W \times 2\pi r N = 6Q \times 2\pi \times \tfrac{1}{4} \times 10 = 30\pi Q \quad ,,$$

$$\left.\begin{matrix}\textit{Change of kinetic}\\ \textit{energy}\end{matrix}\right\} = \frac{2\,\pi^2\,n^2}{g}\left\{Q\,R^2 + W\,r^2 + W_1\,k^2\right\}$$

$$,, \quad ,, \quad = \frac{2\pi^2 n^2}{32}\left\{Q \times 2^2 + 6Q \times (\tfrac{1}{4})^2 + 12Q \times (\tfrac{5}{6})^2\right\} ,,$$

$$,, \quad ,, \quad = \frac{\pi^2\,n^2}{16} \times \frac{305}{24}\,Q \textit{ ft.-lbs.}$$

$$\therefore \quad 40\,\pi\,Q = 30\,\pi\,Q + \frac{305\,\pi^2\,n^2}{16 \times 24}Q.$$

$$\therefore \quad n^2 = \frac{10 \times 16 \times 24}{305 \times \frac{22}{7}} = 4.$$

$\therefore$ $n = 2$ revolutions per second, or 120 per min.

Determination of the Energy of Flywheels.—Before the energy of a rotating body can be calculated at any given speed, it is necessary to know the radius of gyration of that body about the given axis of rotation. We have already shown how this quantity can be calculated in certain bodies which are of regular geometrical form; but many cases occur in the rotating parts of machines where the above methods of calculation would be most difficult, if not altogether impossible. Such is the case with most flywheels. The flywheel is a most important part of an engine, since it is a regulator of the speed. Owing to the great mass of its rim it naturally possesses great inertia, and is, therefore, capable of storing up a considerable amount of the energy developed in the cylinder, and of again imparting this stored energy to the moving parts during those portions of a revolution when the work done in the cylinder is less than the work being done outside. It is important to know the radius of gyration of the wheel, so that calculations relating to the storage and output of its energy can be effected. This radius of the wheel may be determined either approximately by calculation, or accurately by experimenting on the wheel itself, or with another similarly shaped wheel. We shall deal with these cases in turn.

(1) *By Approximate Calculation.*—Most flywheels consist of a heavy rim with comparatively light arms and nave; hence,

in calculations relating to the radii of gyration of such wheels, we may neglect the effects of the arms and nave, and consider only that of the heavy rim. Usually the rim is of a rectangular cross section.

Let R, r = Outside and inside radii of rim.

Then, from Table I., case V., of this Lecture, we get:—

$$k^2 = \frac{1}{2}(R^2 + r^2).$$

Substituting this in the equation for the kinetic energy of the wheel, we may obtain an approximate result.

Many engineers, however, further simplify their formula by taking for the radius of gyration the *mean* radius of the rim, and consider this quite near enough for most purposes. Thus:—

$$k = \frac{1}{2}(R + r).$$

The difference in the kinetic energy, as calculated from those two assumptions, may be shown as follows:—

Let W = Total weight of wheel.
,, ω = Angular velocity of wheel

Then, according to the *first* assumption:—

$$\textit{The kinetic energy} = \frac{W\,\omega^2\,k^2}{2\,g} = \frac{W\,\omega^2}{2\,g} \times \frac{R^2 + r^2}{2}.$$

And, according to the *second* assumption:—

$$\text{The kinetic energy} = \frac{W\,\omega^2}{2\,g} \times \frac{(R + r)^2}{4}.$$

$$\text{Hence, the difference} = \frac{W\,\omega^2}{2\,g}\left\{\frac{R^2 + r^2}{2} - \frac{(R + r)^2}{4}\right\}$$

$$\text{,,} \qquad \text{,,} \qquad = \frac{W\,\omega^2}{2\,g} \times \frac{(R - r)^2}{4}.$$

That is, the kinetic energy in the *first* case is greater than that in the *second* case by $\frac{W\,\omega^2}{2\,g} \times \frac{(R - r)^2}{4}$. This difference, however, becomes less as $R - r$ diminishes—that is, as r approaches R. On the other hand, it gets greater the thicker the rim. The radius of gyration in the first case—viz., $k^2 = \frac{1}{2}(R^2 + r^2)$, is too great; because the effect of the arms

and nave is to reduce that radius, whereas the other result, $k = \frac{1}{2}(R + r)$, may be too small. It sometimes happens that a closer approximation may be obtained by taking the arithmetical *mean* of the above results, thus:—

$$\left.\begin{array}{c}\text{The kinetic energy}\\ \text{of the wheel}\end{array}\right\} = \frac{W\omega^2}{2g} \times \frac{1}{2}\left\{\frac{R^2 + r^2}{2} + \frac{(R+r)^2}{4}\right\}$$

$$\text{,,} \qquad \text{,,} \qquad = \frac{W\omega^2}{2g} \times \frac{\{3(R + r)^2 - 4Rr\}}{8}. \qquad \text{(XIV)}$$

(2) *By Experiment on the Wheel.*—When accurate results are required, we may determine the radius of gyration of the wheel experimentally as follows:—

Disconnect the flywheel and its shaft from all other moving pieces, and see that the shaft runs smoothly in its bearings. Fit a flat pulley on the shaft and wind a few turns of flexible rope in a single layer round the same.* To the free end of this rope attach a weight sufficiently heavy to cause the flywheel to rotate at a uniform speed when started by the hand. This weight should just supply the energy absorbed by the friction of the shaft in its bearings and the bending of the rope. Now rewind the rope on the pulley and add another weight to its free end, so that the wheel will now start rotating when the weights are allowed to fall. Note the time taken by the weights in falling a known distance. The height through which the weights fall, and the diameter of the pulley being known, it is easy to calculate both the speed of the wheel and the falling weights, and hence their kinetic energies at the instant when the latter reach the ground.

Another method of allowing for the friction of the bearings, &c., is to use only one weight. Note the exact number of turns which the wheel makes (after the weight has ceased to act) until it comes to rest. Then neglecting the atmospheric resistance (which will be very small in an experiment of this kind) the work absorbed at the bearings will be equal to the kinetic energy of the wheel at the instant when the weight ceases to act.

These methods will be better understood when stated thus:—

* If the flywheel shaft be of sufficient diameter, this pulley may be dispensed with, and the rope need then be simply wound round the shaft. If a convenient direct drop for the weights cannot be arranged for, then the rope may pass round a guide pulley fixed to the roof, but in this case the kinetic energy of this pulley must be allowed for.

Let W = Weight of flywheel *in lbs.*
„ w = Weight producing motion of wheel.
„ w_1 = Weight required to balance friction.
„ k = Radius of gyration of flywheel *in feet.*
„ h = Height through which w and w_1 fall *in feet.*
„ D = Diameter of pulley keyed to shaft *in feet.*
„ d = Diameter of rope *in feet.*
„ t = Time taken by weight, w, in falling to the ground *in seconds.*
„ n = Number of revolutions *per second* which wheel is making at instant when w reaches the ground.
„ v = Velocity with which w and w_1 strike the ground.
„ N = Number of revolutions made by wheel after w ceases to act.

Firstly.—*When w_1 is employed to balance the frictional resistances.* All the energy exerted by w is employed in giving kinetic energy to the wheel.

But, *Energy exerted* $= w\,h.$

And, $\left.\begin{array}{c}\textit{Change}\\ \textit{of kinetic}\\ \textit{energy}\end{array}\right\} = \left\{\begin{array}{c}\textit{Kinetic}\\ \textit{energy of}\\ \textit{wheel}\end{array}\right\} + \left\{\begin{array}{c}\textit{Kinetic energy of } w \textit{ and}\\ w_1 \textit{ when they reach}\\ \textit{the ground}\end{array}\right.$

$$„ \quad = \frac{W \times 4\,\pi^2\,n^2\,k^2}{2\,g} + \frac{(w + w_1)\,v^2}{2\,g}.$$

$$\therefore \quad w\,h = \frac{W \times 4\,\pi^2\,n^2\,k^2}{2\,g} + \frac{(w + w_1)\,v^2}{2\,g} \quad . \quad . \quad (1)$$

The revolutions, n, and the linear velocity, v, of the falling body at the instant when the latter reaches the ground can be determined as follows, when t, D, and d are known:—

$$\left.\begin{array}{r}\textit{Number of revols. made by}\\ \textit{wheel during action of } w\end{array}\right\} = \frac{h}{\pi\,(D + d)} \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

$$\therefore \quad \left.\begin{array}{r}\textit{Average number of}\\ \textit{revols. per second}\end{array}\right\} = \frac{h}{\pi\,(D + d)\,t}$$

$$\therefore \quad n = \textit{Twice the average,}$$

$$„ = \frac{2\,h}{\pi\,(D + d)\,t} \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

Similarly $\quad v$ = *Twice average linear velocity of w and w_1,*

$$„ = \frac{2\,h}{t} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

$\therefore$ From equation (1) we get :—

$$w\,h = \frac{W \times 4\pi^2 \times 4h^2 \times k^2}{2g \times \pi^2(D+d)^2 t^2} + \frac{(w + w_1) \times 4h^2}{2g \times t^2}$$

$$\therefore \qquad \frac{8\,W\,h\,k^2}{g\,(D+d)^2 t^2} = w - \frac{2\,(w + w_1)\,h}{g\,t^2}. \quad . \quad . \quad \text{(XV)}$$

SECONDLY.—*When the number of turns made by wheel after w ceases to act is known,*

$$\textit{Energy exerted} = w\,h.$$

$$\left.\begin{array}{l}\textit{Work done on friction dur-}\\ \textit{ing last N revolutions of}\\ \textit{wheel}\end{array}\right\} = \left\{\begin{array}{l}\textit{Kinetic energy of wheel at}\\ \textit{instant when } w \textit{ ceases to}\\ \textit{act}\end{array}\right.$$

$$\text{,,} \qquad \text{,,} \qquad = \frac{W \times 4\,\pi^2\,n^2\,k^2}{2\,g}.$$

But, by equation (2), the wheel makes $\dfrac{h}{\pi\,(D+d)}$ revolutions during the action of w.

$$\therefore \left.\begin{array}{l}\textit{Work done on friction}\\ \textit{during action of } w\end{array}\right\} = \frac{W \times 4\,\pi^2\,n^2\,k^2}{2\,g} \times \frac{\dfrac{h}{\pi\,(D+d)}}{N}.$$

From equation (3), we get :—

$$n = \frac{2\,h}{\pi\,(D+d)\,t}.$$

$$\therefore \qquad \textit{Work done} = \frac{W \times 4\,\pi^2 \times \dfrac{4\,h^2}{\pi^2\,(D+d)^2\,t^2} \times k^2}{2\,g} \times \frac{h}{\pi\,(D+d)\,N}$$

$$\text{,,} \qquad = \frac{8\,W\,h^3\,k^2}{g\,\pi\,(D+d)^3\,t^2\,N}.$$

$$\left.\begin{array}{l}\textit{Change of}\\ \textit{kinetic energy}\end{array}\right\} = \frac{W \times 4\,\pi^2\,n^2\,k^2}{2\,g} + \frac{w\,v^2}{2\,g}$$

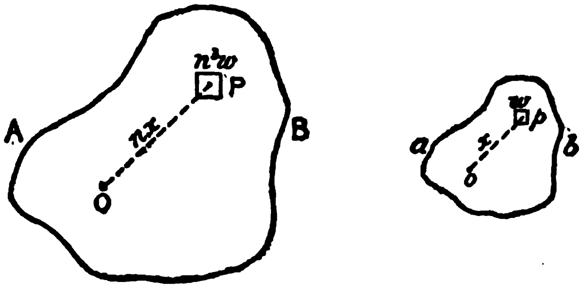
$$\text{,,} \qquad = \frac{8\,W\,h^2\,k^2}{g\,(D+d)^2\,t^2} + \frac{2\,w\,h^2}{g\,t^2}.$$

Hence, $$w\,h = \frac{8\,W\,h^3\,k^2}{g\,\pi\,(D+d)^3\,t^2\,N} + \frac{8\,W\,h^2\,k^2}{g\,(D+d)^2\,t^2} + \frac{2\,w\,h^2}{g\,t^2}.$$

$$\therefore \frac{8\,W\,h\,k^2}{g\,(D+d)^2\,t^2}\left\{\frac{h}{\pi\,(D+d)\,N} + 1\right\} = w\left\{1 - \frac{2\,h}{g\,t^2}\right\}. \quad \text{(XVI)}$$

Equations (XV) and (XVI) enable us to find the radius of gyration, k, when the data are furnished by experiment.

If the wheel whose radius of gyration has to be found cannot be conveniently experimented on, then the radius of gyration of another similar wheel may be determined, and that for the first wheel calculated therefrom. It is easy to show generally that the moments of inertia of two similar bodies rotating about similarly placed axes are as the fifth powers of their like linear dimensions.



MOMENTS OF INERTIA OF SIMILAR BODIES.

Let A B, and $a\,b$, be any two similar bodies whose axes O, and o, are similarly situated. Let the linear dimensions of the larger body be n times those of the smaller. Taking similar parts at P, and p, so that P is n times as large as p in each direction, it is evident that their masses will be in the proportion of $n^3 : 1$.

i.e., Mass of element at P : *Mass of corresponding element at* p $= n^3 m : m$. Also, if $o\,p = x$, then $\text{O P} = n\,x$.

$\therefore$ *Mom. of inertia of* A B *about* O $= \Sigma n^3 m \times (n\,x)^2 = n^5 \Sigma m\,x^2$,

and, *Mom of inertia of* $a\,b$ *about* O $= \Sigma m\,x^2$.

$$\therefore \quad \left.\begin{matrix}\text{Mom. of inertia of A B :}\\ \text{Mom. of inertia of } a\,b\end{matrix}\right\} = n^5 : 1. \quad . \quad . \quad . \quad . \quad \text{(XVII)}$$

Thus, if two flywheels are made from the same drawing, but the scale in the one case be 4 inches to the foot, and in the other $1\frac{1}{2}$ inches to the foot, then their like linear dimensions will be inversely as the scales to which they are drawn, that is :—

Size of first wheel : *Size of second wheel* $= 1\frac{1}{2} : 4 = 3 : 8$.

$\therefore$ **Mom. of inertia of first wheel : Mom. of inertia of second wheel** $= 3^5 : 8^5 = 243 : 32768 = 1 : 134{\cdot}8$ **nearly.**

Centripetal and Centrifugal Force.—If a body is observed to be moving in a curvilinear path, either with uniform or variable speed, we at once infer that it is being continually acted upon by some deviating force directed towards the inside of the curve. In the case of a body moving in a circular path, that deviating force must be directed towards the centre of the circle. Hence, a body may be made to move in a circular path either by having it attached to a fixed point (the centre) by an inextensible string, or by compelling it to move in a circular groove. The necessary deviating force is supplied in the first case by the string attached to the body, while in the second case it is supplied by the sides of the groove. In either case this centrally-directed force is called the **Centripetal Force,** while its reaction is called the **Centrifugal Force.** These terms may be defined as follows :—

DEFINITION.—**Centripetal Force is that force which a guiding body exerts on a revolving body in order to compel the revolving body to move in its curvilinear path, and is always directed towards a fixed centre.**

DEFINITION.—**Centrifugal Force is the force with which a revolving body reacts on the body that constrains it to move in a curved path, and is equal and opposite in direction to the force with which the constraining body acts on the revolving body.**

i.e., **Centripetal Force = Centrifugal Force.**

We stated in Lecture XX. that when the velocity of a body changes, whether in magnitude or in direction, the velocity is said to be accelerated, and we have there shown how to measure this acceleration in the case of a particle moving with uniform speed in a circle. Thus, the radial or centripetal acceleration is there shown to be :—

$$a = \frac{v^2}{r}.$$

Where, v = Linear velocity of the particle in the circle,

and, r = Radius of the circle.

But an acceleration of a body can only be produced by the action of some force on it, and in the last Lecture we have shown how this force is measured when the weight of the body and the acceleration are known. Hence :—

$$F = \frac{w}{g} a.$$

Let w = Weight of particle moving uniformly in a circle.
„ v = Linear velocity of particle in circle.
„ r = Radius of circle.
„ F = Centripetal or centrifugal force.
„ a = Centripetal acceleration $= \frac{v^2}{r}$.

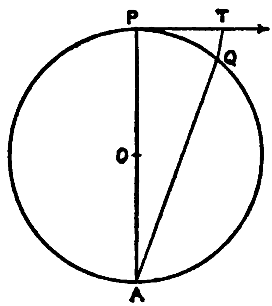
Then, $$F = \frac{w}{g} a = \frac{w v^2}{g r} \quad . \quad . \quad . \quad . \quad \text{(XVIII)}$$

We may, however, establish the same result in a different manner as follows :—

Let P be the position of the particle at any instant, and Q its position after a small interval of time, t. If no force acted on the body during that small interval of time, it would move along the tangent P T, and at the end of the interval be found at T, such that:—

$$P T = v t.$$

But Q is its actual position; therefore T Q represents the deviation due to the centripetal force during that interval of time. Join Q A.



CENTRIPETAL FORCE.

Then, $T Q = \frac{1}{2} a t^2.$

But, since P T and Q T are very small, T Q A will be very nearly a straight line.

$$\therefore \quad P T^2 = T A \times T Q \qquad \text{[Euc. III., 35]}$$
$$\text{,,} \quad = (Q A + T Q) \times T Q$$
$$\text{,,} \quad = Q A \times T Q + T Q^2.$$

In the limit, when t is infinitely small and, therefore, Q infinitely near to P, we may neglect $T Q^2$, and put $Q A = P A = 2 r$.

$$\therefore \quad v^2 t^2 = 2 r \times \tfrac{1}{2} a t^2,$$

$$\therefore \quad a = \frac{v^2}{r}.$$

This is the same result as obtained by means of the Hodograph in Lecture XX.

$$\therefore \qquad \text{Centrifugal force} = \frac{w}{g} a = \frac{w\, v^2}{g\, r}.$$

Let ω = Angular velocity of radius O P.

Then, $v = \omega\, r,$

$$\therefore \qquad \mathrm{F} = \frac{w\, \omega^2\, r^2}{g\, r} = \frac{w\, \omega^2\, r}{g}. \quad . \quad . \quad . \quad . \quad \text{(XIX)}$$

This shows that the centrifugal force is proportional to the square of the angular velocity of the particle, and to its distance from the centre of rotation.

We may now show that a similar expression holds good for the case of an extended rigid body turning about an axis.

Taking any particle of the body of weight w, and at a distance x from the axis of rotation, we get:—

$$\textit{Cent. force of the element} = \frac{w\, \omega^2\, x}{g}$$

$$\therefore \qquad \textit{Cent. force of whole body} = \frac{\omega^2}{g} \Sigma\, w\, x.$$

But, $\Sigma\, w\, x = \mathrm{W}\, r.$

Where W = Weight of body,

And r = Distance of centre of gravity of body from axis of rotation.

$$\therefore \qquad \mathrm{F} = \frac{\mathrm{W}\, \omega^2\, r}{g}. \quad . \quad . \quad . \quad . \quad . \quad \text{(XX)}$$

Hence, if the axis of rotation passes through the centre of gravity of the body, the centrifugal force is *nil*. If, however, the body be unsymmetrical about the axis of rotation, there may be, as explained in the next Lecture, a centrifugal *couple* tending to twist the axis of rotation and make the body rotate about some other axis.

EXAMPLE VIII.—A railway carriage weighing 4 tons is moving at the rate of 60 miles per hour round a curve $\frac{1}{4}$ mile in radius. Find the pressure on the rails due to centrifugal force; also, how much the outer rail should be higher than the inner rail in order that the pressure may be equally distributed on both? The distance between the rails is 4 feet $8\frac{1}{2}$ inches.

ANSWER.—Here, $\mathrm{W} = 4 \times 2240$ lbs.; $r = \frac{1}{4} \times 5280 = 1320$ feet; $v = \dfrac{60 \times 5280}{60 \times 60} = 88$ ft. per sec.

$$\therefore \left.\begin{array}{c}\textbf{Centrifugal}\\ \textbf{force}\end{array}\right\} = \frac{\mathrm{W}\, v^2}{g\, r} = \frac{4 \times 2240 \times 88 \times 88}{32 \times 1320} = 1642{\cdot}7 \text{ lbs.}$$

Hence, if both the inner and the outer rails were on a level, the flanges of the wheels would press on the latter with a force of 1642·7 lbs. By raising the outer line of rails above the level of the inner one, the carriage may be made to lie on an incline, and the outer rails thus relieved of the centrifugal pressure.

Let h = Height of the outer rail above level of the inner rail.
" l = Distance between the rails = 4 ft. $8\frac{1}{2}$ ins. = $56\frac{1}{2}$ ins.
" F = Centrifugal force on carriage = 1642·7 lbs.

Then, as a question on the *Inclined Plane*, we get:—

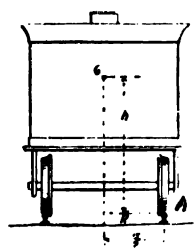
$$F : W = h : l.$$
$$h = \frac{F}{W} \times l = \frac{1642 \cdot 7}{4 \times 2240} \times 56\tfrac{1}{2} = 10 \cdot 4 \text{ inches nearly.}$$

EXAMPLE VIII*a*.—An engine is running on level rails round a curve of radius, r, the distance between the rails is d, and the height of the centre of gravity above the rails is h. Prove that if the velocity exceeds $\sqrt{\frac{g\,r\,d}{2\,h}}$ the engine will fall.

Show whether in general for a particular carriage it would be safer for the engine to push or pull a train.

ANSWER.—When a body of weight, W, or mass, M, is moving in a circle, the normal force acting at every instant to deflect it from its straight-line path (usually called the centrifugal force) is given by

$$F = \frac{W\,v^2}{g\,r}, \text{ or } \tfrac{1}{2}\,\frac{M\,v^2}{r}.$$



(Where W denotes the weight and M the mass of the body, v the velocity of the body in feet per second, r is the radius of the circular path in feet, and g the acceleration due to gravity.) To balance this there must be a force supplied by the flange against the rail, A; denote this by R.

The weight, W, of the engine may be supposed to act at its centre of gravity, G. Then, taking moments about A, the moment of F is $F \times h$, and the moment of W is $W \times \frac{d}{2}$.

So long as the moment of the weight is greater than the

moment of F, equilibrium is stable. When $F h$ is greater than $W \times \frac{d}{2}$, the engine will overturn, and is just on the point of overturning when $F h = W \times \frac{d}{2}$, and this obviously gives the limiting speed.

Thus we have $F \times h = W \times \frac{d}{2}$.

And substituting for F,

$$\frac{W v^2}{g r} \times h = W \times \frac{d}{2}$$

$$\therefore \qquad v = \sqrt{\frac{g r d}{2 h}}.$$

If the engine is pushing a given carriage, the pressure on the outer rail, A, is increased, and, therefore, the tendency to overturn. The reverse occurs during the process of pulling; hence it is safer for the engine to pull than to push.

The preceding is probably the simplest method of treating the problem; more accurately, in addition to the centrifugal action, there is the gyroscopic action of the rotating wheels to be taken into account, this gyroscopic action increasing the pressure on the outer rail and diminishing the pressure on the inner, and, therefore, increasing the tendency to overturn the engine.

Straining Actions due to Centrifugal Forces.—Whenever a body rotates about an axis, the material of that body becomes strained by reason of the centrifugal forces set up. Thus, in the case of a flywheel or pulley, the centrifugal forces set up may be sufficient to tear the rim from the arms, the arms from the nave, or to burst the rim. In Lecture XVIII., Vol. I., we explained the effects of the centrifugal forces acting on a belt when moving over a pulley with a high velocity. We there showed that the tensions in the two parts of the belt were increased by the centrifugal action on that part of the belt which is in contact with the pulley. We shall now show that similar effects occur in a rapidly-revolving flywheel or pulley.

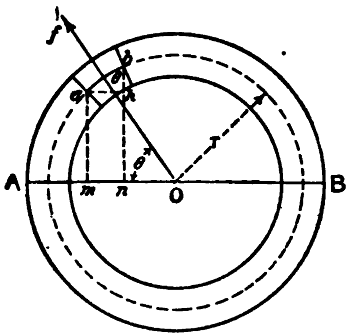
Suppose we have a flywheel built up of segments, each segment being attached to an arm, while they are also attached to each other by dowels and cotters, or bolts, &c. Let the weight of each segment be W; the distance of its centre of gravity from the axis of rotation, r, and the angular velocity of the wheel, ω. Then, neglecting the assistance afforded by the connection between the various segments, it is obvious

that the tension in the arm to which the segment is attached is :—

$$P = \frac{W \omega^2 r}{g},$$

The arm must, therefore, be made strong enough to withstand this stress.

Again, in the case of a solid rim, the effect of the centrifugal forces is to burst it along a section made by a plane containing the axis of the shaft. Let the figure represent the rim of a fly-wheel. Then, in order to calculate the stress in its material at any section, A B, made by a plane containing the axis, O, consider the effects of a thin slice of the rim at $a\,b$.



STRESS IN RIM OF FLYWHEEL DUE TO CENTRIFUGAL FORCE.

Let W = Total weight of rim.

" r = Mean radius of rim.

" x = Length of small arc $a\,b$ of mean rim.

" ω = Angular velocity of wheel.

Then, $\dfrac{\textit{Weight of slice } a\,b}{\textit{Weight of rim}} = \dfrac{\textit{Arc } a\,b \textit{ of mean rim}}{\textit{Circumference of mean rim}} = \dfrac{x}{2\pi r}.$

$\therefore$ *Weight of element* $a\,b = \dfrac{W}{2\pi r} \times x.$

The centrifugal force of the element at $a\,b$ is :—

$$\therefore \qquad f = \frac{W}{2\pi r} \times \frac{\omega^2 r}{g} \times x = \frac{W\omega^2}{2\pi g} x.$$

This force acts through the *c.g.* of the element. Resolve f in directions parallel and perpendicular to A B. The latter component only is effective in producing stress at the sections A and B.

$\therefore$ $\left.\begin{array}{l}\textit{Stress at sections A and B due}\\ \textit{to cent. force on element } a\,b\end{array}\right\} = f \sin\theta = \dfrac{W\omega^2}{2\pi g} \times x \sin\theta.$

Where, $\theta = \angle A O f.$

From a and b drop the perpendiculars $a\,m$, $b\,n$ on A B, and through a draw $a\,h$ perpendicular to $b\,n$. Then $\angle\,a\,b\,h = \theta$ and $x \sin\theta = a\,h = m\,n$.

$$\therefore \quad \left.\begin{array}{r}\textit{Stress at sections A and B due}\\ \textit{to cent. force on element } a\,b\end{array}\right\} = \frac{W\,\omega^2}{2\,\pi\,g} \times m\,n.$$

Continuing this reasoning for all the slices from A round to B, and adding the results, we get :—

$$\left.\begin{array}{l}\text{Total stress over}\\ \text{sections at A}\\ \text{and B}\end{array}\right\} = \frac{W\,\omega^2}{2\,\pi\,g}\,\Sigma\,m\,n = \frac{W\,\omega^2}{2\,\pi\,g} \times 2\,r = \frac{W\,\omega^2\,r}{\pi\,g}.$$

Let A = Area of section of rim at A, or B, in *square inches*.
„ p = Stress in *lbs. per square inch* over section.
„ w = Weight of *a cubic foot* of material of rim.

$$\text{Then,} \quad p = \frac{\textit{Total stress over section at A or B}}{\textit{Area of section}}.$$

$$\text{Or,} \quad p = \frac{\frac{1}{2} \times \dfrac{W\,\omega^2\,r}{\pi\,g}}{A} = \frac{W\,\omega^2\,r}{2\,\pi\,g\,A}.$$

But, $W = \textit{Area of cross section of rim in sq. ft.} \times 2\,\pi\,r\,w.$

$$\therefore \quad W = \frac{A}{144} \times 2\,\pi\,r\,w.$$

Substituting this in the last equation, we get :—

$$p = \frac{w\,\omega^2\,r^2}{144\,g} \quad \ldots\ldots \quad \text{(XXI)}$$

Or, if n = Revolutions of wheel *per second*,
d = Diameter of rim *in feet*,
v = Velocity of rim *in feet per second* = $\omega\,r$.

$$\left.\begin{array}{ll}\text{Then,} & p = \dfrac{w\,\pi^2\,d^2\,n^2}{144\,g} \text{ lbs. per square inch.}\\ \text{Or,} & p = \dfrac{w\,v^2}{144\,g} \quad \text{,, \quad ,, \quad ,,}\end{array}\right\} \text{(XXII)}$$

From this we see that the stress per square inch does not depend on the cross area of the rim nor the diameter of the wheel, but only on the density of the material and its speed. It will also be observed that the centrifugal force in the rim is similar in effect to a hydrostatic pressure on the inside of a cylindrical vessel.

EXAMPLE IX.—A flywheel, 21 feet in diameter, makes 100 revolutions per minute. The weight of a cubic foot of its material is 448 lbs. Find the intensity of stress on a transverse section of rim, assuming that it is unaffected by the arms. If the safe stress permissible in the material is 6,000 lbs. per square inch, what is the greatest speed at which the wheel can be run with safety?

ANSWER.—Here, w = 448 lbs. per cubic foot; d = 21 feet; $n = \frac{100}{60} = \frac{5}{3}$ revolutions per second.

Therefore, from equation (XXII), we get:—

$$\text{Stress in rim} = p = \frac{w\,\pi^2 d^2 n^2}{144\,g}$$

Or,
$$p = \frac{448 \times \left(\frac{22}{7}\right)^2 \times 21^2 \times \left(\frac{5}{3}\right)^2}{144 \times 32} = 1176 \cdot 4 \text{ lbs. per sq. in.}$$

Next, let n = Maximum number of revolutions per second which the wheel can make without bursting.

Then, from the previous formula:—$p = \dfrac{w\,\pi^2 d^2 n^2}{144\,g}$,

We get, $n^2 = \dfrac{144\,g\,p}{w\,\pi^2 d^2}$, or, $n = \dfrac{12}{\pi d}\sqrt{\dfrac{g\,p}{w}}$

Substituting p = 6,000, and the values for the other letters, we get:—

$$n = \frac{12}{\frac{22}{7} \times 21}\sqrt{\frac{32 \times 6000}{448}} = 3 \cdot 76 \text{ revs. per sec.} = 225 \cdot 6 \text{ per min.}$$

Note.—Students should refer to the author's *Text-Book on Steam and Steam Engines*, Lecture XVII., for a discussion of the effects of the inertia of the moving parts of an engine.

LECTURE XXII.—QUESTIONS.

1. Define the terms moment of inertia and radius of gyration of a body. Find the moment of inertia of rectangular lamina—first, with respect to one edge; secondly, with respect to a diagonal.

2. An axis is drawn through the centre of gravity of a body whose mass is M; a second axis is drawn parallel to the former and at a distance, h, from it. If I denotes the moment of inertia of the body with respect to the first axis, show that the moment of inertia with respect to the second axis is $I + M h^2$. A fine wire of uniform thickness is bent into the form of a circle whose radius is r; find its moment of inertia with respect to an axis passing at right angles to the plane of the circle through a point in the circumference. *Ans.* $\frac{3}{2} M r^2$.

3. State and prove the theorem of moments of inertia for parallel axes. Find the moment of inertia of a cylinder about a line perpendicular to its axis through its mid point. (S. & A. Theor. Mechs. Hons. Exam.)

4. A wheel and axle are composed of the same specific gravity. The wheel is 4 feet radius, and 6 inches thick. The axle is 6 inches radius and 4 feet long. Find radius of gyration of the whole about the axis. *Ans.* $k = \sqrt{7 \cdot 125} = 2 \cdot 67$ ft.

5. The rim of a flywheel is rectangular in section, 6 inches wide, outside and inside radii 6 and 5 feet respectively. The nave is cylindrical, 2 feet long and 1 foot in diameter. There are eight cylindrical spokes of 4 inches diameter. Find the radius of gyration of the wheel. *Ans.* 4·8 ft.

6. Show that the kinetic energy of a body revolving with an angular velocity, ω, about a given axis is $\frac{1}{2} I \omega^2$, where I denotes the moment of inertia of the body with reference to the axis. A flywheel has a mass of 30 tons, which may be supposed to be distributed along the circumference of a circle 8 feet in radius; it makes 20 revolutions a minute; find its kinetic energy in foot-pounds. *Ans.* 295,000 ft.-lbs.

7. Find the moment of inertia of a rectangular lamina about an edge. A retangular lamina, whose shorter edges are 4 feet long, turns round one of its longer edges 50 times a minute. It weighs 441 lbs.; find its kinetic energy. *Ans.* 1008·3 ft.-lbs.

8. When a rigid body turns round an axis, what relation exists between its angular velocity and its kinetic energy? A rod of uniform density can turn freely round one end; it is let fall from a horizontal position; what is its angular velocity when it reaches its lowest position? Prove your equations. *Ans.* $\omega = \sqrt{\frac{3 g}{l}}$.

9. How do you estimate the total energy possessed by a body when moving with both translation and rotation? Find the velocity of the centre (1) when a hoop, (2) when a disc, and (3) when a sphere rolls down an inclined plane of height, h. *Ans.* (1) $v = \sqrt{g h}$, (2) $v = 2 \sqrt{\frac{g h}{3}}$, (3) $v = \sqrt{\frac{10 g h}{7}}$.

10. Sketch, and explain the principle of the action of, a fly-press for stamping metals. If a velocity of 5 feet per second is given to the balls of such a press, and their motion is stopped after the screw has made one-quarter of a turn from the time that the die touches the metal, the pitch of the screw being ¼ inch; find the weight of the balls, so that the pressure exerted may be 4,000 lbs. *Ans.* 26·67 lbs. each.

11. Two weights of 100 lbs. each are placed at the ends of the arms of a fly-press, and are moving with a velocity of 12 feet per second. How many foot-pounds of work must be expended in bringing them to rest? Hence explain the mechanical action of the fly-press as a machine for punching or stamping metals. *Ans.* 450 ft. lbs.

12. In a fly-press there are two weights, each of 60 lbs., placed at the ends of an arm which drives the screw; and the velocity of each weight at the instant of striking the blow is 10 feet per second. The die at the end of the screw moves through $\frac{1}{16}$ inch in coming to rest; what mean statical pressure does it exert on the metal subjected to the operation of stamping? *Ans.* 22,500 lbs.

13. In a fly-press for stamping metals a ball of 70 lbs. is placed at each end of the lever attached to the head of the screw. At the moment of striking the blow the weights have a velocity of 550 feet per minute, and the die at the end of the screw indents the metal to a depth of $\frac{1}{16}$ inch before coming to rest. What would be the mean statical pressure exerted on the metal? *Ans.* 26,468·75 lbs.

14. Prove that the kinetic energy of a train of railway carriages moving with velocity, v, is $\left\{W + w\left(1 + \frac{k^2}{r^2}\right)\right\}\frac{v^2}{2g}$ ft. lbs., where w denotes the weight of the wheels and axles; W the weight of the rest of the train; r the radius of the wheels, and k the radius of gyration of a pair of wheels about their axis, the units being feet, lbs., and seconds. Determine the acceleration with which the train would freely descend an incline of inclination, α.

15. Describe and show by the necessary sketches the construction of a fly-press for punching holes in iron plates. In such a press the two balls weigh 30 lbs. each, and are placed at a radius of 30 inches from the axis of the screw, the screw itself being of 1 inch pitch. What diameter of hole could be punched by such a press in a wrought-iron plate of ½ inch in thickness; the shearing strength of the metal being 22·5 tons per square inch? (Consider that the balls are revolving at the rate of 60 revolutions per minute when the punch comes into contact with the metal, and that the resistance of the plate is overcome in the first sixteenth of an inch of the thickness of the plate.) (S. & A. Adv. Exam., 1896.) *Ans.* 1·12 ins.

16 A pendulum bob weighing 20 lbs. is suspended by a wire, the length from the point of suspension to the centre of the bob being 16 feet. The pendulum swings through an angle of 30° on each side of the vertical; find its potential energy when in the highest position, and its velocity when passing the lowest point. (S. & A. Adv. Exam., 1895.) *Ans.* 42·88 ft. lbs.; 11·71 ft. per second.

17. A flywheel weighs 10,000 lbs., and is of such a size that the matter composing it may be treated as if concentrated on the circumference of a circle 12 feet in radius; what is its kinetic energy when moving at the rate of 15 revolutions a minute? How many turns would it make before coming to rest if the steam were cut off and it moved against a friction of 400 lbs. exerted on the circumference of an axle 1 foot in diameter? *Ans.* 55,520 ft.-lbs.; 44·2 turns.

18. The sectional area of the rim of a cast-iron flywheel is 12 square

inches and the mean radius (or radius of gyration) is 25 inches; what is the kinetic energy at 150 revolutions per minute? What moment of constant magnitude, and acting through one-quarter revolution, would increase the speed to 155 revolutions per minute at the end of the quarter revolution? What would be the length of a solid wrought-iron shaft, 5 inches in diameter, rotating at the same speed and having the same kinetic energy? *Ans.* 35,707,000 ft.-lbs.

19. Prove the formula for the energy stored up in a flywheel on the supposition that the whole of the material is collected in a heavy rim of given mean radius. Apply the formula to show (1) the effect of doubling the number of revolutions per minute; (2) the effect of doubling the weight; (3) the effect of increasing the mean radius in the proportion of 3 to 2.

20. The rim of a flywheel weighs 9 tons, and the mean linear velocity of its mass is assumed to be 40 feet per second; how many foot-tons of work are stored up in it? If it be required to store the additional work of 9 foot-tons, what should be the increase of velocity? *Ans.* 225 ft.-tons; 0·79 ft. per second.

21. A flywheel weighs 2½ tons, and its mean rim has a velocity of 40 feet per second. If the wheel gives out 10,000 foot-pounds of energy, how much is its velocity diminished? *Ans.* 1·455 ft. per second.

22. A flywheel weighing 5 tons has a mean radius of gyration of 10 feet. The wheel is carried on a shaft of 12 inches diameter and is running at 65 revolutions per minute; how many revolutions will the wheel make before stopping if the coefficient of friction of the shaft in its bearing is 0·065? (Other resistances may be neglected.) (S. & A. Adv. Exam., 1896.) *Ans.* 354·66 turns.

23. A particle of given mass moves with a given velocity in a circle of given radius; state what is known as to the force which acts on the particle. Prove the statement. (S. & A. Adv. Theor. Mechs. Exam., 1896.)

24. If a locomotive weighing 55 tons runs round a curve of 1,200 feet radius at 20 miles per hour, what is its centrifugal force? How much higher in level should the outer rail be laid than the inner rail in order that the resolved part of the weight of the locomotive should balance this centrifugal force without pressure being exerted by the outer rail, the gauge being 4 feet 8½ inches? *Ans.* 2760·6 lbs.; 1·27 inches.

25. Prove that a railway carriage running round a curve of radius, r, will upset if the velocity is greater than $\sqrt{\frac{g\,r\,a}{2\,h}}$, where a is the distance between the rails, and h the height of the centre of gravity of the carriage above the rails.

26. Show that by raising the outside rail of a railway track in going round a curve the tendency of the train to leave the rails is diminished, and that if θ be the inclination of the floor of the carriage to the horizontal, when there is no lateral pressure, $\tan\theta = \frac{v^2}{g\,r}$, where r is the radius of the curve, and v the velocity of the train. Hence show that on a 5-foot track, round a curve of one-eighth of a mile radius, that for a mean velocity of 30 miles an hour the outside rail ought to be raised 5½ inches above the level of the inside rail.

27. A body moves in a circle with a uniform velocity, show that it must be acted on by a constant force tending towards the centre, and find the magnitude of the force in terms of the radius of the circle, and of the

mass and velocity of the body. A body weighing $2\frac{1}{2}$ lbs. fastened to one end of a thread 4 feet long is swung round in a circle of which the thread is the radius; what will be its velocity when the tension of the thread is a force of 20 lbs.? ($g = 32$). *Ans.* 32 ft. per second.

28. A segment of a flywheel with the arm to which it is attached weighs 3,500 lbs., and the mass of the portion may be taken as collected at a distance of 8 feet from the axis of the wheel, which makes 40 revolutions per minute. What is the force tending to pull away the segment and arm from the boss of the wheel? You are required to write out a proof of the formula which you employ. *Ans.* 15,365 lbs.

29. Show that the stress per square inch on the rim of a flywheel is equal to the momentum of the amount of rim (per square inch of section) which passes a fixed point in the unit of time. Find the limiting speed of periphery, the material being such that a bar of uniform section 900 feet long may be supported by tension. *Ans.* $30 \sqrt{g\,r}$.

30. A flywheel 20 feet in diameter makes 80 revolutions per minute. Find the stress in its rim due to centrifugal forces, assuming that it is unaffected by the connection with the arms. The weight of a cubic foot of the material forming the rim is 500 lbs. What is the maximum speed at which the wheel can be safely run if the tensile strength of the material has not to exceed 6,000 lbs. per square inch? *Ans.* 762 lbs. per sq. in.; 224·5 revs. per min.

31. When the fly-wheel of a certain traction engine lessens in speed from 150 to 140 revolutions per minute, there is a loss of kinetic energy (on the motion of the whole engine as well as the fly-wheel) of 25,000 foot-pounds.

If the speed is 160 revolutions per minute, how far will the engine travel up an ascent of 1 in 100 before coming to rest, if engine and truck together weigh 30 tons, and there is a constant frictional resistance on a level road of 20 lbs. to the ton? (S. & A. Adv. Exam 1897.)

32. The centre of gravity of a body of 100 lbs. is revolving at 15 ins. from an axis, at 250 revolutions per minute. What is its centrifugal force? Prove the Rule. (S. & A. Adv. Exam., 1897.)

33. Describe experiments to compare the speed of a flywheel with the work given to it by a falling weight. If you have not made such experiments, so that the exact method of finding the speed, of correcting for friction, &c., are unknown to you, you had better not attempt this question. (S. and A. Adv., 1899.)

34. A flywheel is required to store 12,000 ft.-lbs. of energy as its speed increases from 98 to 102 revolutions per minute; what is its moment of inertia? The wheel is a solid disc of cast iron, its thickness one-tenth of its diameter, what is its diameter? Prove the formula you use for calculating its moment of inertia. *Ans.* 86,400 ft.-lbs.; 14·3 feet.

(B. of E. H., Part I., 1900.)

35. A flywheel of a shearing machine has 150,000 foot-pounds of kinetic energy stored in it when its speed is 250 revolutions per minute, what energy does it part with during a reduction of speed to 200 revolutions per minute? If 82 per cent. of this energy given out is imparted to the shears during a stroke of 2 inches, what is the average force due to this on the blade of the shears? (B. of E. Adv., 1902.)

36. A flywheel weighs 5 tons and its radius of gyration is 6·30 feet, what is its moment of inertia in engineers' units? It is at the end of a shaft 40 feet long, 5 inches diameter, modulus of rigidity of material 12×10^6 lb.-inches, what is the natural time of torsional vibration of the system, neglecting the inertia of the shaft itself? (B. of E. H., Part I., 1902.)

Lecture XXII.—A.M.Inst.C.E. Exam. Questions.

1. Explain the importance of elevating the exterior rail in railway curves. Calculate the proper slope for a circular bicycle track 100 feet in radius, to prevent any tendency to side-slip at a speed of 24 miles an hour. (I.C.E., *Oct.*, 1897.)

2. In a gas-engine using the Otto cycle the I.H.P. is 8 and the speed is 264 revolutions per minute. Treating each fourth single stroke as effective and the resistance as uniform, find how many foot-pounds of energy must be stored in the flywheel in order that the speed shall not vary by more than one fortieth of its mean value. (I.C.E., *Oct.*, 1897.)

3. A flywheel supported on a horizontal axle 2 inches in diameter is pulled round by a cord on the axle, carrying a weight. It is found that a weight of 4 lbs. is just sufficient to overcome the friction. A further weight of 16 lbs., making 20 in all, is applied, and after 2 seconds (starting from rest) it is found that the weight has gone down 12 feet. Find the moment of inertia of the wheel. (I.C.E., *Oct.*, 1897.)

4. A brake wheel 4 feet in diameter on a horizontal axle is furnished with internal flanges, which, along with the rim, form a trough containing cooling water. What is the least speed that will prevent the water from falling? (I.C.E., *Feb.*, 1898.)

5. A flywheel alters in speed from 99 to 101 revolutions per minute, when its kinetic energy alters by the amount of 500,000 foot-lbs.; what is its moment of inertia? What is its kinetic energy when making 1 revolution per minute? (I.C.E., *Oct.*, 1898.)

6. Find the radius of gyration in a hollow cylindrical column with an external diameter of 12 inches and a thickness of 1 inch. Also in a solid square column 4 inches by 4 inches. (I.C.E., *Oct.*, 1898.)

7. A small flywheel is mounted with its axle vertical. Over a small pin on the axle is hooked the looped end of a stout cord, and the wheel is turned round until there are ten coils of the cord on the axle. The cord is led over a small pulley and a weight of 20 lbs. attached to its end. This weight is now allowed to fall, giving motion to the flywheel. It is found that after the lapse of 8 seconds the cord slips off the pin, the weight having fallen 4 feet during this time. During the fall the force of gravity has done work; in what form does this appear at the instant the cord leaves the axle of the wheel? Suppose that 85 per cent. of this energy is stored in the flywheel, calculate its moment of inertia. Describe the nature of the experiments you would make to measure any loss of mechanical energy which occurs. (I.C.E., *Feb.*, 1899.)

8. Find the moment of inertia of a circular disc about a tangent, and of a square disc about an axis through one corner perpendicular to its plane. (I.C.E., *Oct.*, 1899.)

9. Define moment of inertia. Find the moment of inertia of an equilateral triangle about its base and about an axis through its apex perpendicular to its plane. (I.C.E., *Feb.*, 1900.)

10. A cast-iron flywheel 10 feet in diameter, with a rim 6 inches by 6 inches, is rotating freely on a shaft at the rate of 100 revolutions per minute. A brake, which exerts a frictional retardation of 100 lbs., is applied to its rim for 20 seconds. Find how much the speed of the flywheel is reduced (neglecting the weight of its arms). (I.C.E., *Feb.*, 1900.)

11. Define "Moment of inertia," and prove the formula which expresses the kinetic energy stored in a body rotating about a fixed axis. How much energy is stored in a 3-foot thin rod weighing 4 lbs., and which is re-

volving at 140 revolutions per minute about an axis through its centre and perpendicular to its length? (I.C.E., *Oct.*, 1900.)

12. The flywheel of an engine of 4 H.P. running at 75 revolutions per minute is equivalent to a heavy rim 2 feet 9 inches mean diameter and weighing 500 lbs. Determine the maximum and minimum speeds of rotation when the fluctuation of energy is one-fourth the energy of a revolution. (I.C.E., *Oct.*, 1900.)

13. A small cast-iron flywheel weighs 96 lbs. It is mounted between conical bearings, and so arranged that a falling weight causes it to rotate, the weight being at the end of a cord, which is wound round the spindle of the wheel. The falling weight is 21 lbs., and acts upon the wheel during a fall of $3\frac{1}{4}$ feet, its velocity at the end of the fall being 0·595 foot per second. If the work done in overcoming the friction of the wheel spindle during the fall of the weight is 12·726 foot-lbs., find the kinetic energy of the flywheel at the instant the falling weight ceases to act upon it. (I.C.E., *Feb.*, 1902.)

14. Define "centrifugal force." A locomotive engine weighs 38 tons, and travels round a curve of 800 feet radius at a speed of 50 miles an hour. Find the centrifugal force. Show also how to find: (*a*) the direction and magnitude of the resultant thrust on the rails due to the weight of the engine and the centrifugal force; (*b*) the height to which the outer rail should be raised over the inner rail, in order that the plane of the rails may be perpendicular to this resultant. Gauge of rails 4 feet $8\frac{1}{2}$ inches. (I.C.E., *Feb.*, 1902.)

15. Explain the use of a flywheel. Determine the weight of rim per horse-power which, when running at a speed of 70 feet per second, will have stored in it 10 per cent. of the energy developed per minute. (I.C.E., *Oct.*, 1902.)

16. Explain the meaning of the term "centrifugal force." With what speed must a locomotive be running on level railway lines, forming a curve of 968 feet radius, if it produce a horizontal thrust on the outer rail equal to $\frac{1}{8}$ of its weight? (I.C.E., *Feb.*, 1903.)

17. A factory is driven by an engine working at 100 I.H.P. One machine requiring 3 H.P. is thrown out of gear, and in two minutes the speed of the engine has increased from 40 to 43 revolutions per minute. The engine power remaining constant and the surplus work being accumulated in the flywheel rim, find the weight of the rim, its diameter being 15 feet. (I.C.E., *Feb.*, 1903.)

LECTURE XXIII.

CONTENTS. — Governing of Engines — Watt's Governor — Action of Watt's Governor—Theory of Watt's Governor—Conical Pendulum—Example I.—Common Pendulum Governor—Crossed-Arm Governor—Parabolic Governors—Galloway's Parabolic Governor—Porter's loaded Governor—Theory of Porter's Governor—Example II.—Spring loaded Governors — Proell's and Hartnell's Spring Governors — Pickering Governor—Willans' Spring Governor—Governing by Throttling and Variable Expansion — Shaft Governors — Relays — Knowles' Supplemental Governor—Inertia Governors—Thunderbolt's Marine Engine Governor—Thunderbolt's Electric Governor Regulator—Experiments upon the Action of Engine Governors — Proell Governor — Belliss Shaft Governor—Comparison of Different Governors—Results of Different Governors—To obtain the Controlling Force Graphically—Flywheels—Balancing Machinery—Weston Self-balancing Centrifugal Machine—Questions.

Governing of Engines.*—For many purposes to which engines are applied, it is necessary that they should maintain a uniform speed. Owing to variations of load and of pressure on the piston, they must have some regulating device, in order to accomplish this object. Fluctuations of the speed of a steam engine are of two kinds. (1) Those which occur during the time of a revolution, and are *periodic*, being caused by the varying pressure on the piston, and obliquity of the connecting rod. (2) Those which are due to change of load, or boiler pressure, and are *not periodic*. To control the first of these as far as possible, an engine is fitted with a **Flywheel**, and for the second a **Governor** is also required.

* The following is a list of books and papers treating of governors and governing:—

Paper on "The Electrical Regulation of the Speed of Steam Engines," by P. W. Willans. *Proc. Inst. C.E.*, 1885, vol. lxxxi., p. 166.

Paper on "A New Method of Investigation applied to the Action of Steam Engine Governors," by Prof. Dwelshauvers-Dery of Liége, translated by Michael Longridge. *Proc. Inst. C.E.*, 1888, vol. xciv., p. 210.

Paper on "The Cyclical Velocity-Variations of Steam and other Engines," by H. B. Ransom. *Proc. Inst. C.E.*, 1889, vol. xcviii., p. 357.

Paper on "The Application of Governors and Flywheels to Steam Engines," by Prof. Dwelshauvers-Dery, translated by Bryan Donkin. *Proc. Inst. C.E.*, 1891, vol. civ., p 196.

Paper on "Flywheels and Governors," by H. B. Ransom. *Proc. Inst. C.E.*, 1892, vol. cix., p. 330.

Paper on "Steam Engine Governors and their Insufficient Regulating Action with Extreme Variations of Load," by Prof. Dwelshauvers-Dery, translated by Bryan Donkin. *Proc. Inst. C.E.*, 1892, vol. cx., p. 276.

Paper on "A Method of Testing Engine Governors," by H. B. Ransom. *Proc. Inst. C.E.*, 1893, vol. cxiii., p. 194. Also, 1899, vol. cxxxvii., p. 376.

A governor is a piece of mechanism which regulates the amount of steam supplied to the engine, to suit the work it is doing, whereas, as explained in the previous Lecture, a fly-wheel acts in virtue of its inertia, so as to distribute throughout a whole revolution the energy developed in the cylinder. The governor can have no effect whatever on the periodic variations of speed, since it can only act during the time that steam is being admitted to the cylinder. With regard to the irregular fluctuations of speed, due to a change of load, the flywheel makes them more gradual and thus gives the governor time to act. A great many varieties of governors have been invented since the introduction of the steam engine, such as hydraulic, centrifugal, inertia, and electrical governors. By far the greatest number, however, depend for their action on centrifugal force and inertia, and since these form useful examples of the practical application of the principles enunciated in the previous Lectures, we shall now confine our remarks to such governors.

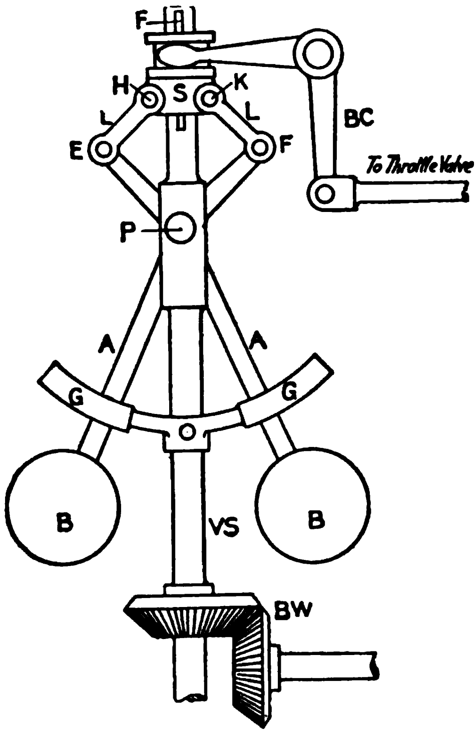
Watt's Governor.—One of Watt's important inventions was his conical pendulum governor, as applied to his double-acting engine.* This governor consists of two arms, A A, carrying heavy balls, B B, and pivoted on a pin, P, passing through the centre of the vertical spindle, V S. The upper ends of these arms are bent, as shown on the figure, and are connected by short links, L L, to the sleeve, S. This sleeve is free to move vertically on the spindle, V S, but is made to rotate with it by a feather, F, and corresponding keyway. This sleeve acts on one end of the bell crank, B C, and thus moves the rod con-

Paper on "The Mechanical and Electrical Regulation of Steam Engines," by John Richardson. *Proc. Inst. C.E.*, 1895, vol. cxx., p. 211.
Paper on "Governing of Steam Engines by Throttling and by Variable Expansion," by Capt. H. R. Sankey. *Proc. Inst. M.E.*, 1895, p. 154.
Paper on "Steam-Engine Governors," read before the Manchester Association of Engineers, by C. F. Budenberg, M.Sc. See *The Practical Engineer*, 17th April, 1891, vol. v., p. 258.
A series of articles on "Engine Governors," by R. G. Blaine, M.E., in *The Practical Engineer*, beginning 13th June, 1890, vol. iv., p. 386, and ending 24th April, 1891, vol. v., p 277.
Article on "A New Shaft Governor," by E. J. Armstrong, in *The Practical Engineer*, 26th July, 1895, vol. xii., p. 71.
Article on "Shaft Governors," by E. T. Adams, in the *Electrical World* of New York. July, 1896.
See Index for Governors in *Gas, Oil, and Air Engines*, by Bryan Donkin, published by Charles Griffin & Co.
The Steam Engine, by D. K. Clark (Blackie & Son), chap. v., on Governors, p. 65, half vol. iii.

* See the Author's *Text-Book on Steam and Steam Engines* for a description of Watt's engines. Also Lecture XIX., Volume I., of this book for an illustration of same.

nected to the throttle valve of the engine. The vertical spindle may be driven by the engine by means of a belt or rope passing round a pulley keyed on it, or by bevel wheels, as shown at B W. In order to relieve the pin, P, the arms are driven by the guides, G G, which are fixed to the vertical spindle.

Action of Watt's Governor.—The governor is so adjusted, that when the engine is working at its normal speed, the balls rotate

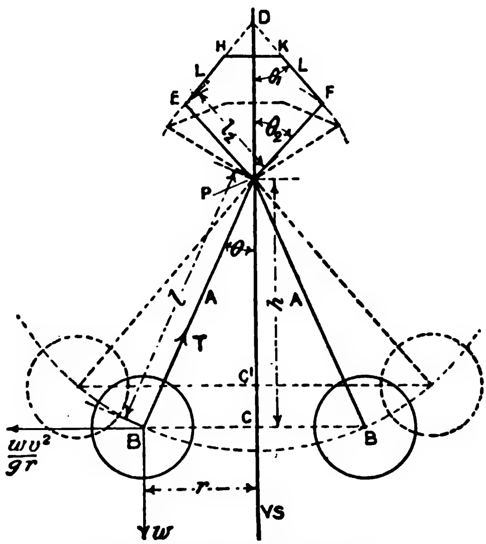


WATT'S PENDULUM GOVERNOR.

at a certain distance from the vertical spindle, and thus the throttle valve is kept sufficiently open to maintain that speed. Should the load be *decreased*, the speed of the engine, and therefore that of the governor balls, naturally becomes greater. This causes an increase of the centrifugal force of the balls, and therefore they diverge further, thereby pulling down the sleeve,

and partially closing the throttle valve, which diminishes the supply of steam and the power developed by the engine. On the other hand, should the load be *increased* the reverse action takes place, the balls come closer together, the sleeve is raised, the throttle valve opened wider, and more steam admitted to the engine. It will thus be seen that a change of speed must take place before the governor begins to act; further, that for any permanent change in the work to be done, there is a permanent alteration of speed. For each particular load on the engine, the throttle valve will be opened by a definite amount, which will be different for different loads, and each position of the valve has a corresponding position of the governor balls. But, as will be shown further on, each position of the balls corresponds to a definite speed, so that there will be a particular speed for each different load.

Theory of Watt's Governor—Conical Pendulum.—Let the balls



THEORY OF WATT'S GOVERNOR.

be rotating about the vertical spindle with a uniform velocity. Then the several forces acting on the different parts of the instrument are in equilibrium with each other. The arms, A, will describe the surface of a cone, B P B, whose height is P C, and for a given velocity of the balls there will be a definite

height of this cone. It will be sufficient to consider one ball and arm, since what is true for one will be true for the other.

Let w = Weight of one ball in lbs.
„ v = Velocity of balls in *feet per second.*
„ h = Height, P C, of cone in *feet.*
„ l = Slant height, P B, of cone in *feet.*
„ r = Radius, B C, of base of cone in *feet.*
„ T = Tension in one arm, A.

There are three forces acting on the ball, B, viz.:—

(1) The weight, w, of the ball acting vertically downward.

(2) The centrifugal force, $w v^2 \div g r$, acting in its plane of rotation, and in the direction C B.

(3) The tension in the arm, A, acting in the direction B P.

These three forces keep the ball in equilibrium, and can, therefore, be represented, in magnitude and direction, by the three sides of a triangle taken in order. If we draw a triangle, having its sides parallel or perpendicular to the directions of these forces, the lengths of the sides of this triangle will be proportional to the forces respectively. Now, such a triangle exists in the figure itself—viz., the triangle P C B—the sides of which are parallel to the three forces:—

Hence, $$h : r = w : \frac{w v^2}{g r} = 1 : \frac{v^2}{g r}.$$

$\therefore$ $$h = \frac{g r^2}{v^2}, \text{ and } \frac{r}{v} = \sqrt{\frac{h}{g}}.$$

If t = time in *seconds* of one complete revolution of balls,
n = number of revolutions per *second,*

Then, $$t v = 2 \pi r, \text{ and } n = \frac{1}{t}.$$

Substituting these in the previous equation we get the following important formulæ:—

$$= 2 \pi \frac{r}{v} = 2 \pi \sqrt{\frac{h}{g}}. \quad . \quad . \quad . \quad . \quad . \qquad \text{(I)}$$

That is, *the period of rotation is proportional to the square root of the height of the cone.*

Also, $$n = \frac{1}{t} = \frac{1}{2 \pi} \sqrt{\frac{g}{h}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(II)}$$

Or, if N be the number of revolutions per *minute* = 60 *n*,

Then, $$N = \frac{30}{\pi}\sqrt{\frac{g}{h}}. \qquad (II_a)$$

That is, *the number of revolutions, or the speed of the engine or governor, varies inversely as the square root of the height of the cone.*

Equation (II_a) may be written in this useful form :—

$$h = \frac{30^2 g}{\pi^2} \times \frac{1}{N^2} = \frac{2936}{N^2} \text{ feet.} \quad . \quad . \quad (II_b)$$

Or, *the height of the cone depends only on the speed of rotation, and varies inversely as the square of the number of revolutions.*

Let the speed of the governor be altered from N_1 to N_2 revolutions per minute, then the heights of the cone corresponding to these speeds are :—

$$h_1 = \frac{2936}{N_1^2}, \text{ and } h_2 = \frac{2936}{N_2^2}.$$

Therefore, for a change of speed from N_1 to N_2 revolutions per minute the height of the cone will be altered by the amount :—

$$h_1 \sim h_2 = 2936\left(\frac{1}{N_1^2} \sim \frac{1}{N_2^2}\right) = \frac{2936\,(N_2^2 \sim N_1^2)}{N_1^2\,N_2^2}. \qquad (III)$$

If, however, the height of the governor be kept constant, and equal to $h = \frac{900\,g}{\pi^2 N_1^2}$, the centrifugal force will change from $\frac{w\,v_1^2}{g\,r}$ to $\frac{w\,v_2^2}{g\,r}$, or from $\frac{w\,r\,\pi^2 N_1^2}{900\,g}$ to $\frac{w\,r\,\pi^2 N_2^2}{900\,g}$, and the difference will produce a tension, or a thrust, in the links L L. If T_2 be the tension, or thrust, in one link L; l, l_1, l_2 the lengths of B P, E D, P E; and θ, θ_1, θ_2 their inclinations to the vertical, then by taking moments about P, we have :—

$$T_2 \times l_2 \cos(\theta_1 + \theta_2 - 90) = \frac{w\,r\,\pi^2}{900\,g}(N_2^2 \sim N_1^2) \times h,$$

Or, $$T_2 = \frac{w\,r\,\pi^2 (N_2^2 \sim N_1^2)\,h}{900\,l_2\,g \sin(\theta_1 + \theta_2)}.$$

Now, the vertical force acting on the sleeve, which is available for overcoming friction, and may be called the *working effort* for that change of speed, is the vertical components of

the stresses in the two links L L. These two stresses are equal.

$$\therefore \textit{The working effort} = 2\,T_2 \cos \theta_1$$

$$\text{,,} \quad \text{,,} \quad = \frac{w\,r\,\pi^2\,h\,(N_2^2 \sim N_1^2)}{450\,l_2\,g} \times \frac{\cos \theta_1}{\sin (\theta_1 + \theta_2)}$$

$$\text{,,} \quad \text{,,} \quad = \frac{w\,r\,\pi^2 \,\dfrac{900\,g}{\pi^2 N_1^2}\,(N_2^2 \sim N_1^2) \cos \theta_1}{450\,l_2\,g \sin (\theta_1 + \theta_2)}$$

$$\text{,,} \quad \text{,,} \quad = \frac{2\,w\,r \cos \theta_1\,(N_2^2 \sim N_1^2)}{l_2 \sin (\theta_1 + \theta_2)\,N_1^2} \quad . \; . \; . \; . \; . \quad \text{(IV)}$$

It is usual for P E and E H to be made equal in length, and then $\theta_1 = \theta_2$ nearly, unless H K be great. In that case:—

$$\textit{The working effort} = \frac{2\,w\,r \cos \theta_2\,(N_2^2 \sim N_1^2)}{l_2 \sin 2\,\theta_2\,N_1^2}$$

$$\left.\begin{aligned} \therefore \quad \text{,,} \quad \text{,,} \quad &= \frac{w\,r\,(N_2^2 \sim N_1^2)}{l_2 \sin \theta_2\,N_1^2} \\ \text{Or,} \quad \text{,,} \quad \text{,,} \quad &= \frac{w\,l \sin \theta\,(N_2^2 \sim N_1^2)}{l_2 \sin \theta_2\,N_1^2} \end{aligned}\right\} \quad . \; . \; . \; . \quad \text{(IV}_a\text{)}$$

If, further, $\theta = \theta_2$, which will always be the case when the sleeve is attached to the arm below the point of suspension, as in the next form of governor, then:—

$$\textit{The working effort} = \frac{w\,l\,(N_2^2 \sim N_1^2)}{l_2\,N_1^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(IV}_b\text{)}$$

It should be noted, however, that this is the effort exerted by the governor when it is just starting to move. The working effort becomes smaller and smaller as the balls rise, until, when the balls have attained the position corresponding to the new speed, it is *nil.*

The movement of the sleeve, corresponding to an alteration in the height of the cone, is best determined graphically by drawing the centre lines of the arms and links to scale for different positions of the balls.

EXAMPLE I.—Find the rise of the balls of a pendulum governor, when its speed is increased from 60 to 62 revolutions per minute. Find also the height of the cone of revolution at the lower speed, and the working effort, if the balls weigh

22 lbs. each, and each arm is jointed to the link at two-thirds of its length below its point of suspension.

Here, $N_1 = 60$, $N_2 = 62$, rise of balls $= h_1 - h_2$, $w = 22$, and $\frac{l}{l_2} = \frac{3}{2}$.

Therefore, from equation (III) we get:—

$$h_1 - h_2 = 2936\left(\frac{N_2^2 - N_1^2}{N_1^2 \times N_2^2}\right) = 2936\left(\frac{62^2 - 60^2}{60^2 \times 62^2}\right).$$

$$\therefore \quad h_1 - h_2 = \cdot 0518 \text{ foot or } \cdot 62 \text{ inch.}$$

Also from equation (II_b):—

$$h_1 = \frac{2936}{N_1^2} = \frac{2936}{3600}.$$

$$\therefore \quad h_1 = \cdot 816 \text{ foot} = 9 \cdot 79 \text{ inches.}$$

And from equation (IV_b):—

$$\textit{The working effort} = \frac{w\,l\,(N_2^2 \sim N_1^2)}{l_2\,N_1^2}$$

$$\text{,,} \quad \text{,,} \quad = \frac{22 \times 3\,(62^2 - 60^2)}{2 \times 60^2}$$

$$\text{,,} \quad \text{,,} \quad = 2 \cdot 237 \text{ lbs.}$$

In this case, as we assume $l_1 = l_2$ and $\theta_1 = \theta_2$, the travel of the sleeve will be twice the rise of the point where the link joins the arm, and this will be two-thirds of the alteration in height.

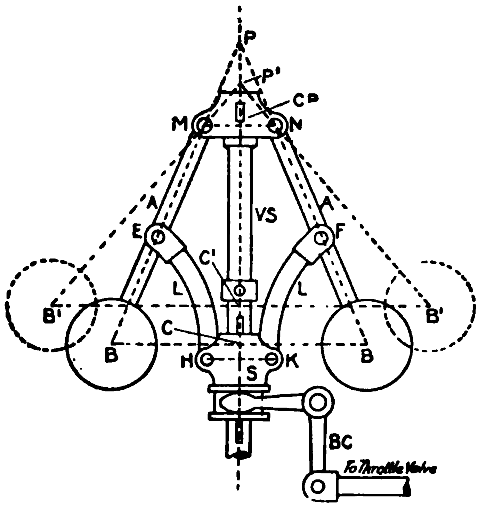
$$\therefore \textit{ Travel of sleeve} = 2 \times \frac{2}{3}(h_1 - h_2) = \frac{4}{3} \times \cdot 62 = \cdot 827 \text{ inch.}$$

Common Pendulum Governor.—A common modification of Watt's governor is shown by the following figure. Here, the arms A A, carrying the balls B B, instead of being jointed together by a pin passing through the vertical spindle V S, are pivoted at M and N to a cross-piece, C P, which is rigidly connected to the spindle. The links L L, carrying the sleeve S, are attached to the arms at the points E and F.

The formulæ deduced for Watt's pendulum governor are equally applicable to this case. The only thing requiring special attention here, is the height of the cone of revolution. The vertex of the cone is always at the point where the centre lines of the arms meet. In this case, the arms terminate at M and N, which are at a short distance from V S, and thus the vertex of the cone will be a variable point on the centre line of

the vertical spindle. When the balls are in the position shown by the full lines, the vertex is at P, and the height of the cone is P C; but, when the balls move into the new position, shown by dotted lines, the vertex of the cone is at P′, and the height of the cone is P′ C′.

The effect of suspending the arms at a short distance from the vertical spindle, is to cause the movement of the sleeve to be less for a given variation in the height of the governor, than would be the case were the centres of suspension in the vertical spindle. It will be apparent from the figure, that the effective variation of height is $CC' = (PC - P'C') - PP' = (h_1 - h_2) - PP'$, instead of $h_1 - h_2$, as in the previous case. Hence,



COMMON PENDULUM GOVERNOR.

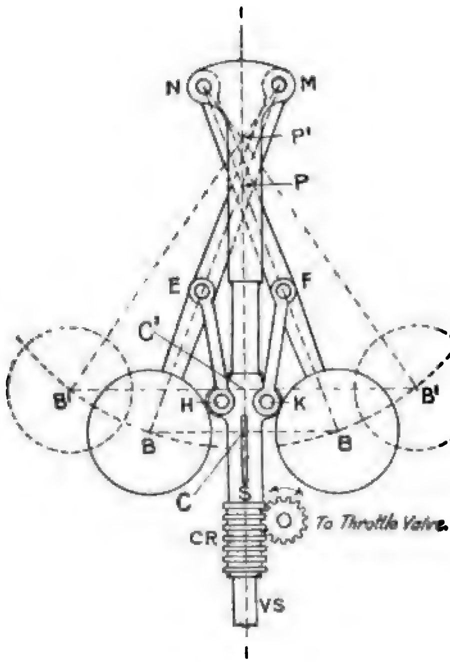
this governor is *less sensitive* than the former, since the speed must vary between greater limits for a given movement of the sleeve. The *sensitiveness* of a governor depends on the movement given to the sleeve for a given variation in speed, and also on the smallness of the time taken by the governor in adapting itself to its new position. In order, therefore, to increase the sensitiveness of this form of governor it is necessary that the points M and N should be as near the vertical spindle as possible.

Crossed-Arm Governor.—The special feature in which this governor differs from the former ones is, that the arms are

suspended from pins placed on the opposite sides of the spindle to that of their balls. From an inspection of the figure, it will be apparent that crossing the arms in this manner causes a greater movement of the sleeve for a given variation in the height than in the pendulum governor. The rise of the balls in this case is :—

$$C\,C' = P\,C - P'\,C' + P\,P' = (h_1 - h_2) + P\,P'.$$

The sensitiveness of this governor is therefore much greater than either of the two previous forms. By properly propor-



CROSSED-ARM GOVERNOR.

tioning the lengths of the arms and N M, so that the balls move out and in, along a curve which is approximately a parabola, this governor may be made almost isochronous, and, therefore, extremely sensitive.* It will be noticed that the sleeve of the

* A governor is said to be *isochronous* when its speed of rotation (and, therefore, the height of the cone) is the same for all positions of the balls within its range.

governor shown, has a circular rack C R, which gears with a pinion on the throttle valve spindle.

Parabolic Governors.*—Governors have been so constructed that their balls were guided to move in a truly parabolic path, and thus be absolutely isochronous, but owing to their complication they have not come into general use. With any centrifugal governor, the speed must increase somewhat before the extra centrifugal force is able to overcome the friction resisting the motion of the links, sleeve, valve, &c., and if it be absolutely isochronous, whenever the friction is overcome the balls would rise right up to the top of their range, and remain there until the speed has fallen sufficiently for gravity to reassert itself and overcome the friction, which would now tend to keep the balls up. They would then come down to the bottom of their range, and there would thus be continual hunting. Such a governor would therefore be wanting in stability or steadiness.

LOADED PARABOLIC GOVERNOR, BY GALLOWAYS, LD.

Galloway's Parabolic Governor.—From the illustration it will be seen, that in this type two cylindrical rollers take the place of the ordinary balls in the previously mentioned governors. These rollers are suspended at each end by links from a crosshead fixed to the top of the governor spindle, and naturally rise and fall in circular arcs with these links as radii. They move along parabolic slots cut in a weight W, which rotates with the spindle, but is free to rise and fall along the same. By this arrangement, the moment of the centrifugal force of the rollers is balanced by that of the weight at nearly the same speed for all positions. Hence, this governor may be considered practically insochronous. To the bottom of the slotted weight there is sometimes attached a sleeve termi-

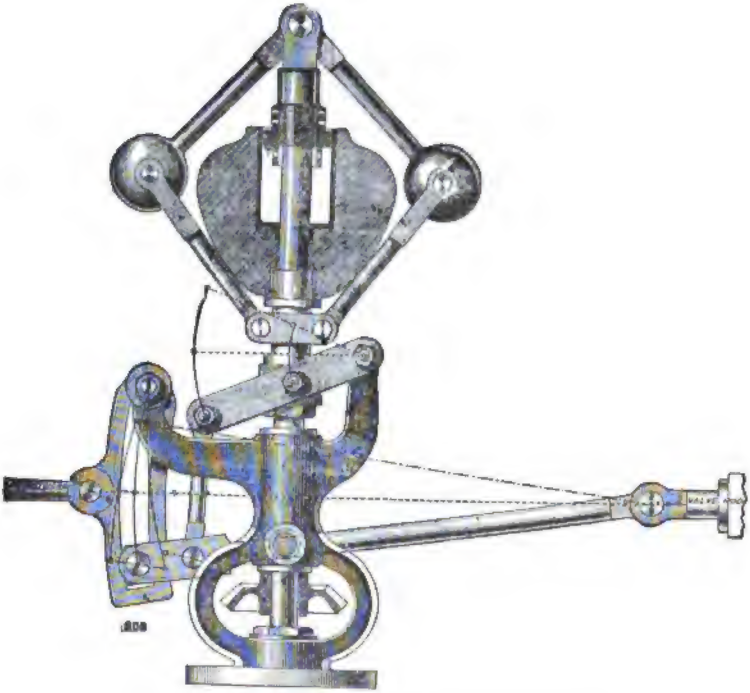
* See the Appendix to *The Steam Engine*, by Prof. Rankine (Chas. Griffin & Co.), and Chapter XV. of *Practical Treatise on the Steam Engine*, by Arthur Rigg (E. & F. Spon), for descriptions of guided parabolic governors.

nating in a collar, which engages the forked lever connected to the throttle valve, or expansion gear; but in this case, a central spindle C S, which is carried up inside the main governor spindle, rises and falls with the weight W, and acts directly on an equilibrium valve. The governor is driven through gearing contained in the cast-iron box seen at the foot of the vertical column.

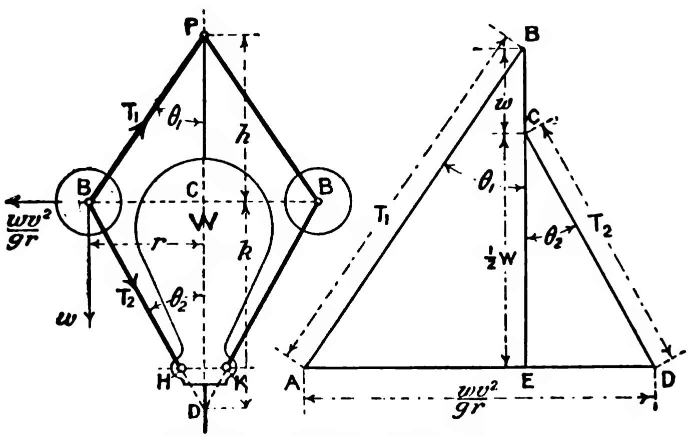
Porter's Loaded Governor.—From equation (IV) and Example I. we see that the simple pendulum governors possess a comparatively small working effort, unless the balls are very heavy. To overcome this objection Porter made the balls smaller, and loaded the sleeve with a heavy weight. This increases the height of the cone, corresponding to any particular speed, and all the forces concerned, and thus gives a greater working effort. It can be used both in connection with throttle valves and some forms of expansion gear. To minimise the oscillations of the ordinary Porter governor, Messrs. Clayton & Shuttleworth have made a cylindrical hole in the top of the central weight, and fixed a piston on the vertical spindle, thus forming a simple air cushion.

PORTER LOADED GOVERNOR, BY TANGYES, LIMITED.

Theory of the Porter Governor.—Each of the balls is in equilibrium under the action of four forces acting in a plane passing through the axis of rotation. These forces are:—(1) the weight of the ball w, (2) the centrifugal force $w\,v^2 \div g\,r$, and (3) the tensions in the two links, T_1 and T_2. Let A B C D A be a polygon representing these forces, A B being parallel and equal to T_1, B C to w, C D to T_2, and D A to the centrifugal force. If B C be produced to meet A D in E, then C E is equal to the vertical component of T_2, and must therefore be equal to half the load W, since this weight is supported by the vertical components of the tensions in the two bottom links.



PORTER GOVERNOR, BY CLAYTON & SHUTTLEWORTH.



ACTION OF THE PORTER GOVERNOR.

If the inclination of the links B P, B H to the vertical, be θ_1 and θ_2 respectively, and the heights C P, C D be denoted by h and k, then, the other letters being the same as before, we have :—

$$T_1 \sin\theta_1 + T_2 \sin\theta_2 = \frac{w v^2}{g r}.$$

$$T_1 \cos\theta_1 = w + \tfrac{1}{2} W, \text{ or } T_1 = \frac{w + \frac{1}{2} W}{\cos\theta_1}.$$

And,
$$T_2 \cos\theta_2 = \tfrac{1}{2} W, \qquad \text{or } T_2 = \frac{\frac{1}{2} W}{\cos\theta_2}.$$

$$\therefore \frac{w + \frac{1}{2} W}{\cos\theta_1}\sin\theta_1 + \frac{\frac{1}{2} W}{\cos\theta_2}\sin\theta_2 = \frac{w v^2}{g r}.$$

But, $\tan\theta_1 = \frac{r}{h}$; $\tan\theta_2 = \frac{r}{k}$; and $v = 2\pi r n$.

$$\therefore \quad (w + \tfrac{1}{2} W)\frac{r}{h} + \tfrac{1}{2} W \frac{r}{k} = \frac{w \times 4\pi^2 r^2 n^2}{g r}.$$

Or,
$$(w + \tfrac{1}{2} W)\frac{1}{h} = \frac{w \times 4\pi^2 n^2}{g} - \frac{\frac{1}{2} W}{k}.$$

$$\therefore \quad h = \frac{w + \frac{1}{2} W}{\dfrac{4\pi^2 n^2 w}{g} - \dfrac{\frac{1}{2} W}{k}}.$$

Hence,
$$h = \frac{(2w + W) g k}{8\pi^2 n^2 w k - W g}$$

Or, since $n = \frac{N}{60}$,
$$h = \frac{450 (2w + W) g k}{\pi^2 N^2 w k - 450 W g} \quad \text{(V)}$$

This governor is usually constructed with all four links of equal length; then $k = h$, and $\theta_1 = \theta_2 = \theta$, very nearly, unless the distance H K is great, and in our further investigations we shall assume that this is so.

In this case we have :—

$$T_1 \sin\theta + T_2 \sin\theta = \frac{w v^2}{g r}.$$

$$T_1 \cos\theta = w + \tfrac{1}{2} W, \text{ or } T_1 = \frac{(w + \frac{1}{2} W)}{\cos\theta},$$

And, $T_2 \cos\theta = \frac{1}{2} W$, or $T_2 = \dfrac{\frac{1}{2} W}{\cos\theta}$

$\therefore$ $(w + \frac{1}{2} W) \tan\theta + \frac{1}{2} W \tan\theta = \dfrac{w v^2}{g r}$.

But, $\tan\theta = \dfrac{r}{h}$ and $v = 2 \pi r n$.

Hence, $(w + W)\dfrac{r}{h} = \dfrac{w \times 4 \pi^2 r^2 n^2}{g r}$.

$$\left.\begin{aligned} \therefore \quad h &= \frac{w + W}{w} \times \frac{g}{4 \pi^2 n^2}, \\ \text{Or,} \quad h &= \frac{w + W}{w} \times \frac{900 g}{\pi^2 N^2} = \frac{w + W}{w} \times \frac{2936}{N^2} \end{aligned}\right\} \text{(VI)}$$

We might also have arrived at this result by putting h for k in equation (V).

If the speed of rotation change from N_1 to N_2 revolutions per minute, the corresponding heights of the governor will be:—

$$h_1 = \frac{w + W}{w} \times \frac{900 g}{\pi^2 N_1^2}; \qquad \text{and } h_2 = \frac{w + W}{w} \times \frac{900 g}{\pi^2 N_2^2}.$$

The alteration in the height of the cone of revolution would therefore be:—

$$\left.\begin{aligned} h_1 \sim h_2 &= \frac{w + W}{w} \times \frac{900 g}{\pi^2} \times \left(\frac{N_2^2 \sim N_1^2}{N_1^2 N_2^2}\right) \\ \text{Or,} \quad h_1 \sim h_2 &= 2936 \, \frac{w + W}{w} \left(\frac{N_2^2 \sim N_1^2}{N_1^2 N_2^2}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad \text{(VII)}$$

With the arrangement of links usually adopted in this governor, the travel of the sleeve is twice the change in the height of the balls and equal to $2 \, (h_1 \sim h_2)$.

Using the simpler equation (VI) we see that:—

$$W = \frac{\pi^2 N^2 w h}{900 g} - w.$$

And, therefore, if the speed alter from N_1 to N_2 revolutions per minute, the load necessary to keep the height of the cone constant, and equal to $h, \left(= \dfrac{w + W}{w} \times \dfrac{900 g}{\pi^2 N_1^2}\right)$, will change from—

$$W = \frac{\pi^2 N_1^2 w h}{900 g} - w \text{ to } W' = \frac{\pi^2 N_2^2 w h}{900 g} - w,$$

But if the actual load be W, the difference $W' \sim W$ is available for overcoming friction and moving the valve. This may be called the *working effort* for that change in speed, and is equal to:—

$$W' \sim W = \frac{\pi^2 w h}{900 g} (N_2^2 \sim N_1^2)$$

$$\text{Or,} \quad W' \sim W = \frac{\pi^2 w \dfrac{w + W}{w} \times \dfrac{900 g}{\pi^2 N_1^2} (N_2^2 \sim N_1^2)}{900 g}$$

$$\therefore \quad W' \sim W = (w + W) \frac{N_2^2 \sim N_1^2}{N_1^2}. \quad \ldots\ldots \quad \text{(VIII)}$$

Example II.—The balls of a Porter governor weigh 4 lbs. each, and the central weight 36 lbs. If all the links are of equal length, find the height of the governor when revolving 240 times per minute. If the speed increase to 248 revolutions per minute what will be the working effort and the rise of the balls and the sleeve?

Here, $N_1 = 240$; $N_2 = 248$; $w = 4$; and $W = 36$.

From equation (VI), $$h_1 = \frac{w + W}{w} \times \frac{2936}{N_1^2},$$

Or, $$h_1 = \frac{4 + 36}{4} \times \frac{2936}{240 \times 240}.$$

$$\therefore \quad h_1 = \cdot 51 \text{ foot or } 6{\cdot}12 \text{ inches.}$$

From equation (VIII), *The working effort*, $$= (w + W) \frac{N_2^2 - N_1^2}{N_1^2},$$

" " $$= (4 + 36) \frac{248^2 - 240^2}{240^2},$$

" " $$= 2{\cdot}71 \text{ lbs.}$$

And from equation (VII), $$h_1 - h_2 = 2936 \frac{w + W}{w} \left(\frac{N_2^2 - N_1^2}{N_1^2 N_2^2} \right),$$

Or, $$h_1 - h_2 = 2936 \times \frac{4 + 36}{4} \times \frac{248^2 - 240^2}{240^2 \times 248^2}$$

$$\therefore \quad h_1 - h_2 = \cdot 0324 \text{ foot or } \cdot 389 \text{ inch.}$$

$$\therefore \quad \textit{Travel of sleeve} = 2 (h_1 - h_2) = 2 \times \cdot 389,$$

" $$= \cdot 778 \text{ inch.}$$

On comparing these results with those of Example I., it will be noticed that while both governors weigh about the same, the loaded governor has a working effort of about 20 per cent. greater than that of the Watt governor, but its travel and height are less. It will also be seen from equations (III) and (VII), by putting $N_2 = cN_1$, where c is a constant, that the change in height corresponding to a given percentage variation in speed gets smaller as the speed increases.

Spring Loaded Governors.—Soon after the introduction of the Porter governor, Mr. John Richardson, of Messrs. Robey & Co., Lincoln, designed one in 1869, in which a spring was substituted for the weight. This improvement produces a greater working effort with less weight, bulk, and cost. A spring has less inertia, and acts much more quickly than a weight, and it has also a certain amount of cushioning action. A governor loaded with a spring can act in any position, whereas one with a weight must work vertically. The equations for a spring governor may be obtained in the same way as for the weighted governor, but the load W will be different for different positions of the balls, owing to the varying compression of the spring.

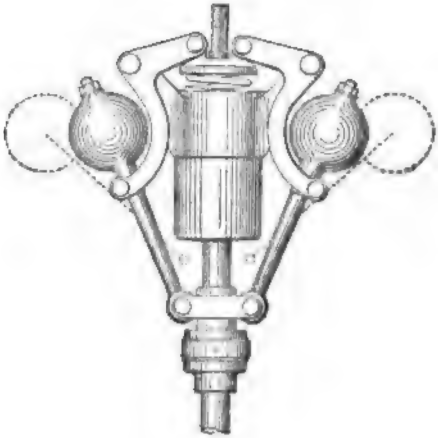
Proell and Hartnell's Spring Governors.*—The first of the two following figures illustrates the well-known Proell spring governor. It will be observed that a helical spring, contained in a cylindrical case, surrounds the governor spindle, and bears upon the inner ends of the two bell crank levers, which are connected to the arms carrying the governor balls. The dotted lines show the positions of the balls for a speed above the normal. As they move out to this position the spring is compressed and the sleeve is raised. It will further be noticed that the links are so proportioned that the balls diverge in nearly a straight line. Consequently, when working vertically, the balls do not move either with or against gravity.

It will be observed that there are no less than three pin joints on each side of the Proell governor above the sleeve, at each of which there must be friction. In Hartnell's governor, illustrated by the next figure, there is but one.†

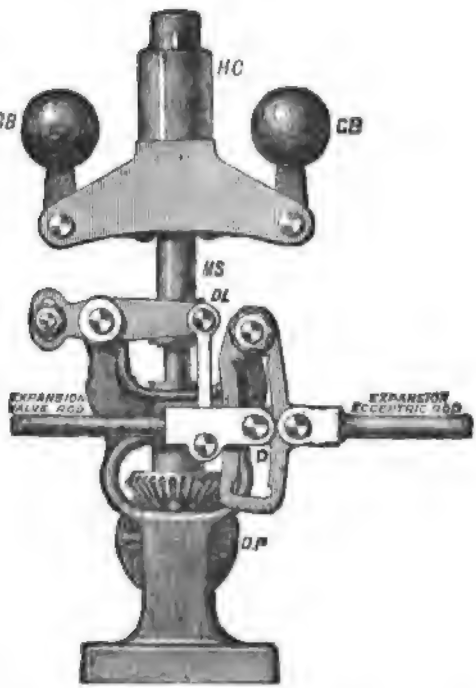
Here, the governor balls are fixed directly to the outer ends of the bell crank levers, the inner ends of which bear upon a collar on the upper end of the movable tube or sleeve, M S.

* The figure of Proell's governor is from *The Proc. Inst. C.E.*, vol. cxx., Session 1895-96, by kind permission of the Council, from a paper read by John Richardson, M.Inst.C.E., on "The Mechanical and Electrical Regulation of Steam Engines," which the student should consult.

† See Lecture XVIII. of the Author's *Text-Book on Steam and Steam Engines* for a description of an engine to which this governor is applied.



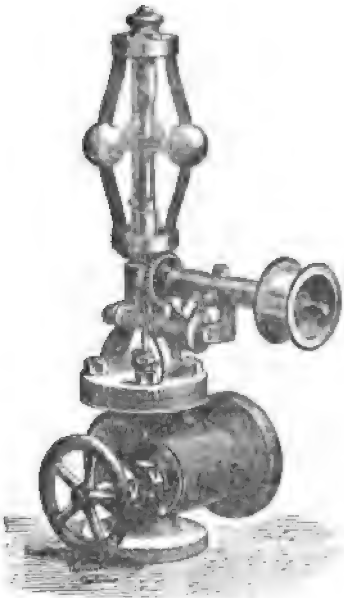
PROELL'S SPRING GOVERNOR.



HARTNELL'S SPRING GOVERNOR, BY MARSHALL, SONS & CO., LTD.

Between the top of this collar and the upper end of the hollow casting H C, which is keyed to the top of the governor spindle, there is placed a strong helical spring. The lower end of M S has a double collar engaged by a forked lever, connected to the drag link D L, and expansion valve rod.

Pickering Governor.—A very simple and direct acting governor which has been introduced for small electric light engines is shown by the next figure. Here the balls are supported by flat springs, which act directly on the throttle-valve spindle. There is also an auxiliary spring, as seen just



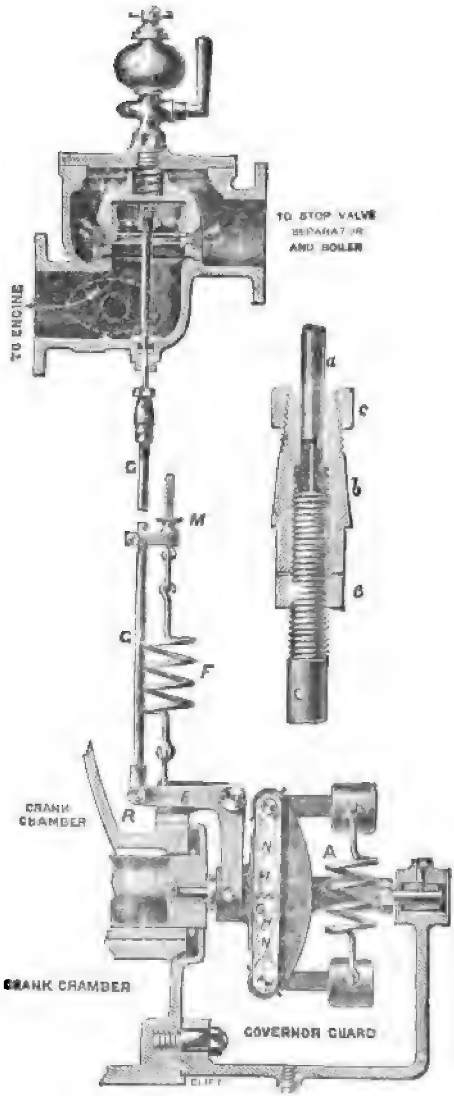
PICKERING GOVERNOR, BY TANGYES, LTD.

below the driving spindle, actuated by a thumb screw and worm wheel, which enables the attendant to adjust the speed of the engine whilst running.

Willans' Spring Governor.*—In the previous cases, the pressure of the spring has to be transmitted through the pin joints of the

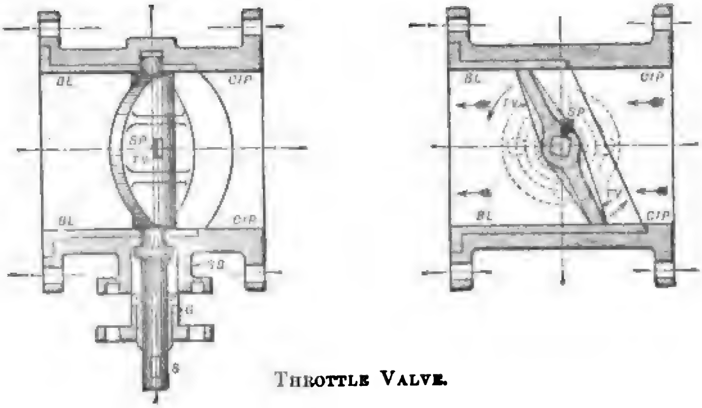
* For a description of Willans' central-valve triple expansion engine, to which this governor is fitted, see the Author's *Text-Book on Steam and the Steam Engine*.

governor arms, and thereby causes more friction and wear and tear than would be the case if the springs were directly connected to the balls. In Willans' governor, as will be seen from the figure, the balls are connected directly by a helical spring A, on each side of the governor spindle. Another spring F, is clamped at its upper end to the throttle-valve spindle G, and hooked at its lower end to the bracket carrying the bell crank lever E. This spring pulls the valve rod downwards, in opposition to the springs A, and thus pushes the sleeve against the toes N N (shown dotted), of the governor arms. By adjusting the tension in F, by the nut M, the governor can be set to the required speed while the engine is running. It will be noticed that this governor works horizontally, and is driven directly by one end of the shaft.



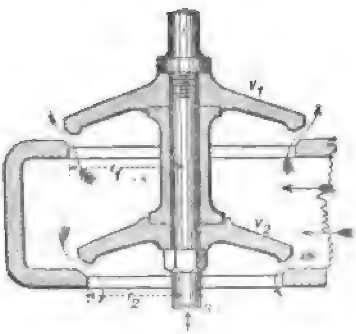
WILLANS' SPRING LOADED GOVERNOR.

Governing by Throttling and Variable Expansion.—Prior to 1876, governors generally controlled steam engines by actuating a butterfly throttle valve of the form shown in the figure. This valve, although simple in construction, is difficult to fit so as to remain steam tight, and hence the double-beat valve shown in



THROTTLE VALVE.

next illustration, or still better, a grating piston valve like that shown attached to the Willans' governor, has been adopted in preference. The ordinary butterfly throttle valve is not, as at one time supposed, a balanced valve, since the action of a fluid rushing past an oblique plane is such, as to cause a greater pressure on the forward edge and thus tend to close the valve. A good throttle valve should be able to entirely stop the admission of steam to the cylinder.



DOUBLE-BEAT VALVE.

Recently, many patents have been taken out for controlling the speed of an engine by altering the point of cut-off. In most cases, this enables the engine to work more economically; but as shown by Captain Sankey in his paper on "Governing of Steam Engines by Throttling and by Variable Expansion" (read before the Institute of Mechanical Engineers, in April, 1895), the indicator diagrams obtained from engines governed by this method are often "cloaks for exaggerated initial con-

densation," and it may be found that the actual feed water used, is less with ordinary throttling than with variable expansion. Throttling the steam varies the amount supplied by varying the pressure, while the volume used remains constant. On the other hand, automatic expansion supplies the steam at a constant pressure but alters the volume used per stroke.

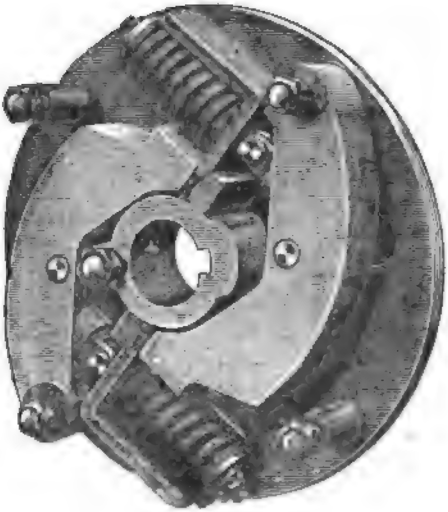
The point of cut-off may be controlled either by means of a separate expansion valve, or by acting directly on the main valve or valves. In the first case, there are two eccentrics which work the main and the expansion valves. As will be seen from the illustrations of the Hartnell, and Clayton & Shuttleworth's governors, the stroke of the expansion valve is altered by a drag link and a block connected to the governor sleeve. In the second case, when a slide valve is used, either the throw or the position of the main eccentric is varied by a shaft governor, and no second eccentric is required. With "trip gear" the governor automatically releases the admission valves sooner or later, according to the load on the engine.*

Shaft Governors.—A large number of these have been designed, but the following illustrations will serve to show their general principle and action. A circular casting is keyed to the crank shaft, and carries on one side a pair of symmetrically arranged weights jointed thereto at one end, but whose other ends are free to move in a plane perpendicular to the shaft against the resistance of the interposed helical springs. On the other side of this casting there is fixed a pair of straps embracing a circular disc carrying the eccentric which works the valve. The centre of this disc is some distance from the centre of the shaft and that of the eccentric. The governor weights have bosses which pass through slots in the circular casting, and are connected by links to studs on the disc. In moving outwards by centrifugal force, these weights compress the springs and rotate the disc, thus changing the position of the eccentric, and varying the cut-off of the slide valve.

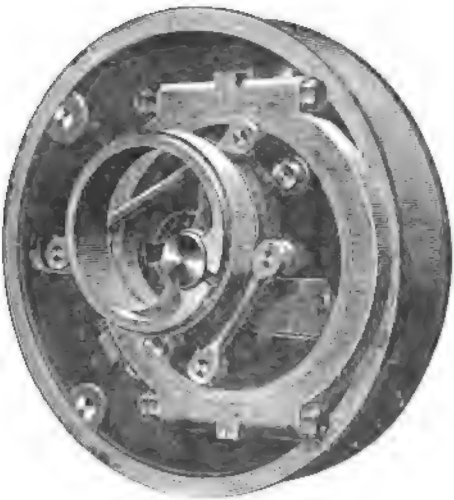
Relays.†—Except in the case of "trip gear," the effort required to work the throttle valve, or expansion gear, may be considerable, and can only be satisfactorily supplied by a relay—that is, by making the engine itself, or steam from the boiler, or water pressure, or electro-magnetic mechanism, move the valve, while

* See Index for page in the Author's *Text-Book on Steam and Steam Engines* for illustrations.

† See *Engineering*, 1st January, 1886, p 4, for a description of Lüdes steam relay governor. Also "Regulation of Steam Engines," by John Richardson, *Proc. Inst. C.E.*, vol. cxx., 1895, Part II., for description and discussion on electrical and other relays for governors.

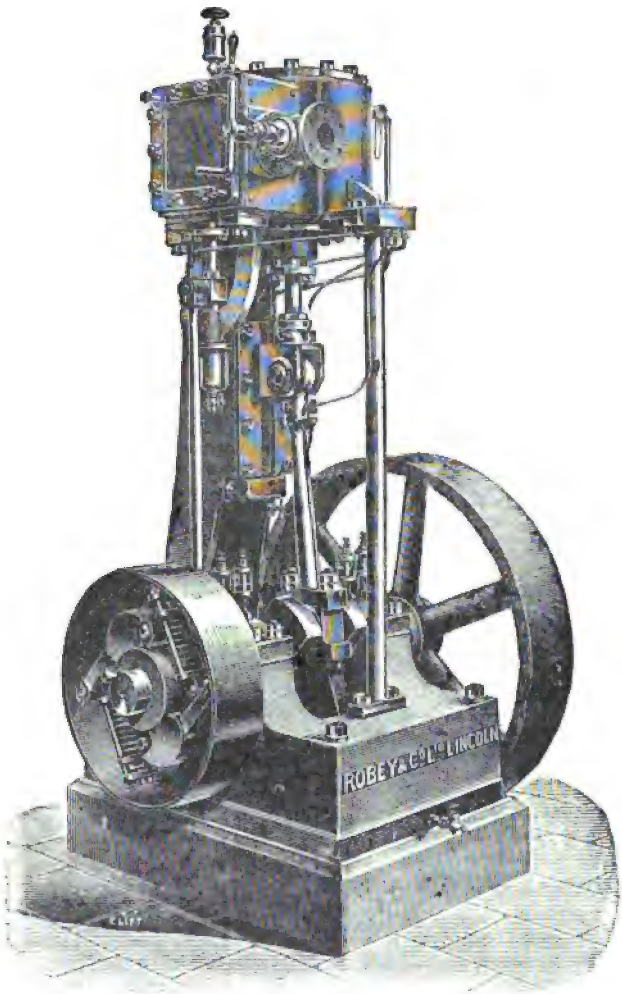


FRONT VIEW.



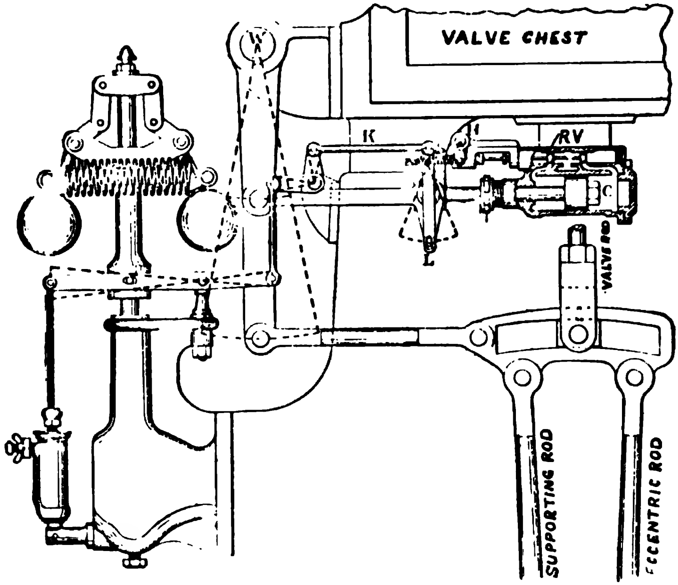
BACK VIEW.

AUTOMATIC EXPANSION SHAFT GOVERNOR,
BY MESSRS. RANSOMS, SIMS & JEFFERIES, LD.



SHAFT GOVERNOR APPLIED TO A HIGH-SPEED ELECTRIC LIGHT ENGINE.

the governor has simply to control the relay. In most relays that have been used for governing, the governor starts the relay, and then the latter goes on without any control, until the position of the governor is altered, and it is set into motion in the opposite direction. All such relays necessarily have the fault of hunting, but this is not so for one of the steering-gear type. The governor, as it were, informs such a relay when to move and how far, and the extent of the change in the height of the governor cone determines the travel of the relay.



GOVERNOR WITH RELAY FOR COMPOUND AND TRIPLE EXPANSION ENGINES, BY DAVEY, PAXMAN & CO.

The steam relay shown is applied by Messrs. Davey, Paxman & Co. to compound and triple expansion engines. The weigh-shaft W, which works the expansion gears of all the cylinders, is connected with the piston of the small relay cylinder C. The relay valve R V, which admits steam to this cylinder, is worked by the floating lever L, and allows steam to enter at its middle and exhaust at its ends. The lower end of the floating lever is attached at L, to the crosshead of the relay piston rod, and its upper end through the links K, &c., to the governor sleeve,

while the small piston valve of the relay is connected to an intermediate point. It will thus be seen that when the governor balls rise, R V will move to the left, and so admit steam to the left of the relay piston. This forces the piston inwards, and pushes the eccentric-rod end of the drag link further from the block attached to the engine valve rod, and therefore reduces the travel of the valve and the power of the engine. In addition, as the relay piston moves one way or the other, it rotates the floating lever L, about its upper end, and brings R V back to its mid position, and so automatically comes to rest. By this means, the relay piston and main slide valves are made to follow all the motions of the governor, and the amount of the motion of the relay piston will depend on the change in the height of the balls. The governor itself has very little work to do, since it has only to move the small valve R V. By means of the weigh shaft W, and levers attached to it the valve rods of all the cylinders are moved simultaneously, in the same way as the one shown. Minor adjustments of the speed may be made while the engine is running by altering the tension in the spring S, by means of a worm and worm wheel on the end of the spindle H.

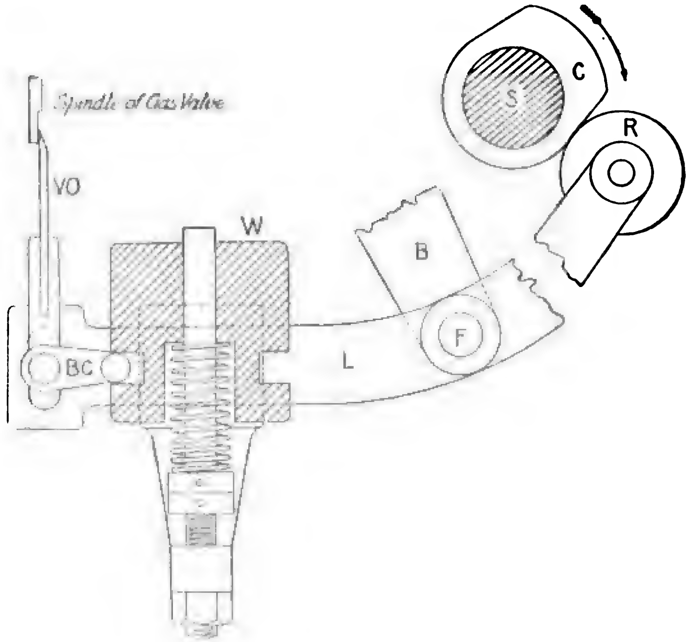
Knowles' Supplemental Governor.*—Another method is that invented by Knowles. Here two governors are used, a large one to control the valve in the ordinary way, and a smaller one to alter the length of the rod connecting the first one to the valve. This is effected by fitting two friction cones to the sleeve of the smaller, or supplemental, governor, and having a third between them, which will gear with one or other if the governor rises or falls by more than a prescribed amount. The valve rod is in two parts, having a right- and left-handed screw respectively at their adjacent ends, and the nut which joins these screws is rotated by the third friction cone. This governor has been extensively employed in spinning mills, where the fluctuations in load are neither great nor sudden, but where the speed must remain very constant.

Inertia Governors.†—For small gas engines, which always receive a full charge of gas during each cycle or none at all, a form of governor known as the inertia governor, has been found suitable. In the one first illustrated, the gas valve is opened by a valve opener V O, which is actuated through the lever L, by the cam C, fixed on the side shaft S. On the lower end of the valve opener there is a bell crank B C, engaging a slot on the

* See the *Practical Engineer*, vol. v., p. 205, March 27, 1891.

† See *Gas, Oil, and Air Engines*, by Bryan Donkin, for other forms of gas engine governors.

governor weight W. The weight is supported by a spring fixed to a bracket on the lever L. As the lever is moved upwards, the inertia of the weight W, causes it to lag behind, and thus compress the spring, but the latter is so adjusted that as long as the speed does not exceed the normal, B C is not moved down sufficiently to cause V O to miss the gas valve spindle.



INERTIA GOVERNOR FOR STOCKPORT GAS ENGINE.

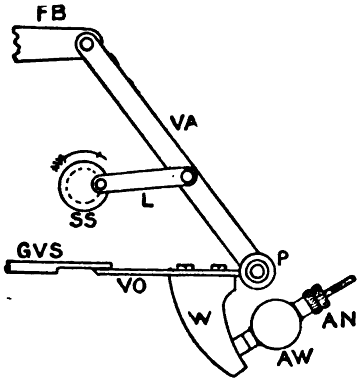
INDEX TO PARTS.

C for Cam.	B for Bracket.
S ,, Side shaft.	W ,, Inertia weight.
R ,, Roller.	B C ,, Bell crank.
L ,, Lever.	V O ,, Valve opener.
F ,, Fulcrum.	

If, however, the speed should rise above the normal, the inertia of the weight is sufficient to press B C down far enough to cause V O to pass to the right of the spindle, and then no gas is admitted for that cycle. As the direction of the thrust necessary to open the valve passes through the centre of the pin supporting

the bell crank B C, it is received direct from the lever L, and does not affect the governor weight W.

Another, and simpler form of inertia governor, as used in small Otto gas engines, depends for its action on the inertia of a small weight W, and adjusting weight A W. The vibrating arm V A, is driven by the link L, from a pin on the end of the side shaft S S, and causes the valve opener V O, and the weight W, to move backwards and forwards. The centre of gravity of the weights and valve opener being to the right of the pin P, the point of V O is pressed against the flat of the gas valve spindle. The effect of the inertia of these weights acts below P, and therefore tends to turn V O downwards when P is moved to the left. The position of A W is so regulated by the adjusting nuts A N, that when the speed exceeds what is desired, the latter tendency will predominate and cause the end of V O to pass below the valve spindle and leave the valve unopened.



INERTIA GOVERNOR FOR OTTO GAS ENGINE.

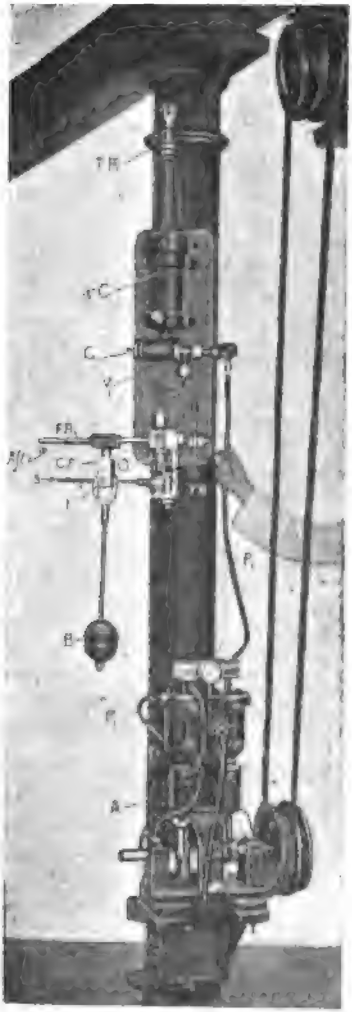
Thunderbolt's Marine Engine Governor.—When a short or a lightly loaded screw-driven steamship encounters heavy weather with "head seas," the screw propeller frequently rises out of the water. This action naturally diminishes its resistance to rotation, since the screw is then revolving more or less in the air, instead of in the denser salt water. Consequently, the engines commence to "race," which is not only very uncomfortable to passengers and crew, but may also be dangerous to the machinery; for, immediately after such a rise, the screw will undoubtedly be as suddenly immersed again in the water, thus alternately subjecting the screw shaft and other moving parts to very severe stresses. To such rough usage may be attributed some of the "delays," "breakdowns," and even total losses of steamships.

Many marine engine governors have been devised and applied, with more or less success, for the object of so automatically adjusting the supply of steam to the engines, that "racing" should not take place. As far as we are aware, one of the most successful of these governors, which has lately been put into

9

extensive daily practice, is that made and supplied by "The Thunderbolt Governor Coy., Ltd.," of Middlesbro'-on-Tees.

General Arrangement.—As will be seen from the accompanying outside view (Fig. 1) this governor consists of—



Index to Parts.

Figs. 1, 2, and 3.

A	for	Air Compressors.
P_1, P_2	,,	Copper Pipes.
R	,,	Regulator.
O	,,	Outlet Port.
V_1, V_2	,,	Valves.
T C	,,	Throttling Cylinder.
T R	,,	Throttle Valve Rod.
G	,,	Gland for V_1 box.
F R	,,	Fixed Rod.
C F	,,	Clamped Fork.
S	,,	Spindle.
F	,,	Fulcrum.
B	,,	Ball of Pendulum.
T S	,,	Thumb Screw.
V_3	,,	Safety Valve.
E G	,,	Emergency Gear.
→	,,	Direction Aft.

Fig. 1.—Thunderbolt's Marine Engine Governor.

Connected-up for Testing Prior to being Fitted on Board Ship.

1. A set of duplex double-acting air compressors, A, driven by rope, belt, or wheel gearing from the main screw shaft.

2. An automatic regulator, R, actuated by a heavy pendulum ball, B.

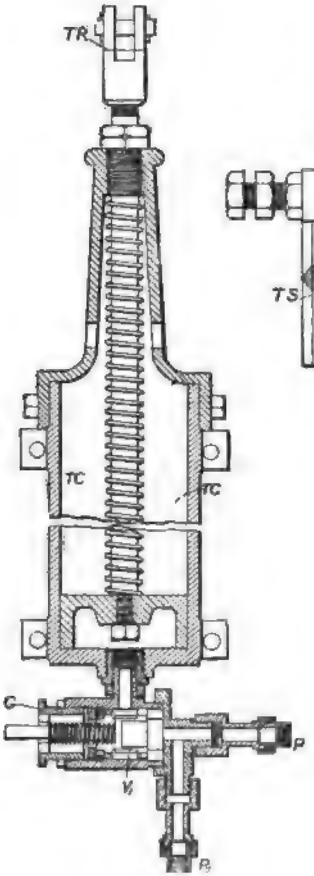


FIG. 2.—THROTTLING CYLINDER.

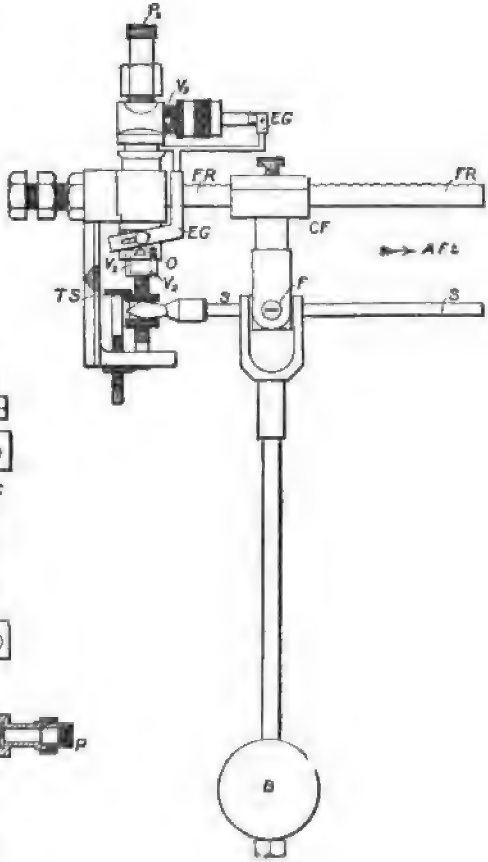


FIG. 3.—REGULATOR.

3. A throttling cylinder, T C, whose piston is connected through, T R, to the main steam-pipe throttle valve.

In this general view, these parts are all shown connected together and fixed to one of the vertical columns of the workshop, where they are made. The whole is just ready for being tested and adjusted, before being fitted on board a vessel. A ½-inch copper pipe, P_1, connects the air compressors, A, with the inlet spring piston valve, V_1, for its cylinder, T C; and another similar pipe, P_2, connects the regulator, R, also with the said valve, V_1.

Action of the Governor.—When the governor is in action and the speed of the main engine is uniform, the compressed air from A passes freely through the pipes, P_1, P_2, and outlet port, O (see Figs. 2 and 3 as well as Fig. 1). Now, should the stern of the vessel rise out of the water, every fore and aft fixture in the vessel, which may have been previously level or vertical, is thereby inclined to its normal position. But the heavy pendulum with its ball, B, being free to swing from its fulcrum, F, as an axis of motion, tends to remain in a *vertical position* due to its inertia. This naturally raises the *forward* conical end of the spindle, S (to which the upper forked end of the pendulum rod is attached), and lifts the valve, V_2, thus partially or wholly closing the outlet port, O. The air from the compressor pump is consequently prevented from escaping as freely as before by the outlet port, O. The pressure of the air in P_2 is thereby very quickly increased, and acting at once upon the piston valve, V_1, against the reaction of its spring, the air passes up to and raises the piston of the cylinder, T C, against the downward pressure of its spiral spring. As this piston rises, it elevates the throttle valve rod, T R, and cuts off the steam supply from the main engines. The speed is automatically prevented from increasing to a dangerous or even inconvenient degree.* On the other hand, when the bow of the ship rises and the screw is thereby suddenly immersed, precisely the opposite action takes place. The heavy pendulum with its ball, B, remains vertical, the conical end of S is depressed and the valve, V_2, uncovers more than the normal amount of the outlet port, O. The air pressure in P_2 and on V_1 is thus reduced below the normal, and the spring in the cylinder, C, presses down its piston, which lowers T R and opens the main steam-pipe throttle valve to the full extent;

* In the case of compound, triple, or quadruple expansion engines, where the steam once admitted naturally applies its force throughout the several cylinders placed in series, then this governor may be also connected to an exhaust throttle valve, as well as to the steam one.

thereby tending to supply the desired quantity and pressure of steam, to keep the engines moving at as uniform a rate as possible, under these erratic circumstances.

When the piston of the throttling cylinder, T C, descends, the air which it contains, escapes freely through holes in the gland, G.

It will be seen, that the action of the pendulum causes the throttle valve of the main engines to synchronise, with the up and down pitching movements of the vessel in such a manner, as to supply steam in proportion to the resistance offered to her screw. Also, the quicker the change in speed of the engines, the quicker will be the alteration in the air pressure, as the compressors are driven by the engines.

With "following seas" the valve, V_2, should be so adjusted by the thumb screw, T S, as to partly close the outlet port, O, and make the governor act more like a stationary or standard speed governor. This governor can be rendered more or less sensitive, or quick in its action (to suit different lengths and kinds of vessels, when running light or loaded), by shifting the fore and aft position of the fulcrum, F, with the suspended clamped fork, C F, along the fixed rod, F R.

Emergency Cut-off.—Should the screw shaft break, or from any other cause the engines "run off," then, the sudden and great increase of air pressure, forces out a small safety valve, V_3, connected directly to the emergency gear, E G, which falls and thus entirely closes the outlet port, O. The pressure of air in, T C, consequently increases and keeps its piston with its throttle valve rod, T R, fully raised, until the engineer can close the main boiler stop valve.

General Behaviour. — From reports of well-known superintending and chief engineers of steamships it is said, that the main stop valve may be left full open, whilst encountering heavy "head seas," when this governor is properly applied and worked, for it has been well designed and made.

Thunderbolt's Electric Governor Regulator.—The same firm make an "Electric Governor Regulator," where a vertical electro-magnetic solenoid, takes the place of the pendulum in the previously described marine form. The terminals of this solenoid are connected to those of the dynamo or switchboard. If an abnormal rise in voltage takes place, then an adjustable movable iron core is attracted further than usual, into the hollow cylindrical centre of the solenoid, thus raising the conical end of the spindle, S, and closing the throttle valve in the manner previously described. Should the voltage fall below the normal then the counter-spring attached to the upper end of the iron

core asserts itself and depresses the conical end of S, which naturally opens the throttle valve and gives the engines more steam.

"**Experiments upon the Action of Engine-Governors.**"*—Although the subject is admitted to be of great importance, comparatively few experiments have been made on engine-governing. The performances of different governors seem to vary so widely, that it is by no means easy to choose the type best suited to an engine working under given conditions. With a view, therefore, to observe in closer mechanical detail the behaviour of different kinds of governors, experiments were made at Mason College, Birmingham, under the superintendence of Professor R. H. Smith, M. Inst. C.E., on nine governors of different types.

From speed-curves and other data connected with any particular governor which you may consider, it is possible to obtain a practical measurement of those quantities which are mainly concerned in the efficient action of a governor—viz., its sensitiveness, its controlling force, and its controlling energy. These quantities must be expressed in terms of some unit of speed-variation, and may be referred to a change of speed amounting to 1 per cent. of the mean speed for which the governor is designed.

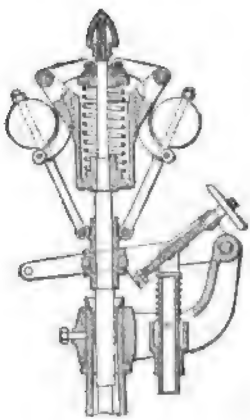


FIG. 1.—PROELL GOVERNOR.

The term "sensitiveness" will be used to indicate the distance in inches through which the sleeve would move in either direction from its mean position in response to a 1 per cent. variation of speed, if there were no frictional resistances to overcome, and when no controlling force is exerted.

The term "controlling force" will be used to indicate the force which would be exerted by the governor upon the controlling gear, if the given variation of speed were to take place without as yet producing any motion of the sleeve.

* I have to thank Mr. W. G. Hibbins, A.M.Inst.C.E., the author of this "Selected Paper," and the Council of the Institution of Civil Engineers for their kind permission to make the following extracts. See *Proc. Inst. C.E.*, vol. cxxxvii., pp. 376 to 401, of 1899, for the complete paper. These detailed experiments will form a useful guide to students, in any similar investigations which they may have to undertake.—A. J.

The term "controlling energy" will be used to indicate the inch-pounds of work that would be done by the sleeve, if, starting from its mean position, it were caused to travel through a distance equal to the sensitiveness, first upwards and back again, then downwards and back again. The speed during each ascent being always 1 per cent. greater, and during each descent 1 per cent. less, than the speed corresponding to the momentary position of a sleeve working without friction. Hence, the "controlling energy" is very nearly the same quantity as the "controlling force," multiplied by four times the distance which represents the "sensitiveness."

Proell Governor.—From tests of this governor (Fig. 1), the curves shown in Fig. 2 were obtained.

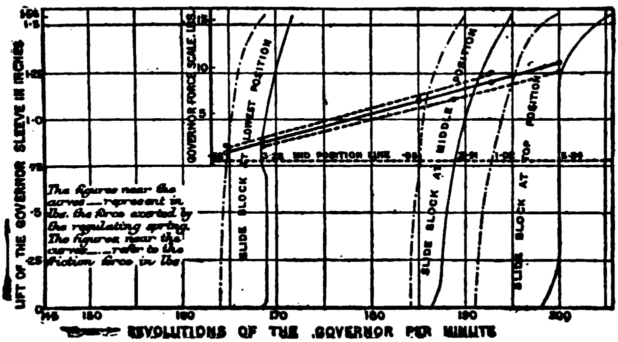


FIG. 2.—SPEED-CURVES FOR PROELL GOVERNOR.

The following are details of this type :—

Weight of each ball, ...	=	4·8 lbs.
Unstrained length of large spring of the governor, ...	=	7¾ inches.
Compressive strength, ...	=	120 lbs. per inch extension.
Pitch of the coils, ...	=	⅝ inch.
Outside diameter of coils, ...	=	2⅝ inches.
Diameter of steel of which the spring was made as given by the micrometer gauge, .	=	0·313 inch.

When placed in the governor, the length of the spring was $5\frac{13}{16}$ inches, which was equivalent to a pressure of 232·5 lbs. on the cap with the governor at rest.

Natural length of small regulating spring,	=	7¾ inches.
Pitch of coils, ...	=	½ inch.
Outside diameter, ...	=	1¾ ,,
Diameter of its steel, ...	=	0·177 inch.
Mean strength, ...	=	16·7 lbs. per inch.

With the slide-block at the bottom of the slide, the regulating spring was compressed $\frac{3}{4}$ inch, when the governor was at rest; the pressure then exerted is 12·52 lbs., which is equivalent to 8·24 lbs. acting vertically upwards at the centre of the governor spindle. With the slide-block at the centre of the slide, the small spring was stretched $\frac{5}{32}$ inch with the governor at rest. The downward pull then exerted is 2·6 lbs., equivalent to 1·33 lbs. at the centre of the governor spindle. With the slide-block at the top of the slide, the small spring is extended $\frac{5}{8}$ inch, the governor being at rest; the downward pull then exerted is 10·43 lbs., equivalent to 3·97 lbs. at centre of governor spindle.

The downward force exerted at the centre line of the governor spindle by the small spring has the following values when the balls are fully extended—

Block in lowest position,	. . . . . .	1·87 lbs.
,, central ,,	. . . . . .	5·2 ,,
,, highest ,,	. . . . . .	6·56 ,,

The pressure exerted by the large spring with the balls fully extended was 326·25 lbs.

Fig. 3 shows the distances from the centre line of the governor spindle to the slide-block in its different positions.

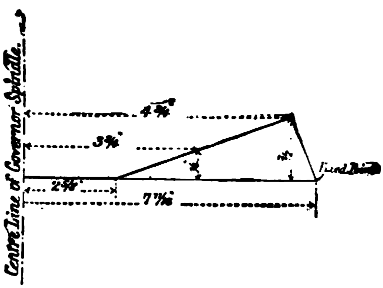


FIG. 3—DISTANCES FROM CENTRE LINE OF GOVERNOR SPINDLE TO SLIDE-BLOCK.

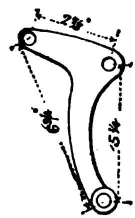


FIG. 4.—BELL-CRANK LEVER.

Diameter of each ball, ...	=	3¼ inches.
Weight of ball and arm, ...	=	6 lbs. 4½ ozs.
Length of governor arm from ball centre to pin centre, ...	=	8 inches.
Full movement of governor-sleeve, . . .	=	1·56 inches.

Fig. 4 shows the leverage exerted by the bell-crank levers which are attached to the governor arms. As shown by the curves in Fig. 2, the governor was exceedingly sensitive, a difference of only 9 revolutions per minute being sufficient to

send the governor sleeve from its lowest to its highest position, the slide-block being at the centre of the slide.

The governor sleeve is at mid-position when 0·78 inch high. The loads placed on the sleeve in this case are those due to the weight of the lever, &c., and the tension or compression of the regulating spring. As it is difficult to estimate the pressure on the governor sleeve due to the weight of the lever, this was measured by a spring-balance attached to the lever at the centre line of the governor spindle, and found to be 3 lbs. There was also a change in the pressure on the sleeve due to the shifting of the slide-block and its attachments.

The pressures exerted at centre line of governor spindle with the block in the three positions were—

	2·01 lbs. when the slide-block was in its	lowest position,
	1·56 ,, ,, ,, ,,	middle ,,
and	1·16 ,, ,, ,, ,,	highest ,,

The loads due to the regulating spring, with the sleeve at mid-position, were—

	3·28 lbs. acting vertically	upwards with slide-block in	lowest position,
	2·91 ,, ,,	downwards ,,	middle ,,
and	5·24 ,, ,,	,, ,,	highest ,,

The total loads for the three positions are therefore—

	(3 + 2·01 − 3·28) lbs.	= 1·73 lbs. acting downwards for	1st position,
	(3 + 1·56 + 2·91) ,,	= 6·47 ,, ,, ,,	2nd ,,
and	(3 + 1·16 + 5·24) ,,	= 9·4 ,, ,, ,,	3rd ,,

These loads have been plotted upwards from the mid-position line, as shown in Fig. 2, and to the given scale. Two straight lines are obtained, as shown; and, measuring their inclination by the divisions of the squared paper as before, the following results are obtained:—

With slide-block in lowest position, friction	= 0·43 lb.
,, ,, middle ,, ,,	= 0·708 ,,
,, ,, highest ,, ,,	= 1·04 ,,

The normal speed of the governor was supposed to be 190 revolutions per minute, and 1 per cent. of this is 1·9 revolutions, equivalent to 1·9 horizontal divisions. Hence, controlling force = 0·50 lb.

Considering the middle pair of curves—*i.e.*, with the slide-block at mid-position—it requires an increase in speed of from 187·5 revolutions to 194·3 revolutions per minute to lift the sleeve from a height of 0·25 inch to 1·5 inch. Therefore—

$$\text{Sensitiveness} = \frac{1\cdot 9}{6\cdot 8} \times 1\cdot 25 = 0\cdot 349.$$

$$\text{Controlling energy} = (4 \times 0\cdot 5 \times 0\cdot 35) = 0\cdot 70 \text{ inch-lb.}$$

The Tangye, Acme, Lüde, and Proell governors were driven by belting from a counter-shaft, which was in its turn driven by means of a vertical steam engine. The Watt, Porter, Pickering, Turner-Hartnell, and Belliss governors as tested, were driven by means of an electric motor, the speed of which could be regulated exactly by means of suitable resistance frames.

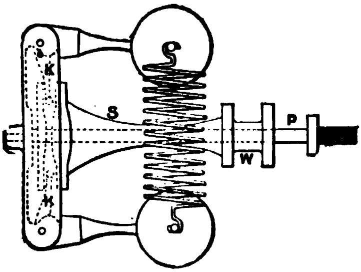


FIG. 5.—BELLISS GOVERNOR.

Belliss Shaft Governor.—The governor of this type (Fig. 5) was supposed to run at about 300 revolutions per minute, and to control an engine of 60 H.P. The brass sleeve, S, is moved by the levers, K K, along the shaft, P. A lever is fixed on to the sleeve at W, which moves the valve-gear. In order to test the governor, it was placed between the centres of a lathe, and driven by the electric motor. The regulating arrangement was attached as shown in Fig. 6.

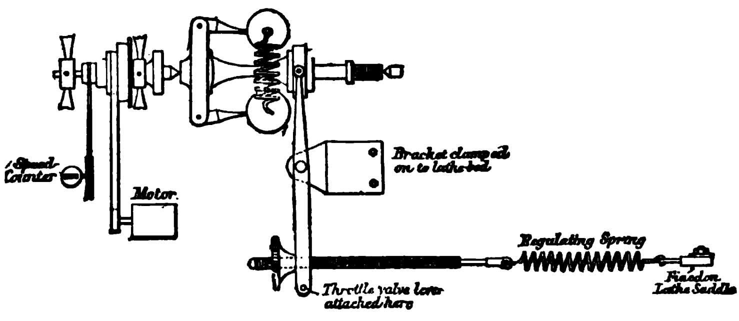


FIG. 6.—REGULATING ARRANGEMENTS WHEN TESTING THE BELLISS GOVERNOR.

Three pairs of curves (Fig. 7) were obtained as follows:—Curves 1, 1, with no regulating spring attached to the lever. Curves 2, 2, with regulating spring attached, but with no tension at starting. Curves 3, 3, with the regulating spring stretched 1·28 inches at the start = 34·12 lbs. pull.

The following are details relating to this governor :—

Mean strength of governor springs	=	5·6 lbs. per inch extension.
,, regulating spring	=	26·66 ,, ,,
Weight of throttle valve and vertical gear, ...	=	4·75 lbs.
Weight of one ball, with arm, &c.,	=	7·42 ,,

This governor appeared to be very powerful, and at the same time, fairly sensitive. Curves 3, 3 give the nearest approach to the actual working curves.* The movements of the sleeve, S, were in this case measured directly, which could easily be done as the sleeve kept very steady, and the speed of rotation could be regulated exactly. The speed was recorded by a speed counter, driven by the cone pulley of the lathe.

The sleeve is at mid-position when about 0·44 inch from its starting point. The loads calculated are zero for curves 1, 1,

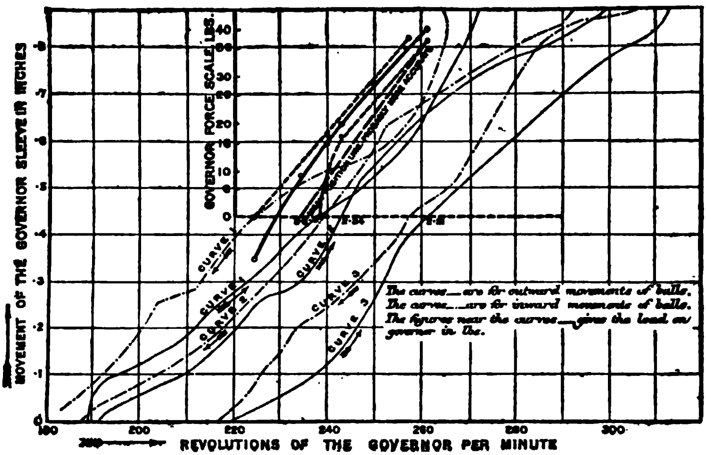


FIG. 7.—SPEED-CURVES FOR BELLISS GOVERNOR.

owing to the fact that no regulating spring is on the governor lever; 17·07 lbs. for curves 2, 2; and 37·76 lbs. for curves 3, 3. These are plotted off above the mid-position line, according to scale; and, tracing the two lines shown in Fig. 7, the following results are obtained by the methods already described :—

For the curves 1, 1, ...	friction	=	8·8 lbs.
,, 2, 2, ...	,,	=	2·34 ,,
,, 3, 3, ...	,,	=	2·21 ,,

* It was intended to obtain curves such that the mean speed of rotation was 300 revolutions per minute, the speed given by the makers for this particular governor, but this was impossible, owing to the shortness of time during which the governor could be spared for the experiments.

The mean speed of the governor is given as 300 revolutions per minute, and 1 per cent. of this is 3 revolutions, equivalent to 1·5 horizontal divisions. Hence, the controlling force of the governor = 3·89 lbs.

Considering curve 3, which is the nearest approach to the working curve, if the speed is altered from 238 to 305 revolutions per minute, the sleeve is moved from 0·1 inch to 0·8 inch; hence, a change in speed of 67 revolutions per minute moves the sleeve 0·7 inch; therefore, the sensitiveness = 0·031. The controlling energy = $(3{\cdot}89 \times 0{\cdot}031 \times 4) = 0{\cdot}482$ inch-lb.

Comparison of Different Governors.—The frictional resistance was reduced to a minimum in each governor, by carefully cleaning and oiling the working parts, but even then it varied considerably in amount, as is shown by the forms taken by several of the curves, and also by the calculations obtained from them. It will be observed that the Proell governor possessed very little frictional resistance in its working parts, as compared with some of the others, and was closely followed in this respect by the Tangye and four-pendulum governors. It should be remembered, however, that the Proell governor had no valve attached, and therefore the frictional resistance of the valve-spindle at the stuffing-box is an element not included in the experiments. The Pickering governor curves exhibit a frictional resistance much greater than might be expected, considering that it possesses no pin joints. It is, therefore, certain that the greater part of the frictional resistance in many governors is produced at the stuffing-box of the valve-spindle. In the governors tested, the stuffing-box was not screwed tighter than is usual in practice. It is also probable, that in a Pickering governor, there is a considerable amount of frictional resistance at the points where the lever of the regulating spring presses on the governor valve-spindle, and also inside the cap at the top of the governor.

The table on the following page shows the chief results obtained from the nine governors tested.

Comparing the values of the controlling force, it will be noticed that the small Tangye governor exhibits a high value; it is, however, lacking in sensitiveness. Another notable feature is the large controlling force of the Turner-Hartnell governor, and the large amount of frictional resistance in its working parts. It is evident that the controlling force depends largely on the weight of the balls, and the central weight or "muff" of the governor, for the highest values are exhibited by the Lüde, Watt, Porter, Turner-Hartnell, and Belliss governors, which all had either heavy balls, weights, or a heavy central load. As regards sensitiveness, the Proell governor appears to be far

superior to the rest. It is striking how favourably the Porter governor compares with some of the more modern types, possessing, as it does, both sensitiveness and great controlling force. The Watt governor also shows large controlling force, due no doubt to its very heavy balls.

Type of Governor.	Mean Speed Revolutions.		Sensitiveness.	Controlling Force.	Controlling Energy.	Approximate Mean Frictional Resistance in Lbs. for Working Curves.
		Per Minute.		Inch-lbs.	Lb .	
Tangye (Soho), .	. . . .	400·0	0·009	1·54	0·056	2·6
Acme, ...	. . . .	400·0	0·016	1·98	0·13	3·22
Lüde (4-pendulum governor), . .	Load = 0 .	200·0	0·056	0·95	0·213	2·62
	,, = 20 lbs.	229·25	0·102	1·96	0·789	1·90
Proell, . . .	. . . .	190·0	0·35	0·50	0·70	0·71
Watt, ...	. . . .	90·0	0·113	4·18	0·189	4·3
Porter, . . .	. . . .	260·0	0·08	1·04	0·33	2·87
Pickering, . .	. . . .	400·0	0·018	0·42	0·03	3·85
Turner - Hartnell (shaft-governor),	. . . .	200·0	0·016	7·99	0·511	...
Belliss (shaft-governor), . .	. .	300·0	0·031	3·89	0·482	2·21

Results.—The following may be regarded as a summary of the results arrived at :—

1. In testing governors by running them steadily at different speeds, great difficulties have to be overcome to obtain the natural characteristics of the governors considered, chiefly owing to frictional resistances, which appear to vary erratically.

2. If the different governors were placed on the engine and tested by suddenly altering the load, there would still be uncertainty in the results, owing to the variations of the frictional resistance in the valve-gear, which is connected with the sleeve (or the eccentric) of the governor.

3. The controlling force of a governor is increased by increasing the weight of the balls or the central load.

4. The controlling force and sensitiveness are increased by providing the governor with powerful springs.

5. Increasing the masses of the revolving balls or weights also increases the amount of frictional resistance.

6. Shaft-governors can be made much more powerful, bulk for bulk, than vertical governors.

7. The advantages of using the more modern types of governor over using the older types, such as well-designed Watt and Porter governors, are not so great as perhaps might be expected.

APPENDIX.

To Obtain the Controlling Force Graphically.

"A method of obtaining graphically the controlling force of a vertical governor is shown in Fig. 8.

Let C = centripetal force in lbs. acting at the centre of the governor ball in order to maintain equilibrium at the normal speed.

W = weight in lbs. of each of the governor balls.

T, t = tension in lbs. on the upper and lower arms respectively.

N = number of revolutions of the governor per minute at the normal speed.

F = the total load on the sleeve divided by the number of balls.

f = the controlling force exerted at the sleeve for each ball in the governor, or the whole controlling force divided by the number of balls.

θ and ϕ = the angles made by the centre lines of the upper and lower arms with the vertical centre line of the spindle when the governor is running at normal speed.

R = radius, in feet, of the circle described by the centres of the governor balls.

The force diagram is constructed as follows :—

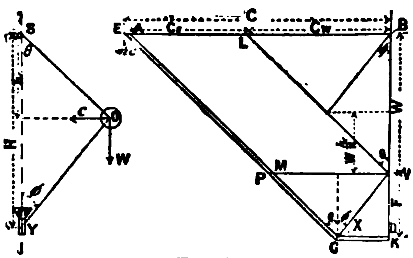


Fig. 8.

Draw a vertical line and mark off B V, V D, representing, to the same scale, W and F * respectively in magnitude and direction.

Draw V L parallel to the upper arm, O S, and V G parallel to the lower arm, O Y.

Draw V M and D X both perpendicular to B D. Through X draw X A parallel to V L, and through B draw B A parallel to V M and D X.

It is evident that the line, B A, represents the force, C, in magnitude and direction.

Now, the force, C, is partly due to the effect of W and partly to the effect of F.

Calling the former part C_w and the latter C_f,

$$C = C_w + C_f.$$

In the diagram, the line B L represents C_w, and L A represents C_f. Now,

$$C = \frac{4\pi^2 N^2 R}{g} . W,$$

* It will be noticed that only half the central weight is here taken into consideration, the other half being supposed to act on the other side of the centre line of the governor spindle.

so that C is proportional to N^2 and 1 per cent. increase of N is equivalent to an increase of 0·02 C in C.

Produce B A to E, making A E equal to 0·02 C.

Draw E G parallel to A X, produce V X to G, and draw G K parallel to X D.

Then the line, D K, represents half the force, f, to the same scale as F and W.

To obtain its magnitude from the similar figures, V D X M, V K G P:—

$$\frac{f}{F} = \frac{0{\cdot}02\,C}{C_f} = 0{\cdot}02\left\{\frac{C_w + C_f}{C_f}\right\}.$$

i.e.,
$$f = 0{\cdot}02\,F\,\frac{C_w + C_f}{C_f} = 0{\cdot}02\left\{1 + \frac{C_w}{C_f}\right\}F.$$

f is the force for 1 ball only. The controlling force for 2 balls is $2f$, and for 4 balls it is $4f$.

This expression for the magnitude of the controlling forces is applicable to any vertical governor, no matter what may be the train of mechanism between the balls and sleeve.

For the special kind of governor shown in Fig. 8, f can be obtained as follows:—

$$\frac{f}{0{\cdot}02\,C} = \frac{F}{F(\tan\theta + \tan\phi)} = \frac{1}{\tan\theta + \tan\phi}.$$

i.e.,
$$f = \frac{0{\cdot}02\,C}{\tan\theta + \tan\phi} = 0{\cdot}02\,\frac{W\tan\theta + F(\tan\theta + \tan\phi)}{\tan\theta + \tan\phi}$$

$$= 0{\cdot}02\left\{F + \frac{W}{1 + \dfrac{\tan\phi}{\tan\theta}}\right\} = 0{\cdot}02\left\{F + W\frac{H - h}{H}\right\}."$$

Flywheels.—We have already mentioned that the function of the flywheel is to take up and give out energy so as to minimise the fluctuations in speed due to the periodic changes in the crank effort, and also to reduce the suddenness of other changes in the speed of the engine.

In the previous Lecture we found an expression for the tension per square inch in the rim of a flywheel due to centrifugal force, and saw that it was independent of the cross area of the rim. The highest speed at which a flywheel can be run with safety, will therefore depend upon the tensile strength and the density of the material of which it is made, for it is evident that we cannot make it able to go faster by enlarging the cross area of the rim, since that increases the total stress in exactly the same proportion as it increases the total strength. Consequently, for very high speeds it is necessary to select a material, and so dispose of it, as to have the greatest possible strength for a given mass. Flywheels are usually made of cast iron, either moulded and cast in one piece, or built up in several segments; but sometimes,

they have been made of wrought iron or steel, so as to offer a much greater resistance to bursting, or by winding steel wire into an annular trough made of steel plates at a definite distance from the crank shaft.*

Balancing Machinery.†—If a flywheel be not accurately balanced, it will cause wobbling stresses, which produce vibration and wear, and which become greater as the speed is increased. Owing to the recent demand for high-speed machinery, such as sugar-drying, cream separating, hydro-extracting, and electric light machinery, the attention of engineers has been specially directed of late to the necessity for more perfect balancing, with a view to reducing vibration and its attendant noise, tear, and wear. Even in the case of express trains, it has been found advisable to balance the carriage wheels. This is done by placing their axles and their wheels on a framing with springs of exactly the same kind as those to be used on the carriage for which they are intended, and running them at their highest speed of, say, 60 to 70 miles per hour. Pieces of clay are placed upon the inside of their rims until they run perfectly smoothly. These lumps are then replaced by pieces of cast iron or lead of the same weight, and the process repeated until as perfect a balance as possible has been obtained.

In works where the importance of balancing machinery is recognised, the machine to be balanced is placed upon a testing table and run at gradually increasing speeds. At each speed the balance is made as perfect as possible, by trial, in a manner similar to that just described for railway carriage wheels, until the maximum working speed has been reached, and the whole is capable of running practically free from vibration even when not secured by bolts or clamps.

A common method of balancing pulleys in the workshop is to mount them on a shaft, or mandril, which is then placed on two parallel and perfectly level straight edges. This is a delicate method of procuring a *statical* balance, but it does not follow that there is a true *dynamic* balance, as there may be a *centrifugal couple*, which will cause vibration, and needless pressure on the bearings. To take a simple case, con-

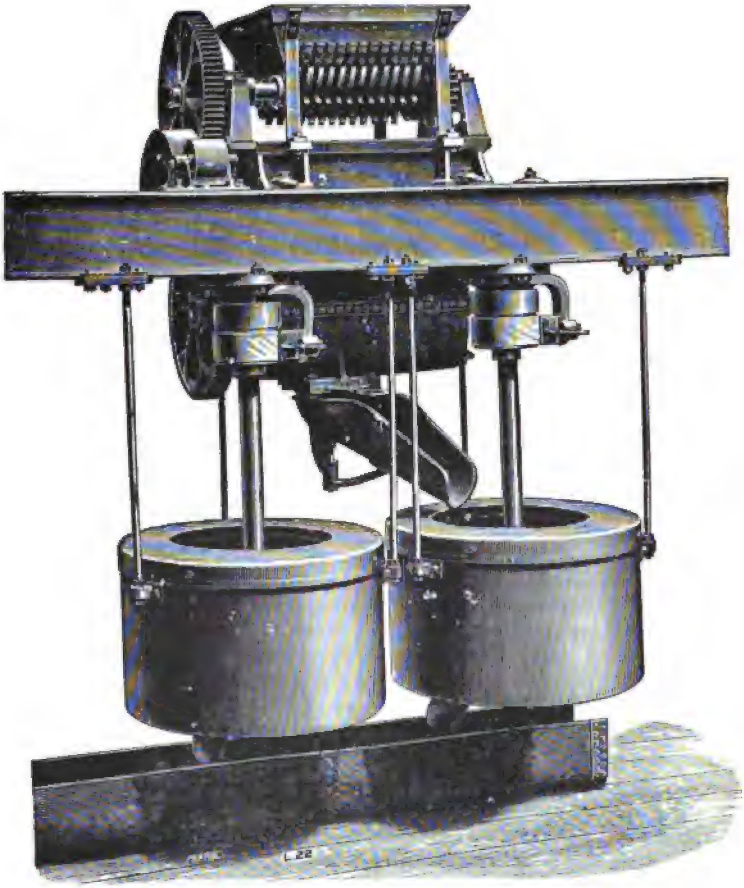
* See Prof. Sharp's pamphlet on "A New System of Wheel Construc tion" (Technical Publishing Co., Manchester). Also, "Flywheels for Slow-Speed Electric Traction Steam Dynamos," by A. Marshall Downie, B.Sc., and the full discussion in vol. xlv.—Parts I. and II., Nov. and Dec., 1901, *Trans. Inst. Engs. and Shipbuilders in Scotland.*

† See *Proc. Inst. Eng. and Shipbuilders in Scotland*, vol. lxiii., 1899, for a paper on "The Mechanics of the Centrifugal Machine," by C. A. Matthey. Also, *Proc. N.E. Coast Inst. of Eng. and Shipbuilders*, vol. xii., 1896, for a paper on "An Investigation into the Force tending to produce Vibration in High-Speed Engines," by J. M. Allan.

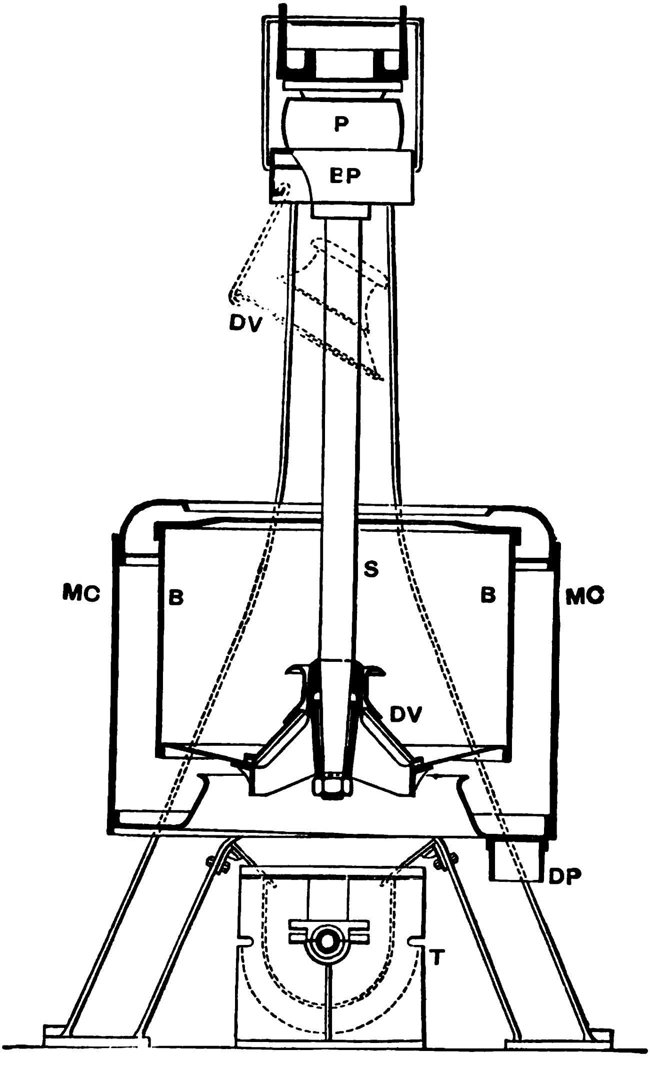
sider a crank shaft with two cranks 180° apart. The static balance may be perfect if the cranks are similar; yet, it is clear that the centrifugal forces of the two cranks, although equal, parallel, and opposite in direction, are not in the same straight line, and therefore form a couple in a plane passing through the axis of the shaft. The plane of this couple revolves with the cranks, and it consequently tends to make the axis describe a double cone in space, the common vertex of this cone being at the centre of gravity of the whole rotating mass. Similarly, with a pulley there may be an excess of material on one side at one extremity of a diameter, and at the other extremity an excess on the other side, which, while the static balance is perfect, cause a centrifugal couple, and set up objectionable vibrations at a high speed. The final adjustment of the balance of a wheel or pulley should therefore always be made at the highest speed at which it is intended to run. In order to have a *statical* balance about an axis, it is sufficient that the axis should pass through the centre of gravity of the *whole* mass, but for a perfect *dynamic* balance, it must *also* pass through the centre of gravity of *every* section taken at right angles to the axis. It is possible, however, in some cases to have the body as a whole balanced without this last condition, but in such cases there will be several centrifugal couples whose resultant is zero, but which tend to bend the shaft at several places.

Weston Centrifugal Machine. — As a useful application of centrifugal force, and an example of a self-balancing high-speed machine, we here illustrate the Weston centrifugal for drying sugar. The first figure gives a general view of a pair of 30-inch centrifugals suspended from the house framing, with sugar-breaker, pug mill, swivel shoot, and molasses gutter. The baskets of these machines are driven at 1,200 revolutions per minute, and give an output of 12 to 16 tons of dried raw sugar, or 12 to 20 tons of dried refined sugar, per day of ten hours, and require about seven horse-power to drive them.

In order to charge the machine, the valve at the bottom of the pug mill is opened, so as to allow the sugar to gravitate down the scoop into the basket B, seen in the vertical section. When a sufficient charge has been given, the pug mill valve is closed, and the basket started rotating by a friction pulley of the kind shown in Lecture VIII., p. 158 of Vol. I. The belt which drives the pulley P, connected to the spindle S, thus gradually brings the speed up to its normal. The centrifugal force causes the water and molasses to pass through the numerous holes in the periphery of the basket into the monitor case M C, from whence it escapes by the discharge pipe D P; while

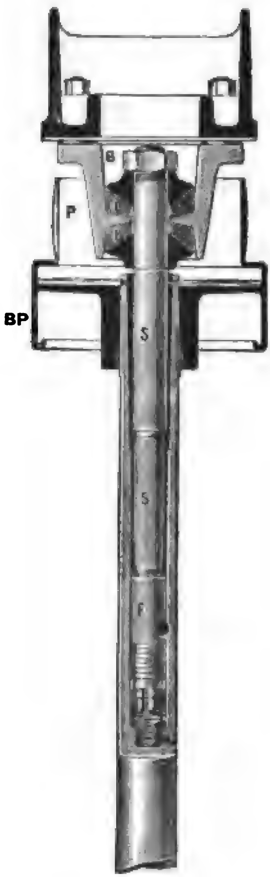


WESTON CENTRIFUGAL SUGAR-DRIERS, BY WATSON, LAIDLAW & Co.



VERTICAL SECTION OF WESTON CENTRIFUGAL.

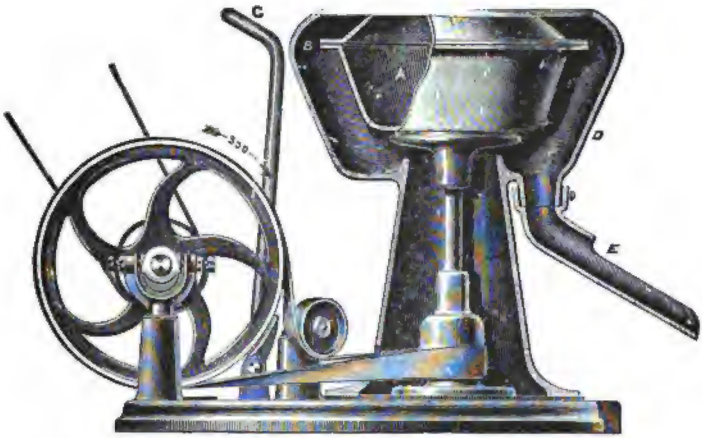
the sugar forms a wall around the inside of the basket. When the sugar is dried, the friction grip of the driving pulley is relieved, and the brake-strap applied to the brake pulley B P, so as to bring the basket and its contents quickly to rest. The conical cover, or discharge valve D V, is then raised and hung on the brake pulley, as seen in dotted lines. The wall of sugar is broken down and swept through the central opening into the conveying trough T. It is then forced along this trough by a large screw to wherever it may be wanted.



Section of Spindle and Bearings for Weston Centrifugal.

The basket is not compelled to revolve about a fixed axis, but is permitted to choose its own centre of rotation by the use of elastic bearings. By allowing the revolving basket to oscillate within certain limits, it assumes as its centre of gyration the centre of gravity of the basket and its contents, and so becomes self-balancing. This reduces to a minimum the power required to drive the machine, severe stresses, wear and tear, and the vibrations transmitted to the building. By referring to the sectional view of the spindle, it will be easily understood how this is accomplished. A strong block B, is bolted to the overhead beam, and inside this block are placed two india-rubber buffers I I, the upper of which sustains the suspended spindle S, by a nut and washer. This spindle does not rotate, but it carries, at its lower end, a series of washers which support the bearing F, fixed to the outer revolving spindle. The pulley P, and brake pulley B P, are attached to the upper end of this outer spindle, and the perforated basket to its lower end. The hollow portion of this spindle is filled with oil, so that the bearing F runs in an oil bath, and is always well lubricated.

There are many other applications of this principle, such as in hydro-extractors, cream-separators, &c. The next illustration shows a modification of the above machine adapted for extracting oil from engine waste, turnings, screws, &c., or drying crystals and ores. The material to be dried is put into the hollow pan A, which is then rotated at about 2,000 revolutions per minute. The oil, or water, escapes through the narrow opening between the upper and lower parts of the pan at B, into the outer casing D, and thence to the spout E. The pan is emptied by lifting it



UNDER-DRIVEN CENTRIFUGAL EXTRACTOR, BY WATSON, LAIDLAW & CO.

bodily from the top of the spindle and turning it upside down. It rests on a leather-faced disc on the top of the spindle, and is kept central by a continuation of the same, which fits easily into a recess in the bottom of the pan. This arrangement permits of a little slip at starting, by which the driving belt is relieved from any sudden or severe stress. The spindle is similar in construction to the one just described, but inverted, so that this machine is also self-balancing.

Lecture XXIII.—Questions.

1. The governor and flywheel of an engine have both the purpose of regulating its speed. Explain how their actions in this respect differ.

2. Explain the principle of *Watt's* pendulum governor, and state its advantages and defects. Various methods have been proposed for improving this form of governor; discuss the action of any such modern apparatus with which you are acquainted.

3. What are the principal essentials of a good steam engine governor? Sketch, in outline, any one form of governor with which you are acquainted, and explain to what extent it is satisfactory according to the conditions which you have laid down, or how it might be improved.

4. Sketch the ordinary pendulum or ball governor of a steam engine. Mark on your drawing some particular line whose length is related to the number of revolutions of the balls. State the relation as nearly as you know it. If the line referred to be shortened in proportion of 2 : 3. how much would the number of revolutions be increased? *Ans.* $\sqrt{3} : \sqrt{2}$.

5. Sketch an ordinary Watt's governor, and explain its action upon the valve with which it is connected. Why is it an improvement to shift the points of suspension so that the arms cross each other?

6. Explain the advantages of the crossed-arm governor for a steam-engine. Find the height of the cone when the engine is making 40 revolutions per minute, and prove the formula on which you rely. *Ans.* 1·8 ft.

7. Define the term "isochronous" as applied to governors. How may isochronism be approximately obtained? Prove the formula, connecting the height of the cone of revolution and the number of revolutions per minute, for a simple pendulum governor.

8. Sketch the pendulum governor as Watt made it. From the balls of a common governor, whose collective weight is A, there is hung by a pair of links (of lengths equal to the ball-rods) a load, B, capable of sliding up and down the spindle. Compare the loaded and common governor as regards sensitiveness, the weights of the arms or links being neglected.

9. Find an expression for the height of the cone in a loaded governor when rotating at a given number of revolutions per minute. Show, by a sketch, the connection of the governor with a throttle valve. By what arrangement may the tendency to over-sensitiveness be corrected?

10. Find the height of a simple or "Watt" governor revolving at 80 revolutions per minute. If the same governor had a weight of 40 lbs. attached to the sleeve, the balls weighing 3 lbs. each, what should be its height, supposing the same speed to be maintained, and the link work to be such that the sleeve rises twice as fast as the balls? Neglect the weight of the connecting links. *Ans.* 5·5 ft. and 6·5 ft.

11. Find the height of a simple conical pendulum revolving at 80 revolutions per minute. If a loaded governor, making 240 revolutions per minute, had a weight of 20 lbs. attached to the sleeve, the balls weighing 2 lbs. each, what would be its height, the vertical motion of the balls being half that of the sleeve?

12. Sketch and describe any spring-loaded governor, and compare the action of the spring with that of a weight.

13. What objection is there to regulating the speed of an enging by the throttle valve?

14. Explain what is meant by automatic expansion gear, showing wherein lie its special advantages in the economic working of an engine. Sketch such an arrangement and its connections.

15. Explain by the aid of the necessary sketches the construction of either the Armington-Sims or the Westinghouse high-speed flywheel governor and valve gear. Show clearly how in these arrangements the throw and angle of advance of the eccentric are varied, whilst the lead is kept constant. (Robey's and Ransoms, Sims & Jefferies' shaft governors are similar to those asked for.)

16. What special benefit is obtained by adding a relay to a governor? Sketch and describe a relay which automatically follows up the motion of the governor.

17. Sketch Knowles' supplemental governor and describe its action.

18. Describe the pendulum governor of the Otto engine, and point out, by reference to sketches, the manner in which it acts.

19. Explain clearly the arrangement by which the speed of an Otto engine is regulated.

20. Describe any form of inertia governor used for regulating the speed of a gas engine.

21. Describe, with proper sketches, a form of vibrating pendulum regulator as fitted to an Otto gas engine, and explain how it acts, and is made adjustable. Assuming that the pendulum is actuated by the rotation of the gas and air valve, describe the mechanism connecting the end of the valve with the pendulum, showing that it forms a well-known combination in linkwork.

22. Explain why it is necessary to balance high-speed machinery, and describe the most approved method of doing so.

23. What primary law in mechanics asserts itself when some revolving piece of machinery moves at a high velocity, and is unbalanced? A weight of 1 lb. is placed on the rim of a wheel 2 feet in diameter, which revolves upon its axis and is otherwise balanced. The linear velocity of the rim being 30 feet per second, what is the pull on the axis as caused by the weight of 1 lb.? *Ans.* 28·1 lbs.

24. Explain by sketches and description how railway carriage wheels for express trains and their axles are balanced. Give your reasons for and against the common workshop expression that a perfect statical balance is not one when the machine is run at a high speed.

LECTURE XXIII.—A.M.INST.C.E. EXAM. QUESTIONS.

1. In a loaded governor, the four equal links of which form a parallelogram with sides 8 inches long, the balls weigh 5 lbs. each, and the load is 20 lbs. Find the speed when the links stand at right angles. Find also what increase of speed is required to raise the load ½ inch higher. (I.C.E., *Feb.*, 1898.)

2. Describe exactly what is the object of using a load on a watt governor. Prove what you say to be correct. (I.C.E., *Oct.*, 1898.)

3. The rods and links of a Porter's loaded governor are each 1 foot long, the balls each weighing 2 lbs., and the load 12 lbs. The valve is full open when the arms are at 30° to the vertical, and shut when they are at 45°. The velocity ratio between the engine-shaft and the governor-spindle is 2. Find the extreme working speeds of the engine. (I.C.E., *Oct.*, 1900.)

4. Show that the height of a simple revolving pendulum when making n revolutions per minute is, approximately, $\frac{35,000}{n^2}$ inches. The balls of an unloaded governor each weigh 20 lbs., the arms and slide links are all equal, and, when running at 80 revolutions per minute, the actual height of the governor is 7 inches. Calculate the pull on the slides at this speed. (I.C.E., *Oct.*, 1902.)

5. What are the difficulties to be overcome in governing turbines to run at constant speed? Describe two methods of governing. (I.C.E., *Oct.*, 1902.) (*See also Lecture XXXVI. on Turbines.*)

6. The rods and links of a Porter's loaded governor are each 1 foot long, the balls each weighing 2 lbs. and the load 12 lbs. The valve is full open when the arms are at 30° to the vertical, and shut when they are at 45°. The velocity ratio between the engine-shaft and the governor-spindle is 2. Find the extreme working speeds of the engine. (I.C.E., *Feb.*, 1903.)

PART IV.—GRAPHIC STATICS AND APPLICATIONS TO ROOFS. CRANES, BEAMS, GIRDERS, AND BRIDGES.

LECTURE XXIV.

Graphic Statics is the Science and Art of determining by scale drawings the total stresses in the various parts of a structure. The forces transmitted through each part of a structure may be ascertained either by calculation or by graphical construction. The former method is extremely tedious, except in very simple cases, whereas the latter is not only rapid, but also affords a self-evident means of checking the accuracy of the solution.

DEFINITION.—**A Framed Structure consists of an assemblage of rigid bars, so arranged, that the stresses in them are principally push or pull and by the use of which, external forces may be transmitted or modified.**

A structure is different from a machine in so far as, the former transmits force while the latter transmits energy. This means that the parts of a structure are assumed to be at rest while those of a machine must be in motion.

In this section we assume, unless otherwise stated :—

(1) That the point of crossing of two or more bars is a frictionless joint, and that the external forces act *only* on the joints of the frame.

(2) That all the members, bars, or links are able to withstand either push or pull, and are consequently termed "rigid bars."

(3) That each bar is incapable of being appreciably deformed under the action of the stress it may have to carry.

For the complete specification of a force, we must know the following four elements :—

(1) The point or place of application.
(2) The line of action—*i.e.*, the line along which the force is acting.
(3) The direction or way the force acts along its line of action.
(4) The magnitude—*i.e.*, the number of units of force.

Classification of Frames.—(1) **Firm Frames** are those which have *just* sufficient bars to prevent change of shape, and any bar may therefore be lengthened or shortened without stressing any of the other members.

(2) **Deficient Frames** are those which have *not* sufficient bars to prevent deformation, and the joints must therefore be made stiff in order to resist change of form.

(3) **Redundant Frames** are those which have *more* bars than are necessary to resist distortion. In frames of this kind we cannot alter the length of certain bars without stressing one or more of the other members. Further, the frame may be self stressed if the redundant bars be badly fitted, and the stresses in the various members are indeterminate unless their yieldingness be taken into account.

Conditions of Equilibrium.—There must be no translation. This is assured if the diagram of external forces is a closed polygon. In other words, their "*Vector Sum*" must = 0.*

There must be no rotation. This is satisfied if:—

(1) The external forces have no resultant movement round any point that may be chosen.

(2) The line of action of the resultant of all except two of the forces passes through the point of intersection of the lines of action of these two forces.

If a number of external forces act upon a structure and keep it at rest, and, if we have to determine graphically the relations among these external forces, we must know at least:—

Either.—**All the elements of all the forces except one and nothing about that one.**

Or.—**All the elements of all the forces except two, and about one of these two its line of action. About the other, one point in its line of action.**

In the former case, we determine the resultant of all the given external forces by any method, and the last or balancing force (that is, the one we know nothing about) has (1) its point of application anywhere in the line of action of the resultant, (2) its line of action coincident with the line of action of the resultant, (3) its direction or way opposite to that of the resultant, and (4) its magnitude is the same as that of the resultant.

The second case will be clear by a reference to Fig. 1.

B A is the resultant of the external forces, 1.2.3...(n - 2), acting on the body or frame. D C is the line of action of the

* See the note at the end of this Lecture for a definition of Scalar, Vector, and Rotor.—A.J.

$(n-1)^{th}$ force, and E the point chosen as a point in the line of action of the n^{th} force.

If three forces act upon a body and keep it at rest their lines of action must all pass through one point. Thereby, the line of action F E, of the n^{th} force may be determined, since it must pass through O the point of intersection of B A with D C. Then by an application of the triangle of forces the magnitudes and ways or directions of the $(n-1)^{th}$ and n^{th} forces may be determined.

Bow's Method of Lettering.—In Fig. 2 we have an example of Bow's method of lettering a system of forces. It will be seen

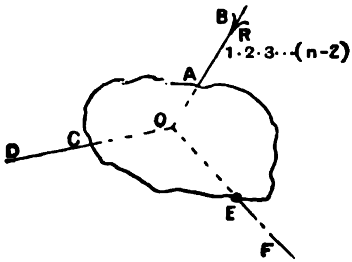


Fig. 1.—Relation among External Forces.

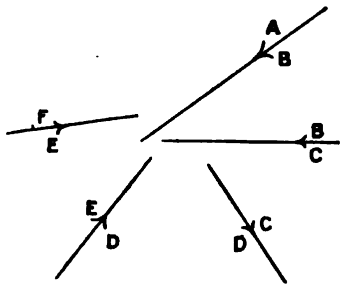


Fig. 2.—Illustration of Bow's Method of Lettering.

that every force has one letter on each side of its line of action. This is in order to name the force. Thus, we speak of the forces A B, B C, C D, D E, and E F. Again, each letter has been used twice, excepting A and F. This would indicate that one force was awanting or required to be determined:—viz., the force A F. This force may be the resultant or the equilibrant as the case may be; or, if on drawing the polygon of forces, F coincides with A (that is, the magnitude of F A is zero), then the system is in translationary equilibrium.

In Fig. 3, we have the forces acting at the joints of the triangular frame X Y Z, named by Bow's method.

The forces which keep in equilibrium the joints X, Y, and Z, are:—

For the joint X—

The force B C (all the elements of which are known).
The action of the stress * C D; and,
The action of the stress D B.

* As is usual, in treatises on this subject, the word *stress*, throughout Part IV., means the *total force* transmitted by the bar, and not the force per unit of cross area.

For the joint Z—

The action of the stress B D;
The action of the stress D A; and,
The supporting force A B.

And for the joint Y—

The action of the stress A D;
The action of the stress D C; and,
The supporting force C A.

The supporting force C A, has been represented by a curved dotted line to indicate that all we know about it is, its point of application.

The point X might be called the joint B C D; the point Y the joint C A D; and the point Z the joint A B D, since the letters naming a joint have been used to name the forces acting at that joint.

In Fig. 3, we have used the letters A, B, and C each four times and the letter D six times. In practice, this is avoided by lettering, as indicated in Fig. 4. Then the forces and bars will have the same names as before. Success in graphic solu-

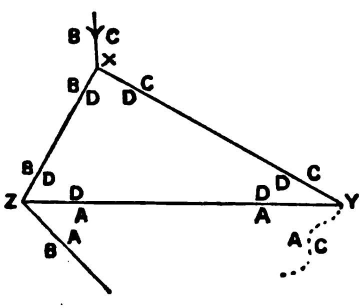


Fig. 3.—Bow's Method of Lettering a Triangular Frame.

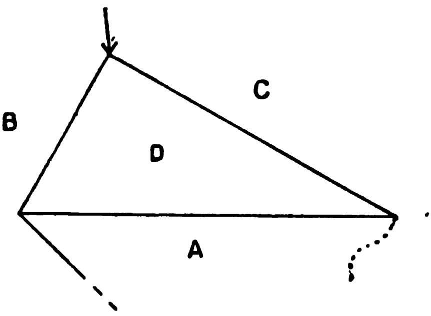


Fig. 4.—Bow's Method in Practice.

tions depends in a great measure on correct lettering, and on correct assumptions having been made, first, with regard to the total number of external forces that act on the frame, and second, with regard to what is known about the various elements of these external forces.

One great advantage of the Graphic Method of Solution is, that our attention is always being directed to the correctness of any assumptions that have been made. If the Stress Diagram

closes, we may safely consider the solution to be correct for the assumptions made; but, if it does not, some assumption is wrong or something has been left out.

Correct lettering is accomplished when every external force and every bar has one letter and only one on each side of it.

All the external forces must be applied at the joints of the frame, but if any should act at a point other than the end of a bar, then two equivalent parallel forces must be applied to the member under consideration, one at each end. By equivalent parallel forces is meant two forces which, applied as stated, would have the given force as their resultant.

The lines of action of the external forces must not fall inside the frame, but must be drawn outside, as in Fig. 4.

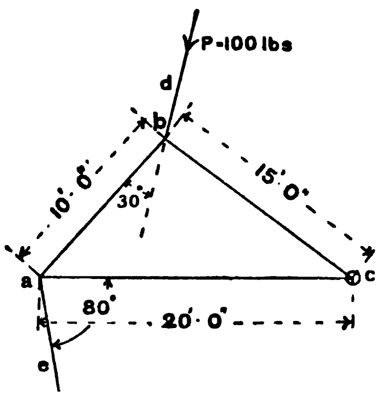


Fig. 5.—Sketch of Frame.

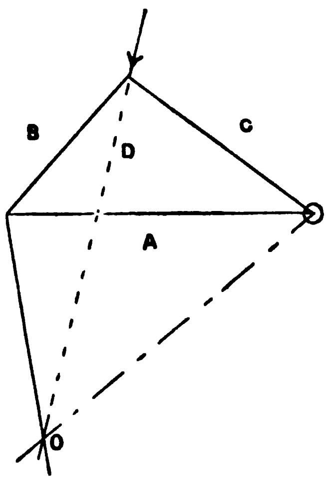


Fig. 6.—Frame Diagram.

Solution of a Triangular Frame.—*Given*, the triangular frame *a b c*, and the force P, completely specified as follows, viz. :—

Its point of application, *b*;
Its line of action, *d b*;
Its way or direction from *d* towards *b*; and,
Its magnitude, P lbs. Also,
The line of action of one of the supporting forces, *a e*.
And finally, a point *c*, in the line of action of the other supporting force.

It is required to find the remaining elements of the supporting forces, also the magnitudes and kind of stresses in the bars.

We begin by drawing the Frame Diagram (Fig. 6) to scale. This scale should be as large as possible, say, in this case, ¼ inch representing 1 foot. Then letter the Frame Diagram by Bow's method. Now, let the lines of action of the forces A B and B C, on being produced meet in O. Then *for no rotation*, the line of action of the other supporting force C A (as indicated by the chain dotted line) must pass through O, and also through the joint D C A, as given. Thus the line of action of the force C A is determined.

For no translation, the triangle of forces is applied, and will give the magnitudes and ways of the supporting forces, as indicated by Fig. 7. The scale used should be as large as convenient, say 1 inch representing 40 lbs.

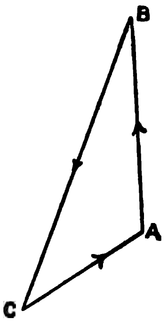


Fig. 7.—Diagram for External Forces.

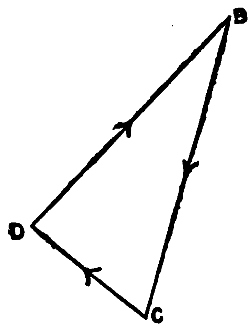


Fig. 8.—Reciprocal Figure for Joint B C D

Definition.—If from a point, a number of lines radiate, and if a polygon be drawn which has its sides either all parallel to, or all at right angles to corresponding radiating lines, then this Polygon is called the Reciprocal of the Point.

Thus, the triangle B C A, Fig. 7, may be called the reciprocal of the point B A C or O, in Fig. 6.

We can now draw the reciprocal figure for any one of the joints of the triangular frame. Because, we know all about the external forces acting at each of these joints; and further, not more than two bars meet at each joint.

If more than two bars meet at a joint, then we must know, in addition to all the external forces acting at that joint, the stresses in all the bars except two, before the reciprocal can be drawn.

Fig. 8 is the reciprocal figure for the joint B C D. It is drawn to the same scale and in exactly the same manner as Fig. 7, viz.:—B C parallel and equal to the external force B C; C D parallel to the bar C D; and D B parallel to the bar D B.

The length of the lines CD and B D, in Fig. 8 (measured to the same scale as that used for B C) determine the magnitudes of two forces which, acting in conjunction with the external force B C, would keep the joint B C D, at rest. The two forces C D and D B, are the actions on the joint of the stresses in the bars C D and D B, and therefore measure the magnitudes of the stresses in these bars. The arrow-heads give the ways or directions along the line of centres of the bars of the actions C D and D B.

Figs. 9 and 10 are drawn to the same scale and in the same way as Fig. 8, and represent the reciprocals for the joints B D A and A D C respectively.

From Figs. 9 and 10 we get similar information regarding the bars A D and D B, and their actions on the joint D B A, and the bars C D and D A, and their actions on the joint A D C, to that derived from Fig. 8 regarding the joint B C D.

In the reciprocal figure for the joint B C D, Fig. 8, the way

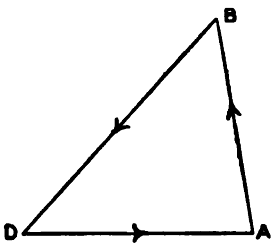


Fig. 9.—Reciprocal for Joint B D A.

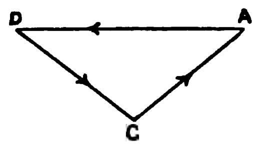


Fig. 10.—Reciprocal for Joint A D C.

of the action on the joint B C D, of the stress in the bar C D, is towards the left and upwards, while in Fig. 10 the way of the action of the stress in the same bar on the joint A D C, is towards the right and downwards.

We will now explain the cause of this apparent contradiction in the two reciprocals. The reciprocal for the joint B C D, shows that the way of the action on the joint B C D, of the stress in the bar D C, is towards the pin B C D—that is, pushing it. (This is indicated in Fig. 11 by the small arrow.) Then from Newton's third law the pin B C D, must push the bar with an equal and opposite force.

If the bar D C, pushes the pin B C D, it must also push the pin C A D. For this reason, that no bar can simultaneously push a pin at one end of itself and pull one at its other end. This is what the reciprocal for the joint A D C, indicates.

Definition of a Strut.—When the reciprocal, for a joint indicates that the way of the action of the stress in a bar is towards the joint, then that bar is under compression and is called a strut.

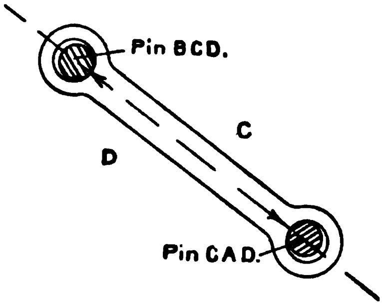


Fig. 11.—The Bar C D.

On reference to the reciprocals for the joints B D A and A D C, it will be seen that the bar D A, is pulling at the pins B D A and A D C. But, by the action and reaction law, the pins will pull at the ends of the bar, and this means that the bar D A, is under tensional stress of an amount measured by the length of the line D A, in the reciprocal figures.

Definition of a Tie.—When the reciprocal for a joint indicates that the way of the action of the stress in a bar is away from the joint, then that bar is under tension, and is called a tie.

In Fig. 12, the reciprocals for the three joints of the frame and the one for the point O, have been combined into one diagram, which may be called either the Stress Diagram or the reciprocal of the Frame Diagram.

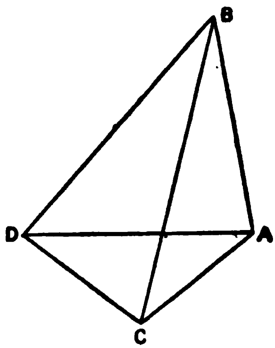


Fig. 12.—Combination of the Reciprocals, or Stress Diagram.

Definition.—Two figures are reciprocal when every point in the one has a corresponding reciprocal in the other.

For example, the point C, in Fig. 12 has the lines D C, A C, and B C, meeting in it. If we refer to the Frame Diagram, Fig. 6, we find that, the bar C D, the force B C, and the force A C, form the reciprocal for this point C in Fig. 12.

We do not put arrow heads on the Stress Diagrams; they would lead to confusion, and are quite unneces-

sary. Take for example the line A D, Fig. 12, from the reciprocal of the joint A B D, we would require an arrow head pointing from D towards A; while, from the reciprocal of the joint A D C, an arrow head would require to point the other way. (The double arrow on the line A D, points to the fact that a stress has no way.)

It will have been observed :—

(1) That when Figs. 7, 8, and 9 have been drawn they give all the information that was intended to be derived from drawing Fig. 10—viz., Fig. 8 gave the magnitude of C D, and Fig. 9 that of D A.

(2) That when we place the reciprocal for the joint A B D, on the reciprocal for the point O, as in Fig. 12, we have only to join D to C in order to complete the diagram.

These two observations point out that we have too much information; the excess is due to the finding of the point O. This frame is one of a class where we may find the stresses without first finding all the elements of the reactions or supporting forces.

Stress Diagram.—We shall now show how to determine the Stress Diagram direct from the Frame Diagram—*i.e.*, without first finding the reciprocals for the joints, and combining them into one.

It is quite immaterial as to which way we go round a structure —*i.e.* (referring to Fig. 6), whether we go from A to B and then to C, or the other way round. We shall find it to be an advantage to go round every structure in the same way as the hands of a watch. By doing so we shall find that the Stress Diagram will always lie to the left hand of the external force polygon. This will enable us to know where to begin the external force polygon in order to leave room for the Stress Diagram.

Referring to Fig. 6, where we are not supposed to know either the point O, or the line of action of the force C A, let us plot out therefrom the Stress Diagram, Fig. 12.

(1) Draw BC, in Fig. 12, parallel to the line of action of the external force B C in the Frame Diagram, Fig. 6, and containing 100 units, to some convenient scale, say 1 inch to represent 40 lbs. The correct lettering of this line is a very important part of the work. Since we are going round the frame in the direction of the hands of a watch—that is, from B to C—then B must be put at the top end and C at the bottom end of the line just drawn so as to indicate the way of the force correctly. If this point is attended to, little trouble will be experienced in drawing the diagrams.

(2) Draw from the last point found (viz., C) a line parallel to some force or bar which has C as one of the letters for its name; for example C D.

(3) Draw from the other end B of the line B C, a line parallel to the bar B D, and mark the point of crossing of the two lines D.

(4) Through D, the last point determined, draw D A parallel to the bar D A, and from some of the other points found draw a line parallel to a force whose line of action is known. Now, C A cannot be used because we only know its point of application, but we know the line of action of B A. Then drawing from B, in Fig. 12, a line parallel to the line of action of the supporting force A B, we determine the point A.

(5) On joining C with A we obtain a line parallel to the line of action of the supporting force C A, and the length of this line, C A, measures the magnitude of the force.

(6) Measuring the lines in Fig. 12 with the scale used to draw down the line B C, we obtain the magnitudes of the stresses in all the bars and of the two supporting forces.

How to Determine the kind of Stress in a Bar.—We will begin with the consideration of the forces which act on the left-hand joint—viz., the joint B D A, in Fig. 12.

Success in this part of the work depends almost entirely upon giving to each bar meeting in the joint under consideration its proper name—*i.e.*, by letters in their proper order.

Since we have gone round the external forces in drawing the Stress Diagram from B to C, &c.—that is, in the direction of the hands of a watch—we must go round each joint of the structure in the same way when naming the bars meeting in that joint.

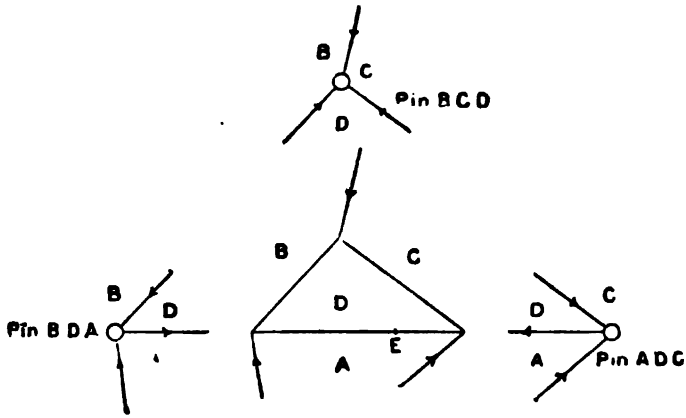
The bars meeting in the joint B D A, would therefore be called, the bar B D, the bar D A, and the supporting force or reaction A B. Having thus determined the name of the bar, we then refer to the Stress Diagram in order to find the way in which the stress in that bar acts with regard to the joint.

Take for example the horizontal member in the Frame Diagram, Fig. 13 (this member is called the tie rod or tie beam, since it ties the lower ends of the rafters together), its name with reference to the joint B D A, is D A. Now, in the Stress Diagram D is on the left of A, and, therefore, the stress in the bar D A, acts from left to right (*i.e.*, from D to A) with respect to the pin at the joint B D A. This means that the bar D A, is pulling at the pin B D A, and therefore the pin B D A, pulls at the bar, thereby putting the bar into tension.

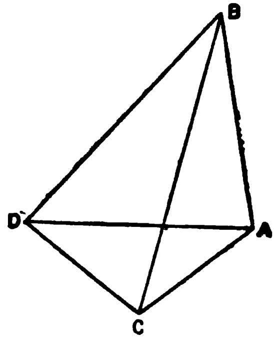
Similarly the stress in the bar B D (called a rafter) acts, so far as the joint B D A is concerned, in the direction indicated by B D in the Stress Diagram—that is, from B to D. The bar B D, is therefore pushing at the joint B D A, and is thus put into compression by the reaction of the pin B D A.

RULE TO DETERMINE THE KIND OF STRESS IN A BAR.—Take

the letters on each side of the Bar in the Frame Diagram, in the same order with respect to the joint on which the Bar acts, as we have taken the letters on each side of the External Forces acting on the Frame. Then along the line in the Stress Diagram, which is named after the Bar, from the first letter



Frame Diagram.



Stress Diagram.

Fig. 13.—Stress Action on Pins.

towards the second, gives the way of the stress' action with respect to the joint under consideration. If the way is towards the joint the stress in the Bar is Compression or Push, and the Bar is called a Strut. If the way is away from the joint the stress is a Tension or Pull, and the Bar is called a Tie.

The above rule may also be applied to a point in a bar. Take, for example, the point E, in the tie rod, and suppose we want to find how the left-hand portion of the tie rod acts upon the section at E. Then the name of the left-hand portion of the tie rod with respect to E is A D, and from the Stress Diagram this acts from right to left—that is, away from E—and is therefore pulling at the section.

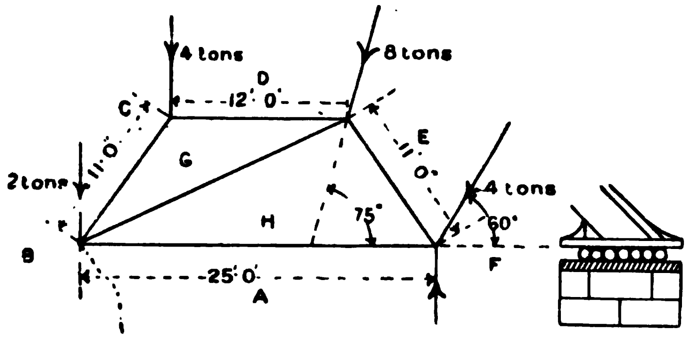


FIG. 14*a*.—FRAME DIAGRAM.
FRAME SOLVABLE WITHOUT KNOWING ALL ABOUT REACTIONS.

Notice that the tie rod, with respect to the left-hand joint, is called D A, and with respect to the right-hand joint would be called A D, and similarly with any other bar in the frame.

The above rule for the kind of stress does away with the use of arrow-heads and of supplementary diagrams.

The action of all the bars on the pins of the frame are shown in the small diagrams surrounding the Frame Diagram of Fig. 13.

Firm Quadrilateral Frame.—This frame is one of a type which allows a solution to be found without having first determined all the elements of the reactions.

We shall assume that the right-hand end rests on rollers, as indicated in Fig. 14*a*. Consequently the line of action of the reaction is practically vertical. If it simply slides instead of rolling, then the reaction is inclined to the normal at an angle equal to the angle of friction, and inclined

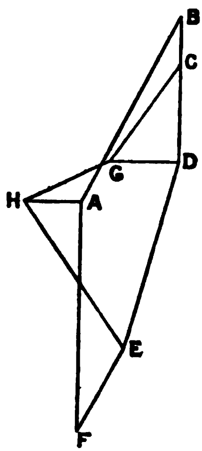


Fig. 14*b*.—STRESS DIAGRAM.

to that side of the normal which will oppose the motion of the frame. The left-hand end of the frame in Fig. 14*a* is assumed to be anchored to the wall by bolts, &c. All we know about the left-hand reaction is its point of application.

If we suppose both ends of a frame to be anchored, then we only know the points of application of the reactions, and we assume that their lines of action are parallel to each other and to the line of action of the resultant of the external forces.

We begin by drawing the Frame Diagram to as large a scale as possible, and then indicate the external forces at the joints by their lines of action. The right-hand reaction is indicated by vertical line, and the left-hand reaction by a dotted curved line, as shown in Fig. 14*a*. We then letter the diagram according to Bow's method.

In drawing the Stress Diagrams, we shall always go round the Frame Diagrams clockways.

We begin by drawing a line parallel to the line of action of the first force or load B C. This line should contain as many units of length as B C contains units of force, which in this example is 2 tons.*

Then draw C D parallel to the line of action of the load C D, and containing 4 units of length corresponding to the 4 tons load. Next draw D E parallel to the line of action of the load D E, and E F parallel to the line of action of the load E F, representing 8 units and 4 units, respectively.

The line B C D E F is called the Line of Loads.

In order to complete the Stress Diagram we shall begin with the joint C D G, which is the only joint of which we have sufficient data. Draw from the point C in the "Line of Loads" a line parallel to the bar C G, and from D a line parallel to the bar D G. The intersection of these two lines is called the point G. From G draw G H parallel to the bar G H, and from E draw E H parallel to the bar E H. This determines the point H. Then draw H A parallel to the bar H A, and from F draw a line parallel to the line of action of the reaction F A. The intersection of these two lines fixes the point A. Joining A with B gives the finishing line of the Stress Diagram. The line A B in the Stress Diagram is parallel to the line of action of the left-hand reaction.

By applying the rule for the kind of stress, we can determine from the diagram all we may wish to know—*e.g.*, with respect to the top right-hand joint, the diagonal member is called H G.

* The scale for the diagram should be as large as convenient. A rough guess may be made by adding all the loads together, and assuming that this will be the total vertical length of the diagram.

On reference to the Stress Diagram we see that the way of its action is from H to G which means pushing at the joint. Therefore, the diagonal member is in compression, and so on for the other members.

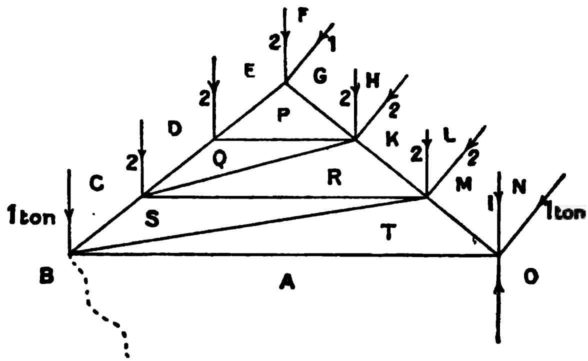


FIG. 15*a*.—FRAME DIAGRAM.
FRAME WITH WIND PRESSURE.

The magnitudes of the stresses are measured by the lengths of the lines in the Stress Diagram.

The polygon B C D E F A is called the polygon of external forces.

Firm Triangular Frame.—This frame, Fig. 15*a*, can also be solved without knowing all about the reactions.

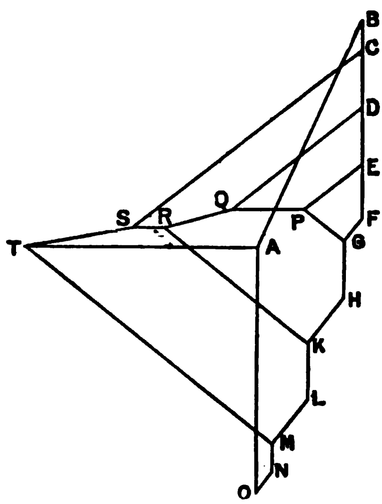


FIG. 15*b*.—STRESS DIAGRAM.

The right-hand end of the frame is assumed to be resting on rollers, while the left-hand end is anchored to the wall. The vertical loads on the Frame Diagram represent the action of gravity on the roofing, such as slates, &c., which is assumed to be uniformly distributed over the surface.

In Fig. 15*a*, the rafters are shown divided into three equal

parts called bays; and, since the joint at each end of a bay must carry one half of the uniformly distributed load over that bay,

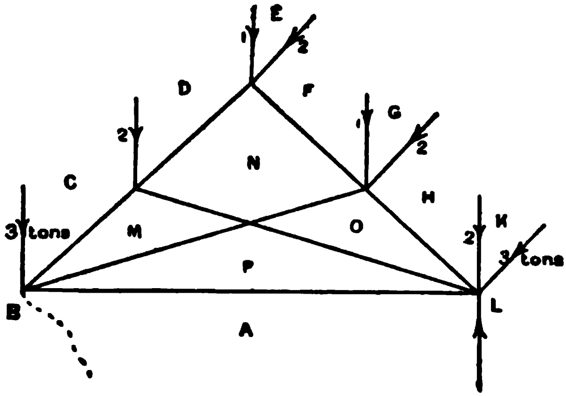


FIG. 16a.—FRAME DIAGRAM.
FIRM FRAME WITH A QUADRILATERAL PART.

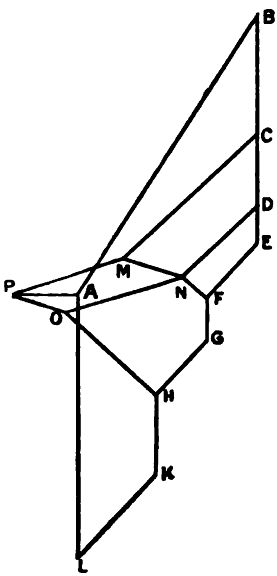


FIG. 16b.—STRESS DIAGRAM.

the vertical loads will have the proportions shown by the numbers on the Frame Diagram. Wind pressure is also assumed to be uniformly distributed, and is reckoned as so many lbs. per square foot normal to the rafters. This is indicated on the right-hand side of the Frame Diagram.

Note.*—When wind pressure acts on the rafter which is anchored, the stresses in the members of the frame are more severe than when it acts on the free rafter. This should be remembered when designing a roof.*

Since we know all the elements of the external forces, the line of loads may be drawn as in Fig. 14b.

Therefore, in order to complete the Stress Diagram we can begin at the top joint of the Frame Diagram where only two members meet. This enables us to find first the point P in the Stress Diagram, then the point Q, and so on.

Firm Frame.—The frame represented in Fig. 16*a* is of the same class as the two preceding. In drawing the Stress

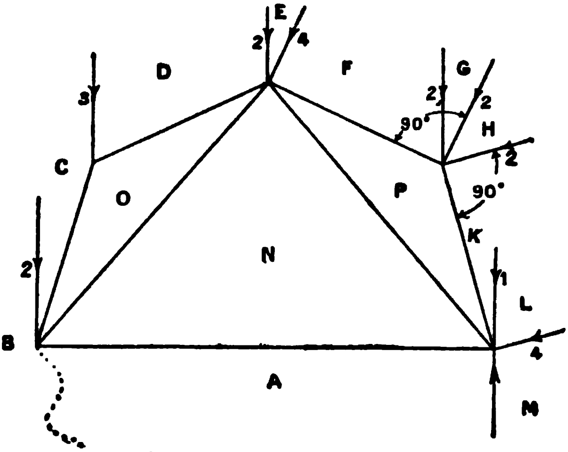


FIG. 17*a*.—FRAME DIAGRAM.

FIRM FRAME WITH MANSARD OUTLINE.

Diagram, although we have determined the point N and the point M, we cannot fix the point P, until we obtain the point O. After that, the diagram closes in the usual way.*

Firm Frame with Mansard Outline.—In Fig. 17*a* we have illustrated a frame having the double-sloped outline of the Mansard Roof. It is of the same type as Fig. 16*a*, and presents the same peculiarity in the drawing of the Stress Diagram. Wind pressure is indicated on the right-hand rafters. The forces E F and G H are both normal to the bar F P. *They should, however, be of equal value.* Also, the forces H K and L M are both perpendicular to the bar K P.

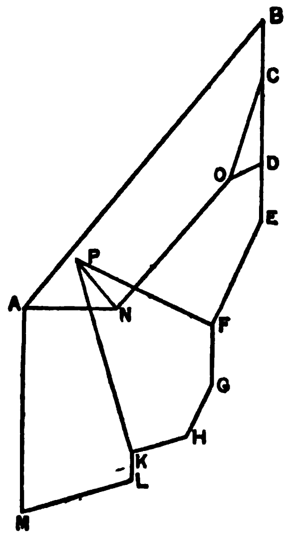


FIG. 17*b*.—STRESS DIAGRAM.

*It may be objected, that we cannot fix the point P, since M N O P is not a fixed point, for M N and O N act independently of each other.

Note for page 155.—The word *Vector* was used in defining the conditions of equilibrium in frames, consequently it may be as well to define the following terms here: —

Scalar.—A quantity which has no relation to definite direction in space, or which is considered apart from such direction, is called a "Scalar" or "Scalar-Quantity."

Vector.—A geometrical quantity which is related to a definite direction in space is called a "Vector" or "Vector-Quantity."

Vector-Quantity.—This requires for its complete determination (1) the magnitude, (2) the direction, and (3) the *sense* to be given.

A vector-quantity may be geometrically represented by a line, if—

(1) The length of the line represents to scale the magnitude of the quantity.
(2) The line be placed in the proper direction.
(3) The proper sense or way be given to the line.

The sense is usually indicated by an arrowhead on the line.

The line itself with its direction and sense is called a *Vector* $\rightarrow$

Suppose that a force of known magnitude acts along a line from P to $\rightarrow$ Q; then, the Vector is written down as $\overline{PQ}$, with a bar-line over the two letters P and Q.

Any quantity, whether *scalar* or *vector* (considered as occupying a definite position in space), is said to be *localised*. Thus the mass of a body in a given position is a *localised scalar*, and a force acting on a body at a definite point is a *localised vector*.

Vector Sum.—The sum of a number of vectors is often called the Resultant Vector, and in relation to this resultant the other Vectors are called Components.

To add a number of vectors, place the first anywhere, the beginning of the second to the end of the first, and so on, then the vector from the beginning of the first to the end of the last is the SUM OF THE GIVEN VECTORS (*Henrici and Turner*).

Rotor.—A localised vector is called a *Rotor* (*Clifford*).

LECTURE XXIV.—QUESTIONS.

1. What is a frame or framed structure? Distinguish between the three different kinds of frames.

2. Explain in your own words Bow's method of lettering a system of forces, with two examples.

3. What is meant by the reciprocal of a point, and a pair of reciprocal figures?

4. Explain how you would represent forces in a diagram so as to determine those in each part of a structure, and explain the principles upon which the construction depends.

5. State a rule for determining the kind of stress in a bar.

6. Illustrate and explain how you would find the stresses in a firm quadrilateral frame.

7. Illustrate and explain how you would find the stresses in a firm triangular frame.

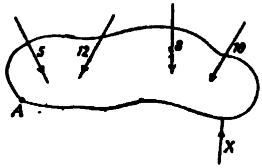
8. Illustrate and explain how you would find the stresses in the outline of a Mansard frame.

9. Draw a parallelogram, A B C D. The side A B is 2 inches, and the side A D is 3 inches; the angle B A D is 75°. The point E bisects the side C D. There are forces, in the direction D A of 15 lbs.; in the direction A B of 20 lbs.; in the direction E B of 23 lbs.; find the resultant, giving its magnitude, the angle which it makes with A D, and its sense.
(B. of E. Adv., 1900.)

10. Draw a parallelogram, A B C D. The side A B is 2 inches, and the side A D is 3 inches; the angle B A D is 75°. The point E bisects the side C D. There are forces in the direction D A of 15 lbs., in the direction A B of 20 lbs., in the direction E B of 23 lbs. What two forces, acting one along D C and the other passing through the centre of the parallelogram, will produce equilibrium? (B. of E. H., Part I., 1900.)

LECTURE XXIV.—A.M.INST C.E. EXAM. QUESTIONS.

1. The figure shows forces acting upon a structure in a plane; the direction and position of one supporting force, X, are shown; you are informed that the other acts through the point A. Find these supporting forces. (I.C.E., *Oct.*, 1898.)



2. The following forces act on a particle situated at the origin of co-ordinates in the plane of the paper:—A force of 5 lbs. making an angle of 30° with the axis of x, 9 lbs. at an angle of 90°, 7 lbs. at 135°, 10 lbs. at 225°, and 3 lbs at 300°, find graphically the resultant in magnitude and direction. (I.C.E., *Oct.*, 1899.)

3. Write down the conditions of equilibrium for a particle and for a rigid body. (I.C.E., *Oct.*, 1899.)

4. What are the graphical conditions of equilibrium for a system of coplanar forces? (I.C.E., *Oct.*, 1899.)

5. Show how to draw the link polygon for a system of coplanar forces acting on a body. If the force polygon closes and the link polygon does not, show that the forces acting on the body are equivalent to a couple. (I.C.E., *Feb.*, 1900.)

6. A masonry wall 10 feet in height, with a uniform thickness of 2 feet, is subjected to a horizontal wind-pressure of 28 lbs. per square foot on one side. Taking the weight of the masonry at $1\frac{1}{4}$ cwts. per cubic foot, find the point at which the resultant line of pressure will intersect the base of the rectangular section of the wall. (I.C.E., *Feb.*, 1901.)

LECTURE XXV.

CONTENTS.—Substituted Frames—King Post Truss—Right-Angled Strut Truss—Roof Truss—Load at an Internal Joint of a Frame—Modified French Truss—Bowstring Truss—Questions.

Substituted Frames.—The types of frames illustrated in the previous Lecture, although not practical examples, are intended to be substituted for some other actual form in order to determine the reactions therein, since **the reactions do not depend upon the form of frame carrying the roofing, but merely on the distribution of the loads.** In substituting one of the above frames for a practical one, we must have the joints of the substituted frame coincident with those of the given one. This will be illustrated by the following examples:—

King-Post Truss.—In Fig. 18 we have the Frame Diagram of a king post truss with wind pressure on the right-hand rafter. In this case, we assume both rafters to be anchored to the walls. Therefore, all we know about the elements of the reactions are

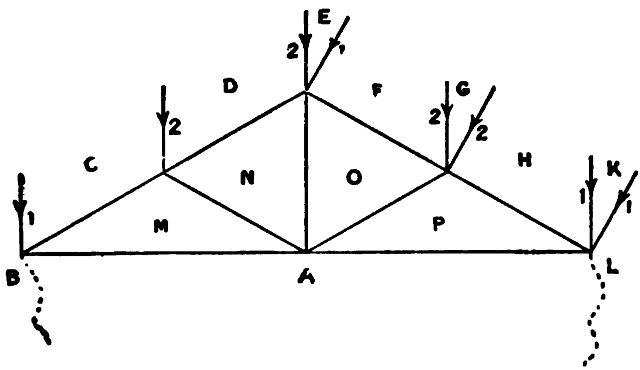


FIG. 18.—FRAME DIAGRAM OF KING POST TRUSS.

their points of application, and that their lines of action are parallel to each other, as well as to the line of action of the resultant of the external forces.

Before we can determine the Stress Diagram for this frame we must first determine the reactions, because more than two bars

meet in each of the joints except the two where the reactions act. Consequently, until we determine all the elements of the reactions we cannot begin at either of these two joints.

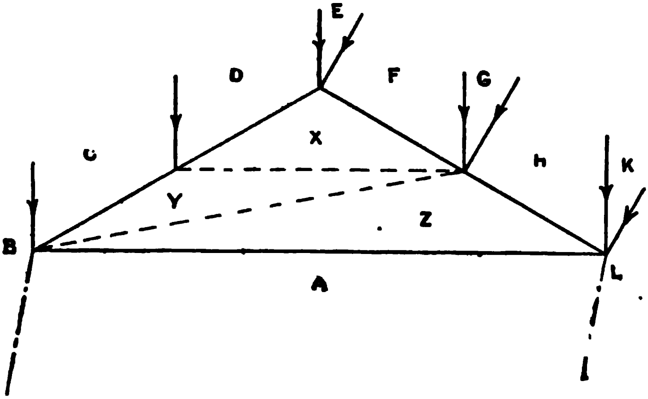


FIG. 19.—SUBSTITUTED FRAME.

In order to determine the reactions, we shall substitute a frame similar to that illustrated in Fig. 15*a*. This substituted

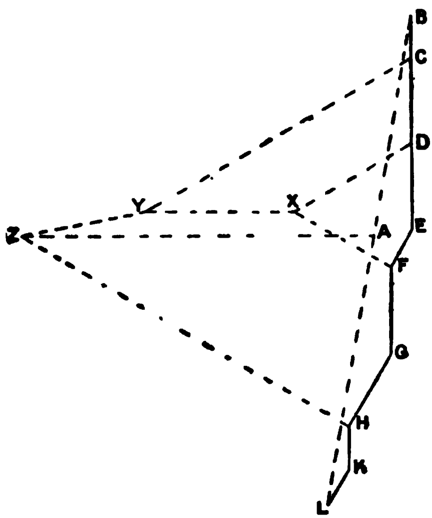


FIG. 20.—STRESS DIAGRAM FOR SUBSTITUTED FRAME.

frame is shown in Fig. 19. In practice, this frame is merely sketched in order to apply the proper letters. The dotted lines are drawn in the Frame Diagram, or the set square is simply made to pass through the requisite joints, and then the lines are drawn parallel thereto in the Stress Diagram. To obtain Fig. 20 we begin by drawing the line of loads. Then, we find the point X, when a line from the point X drawn parallel to the substituted bar X Y, and one from the point C parallel to the rafter C Y fix the point Y. Next we find the point Z. Now, the line of action of the resultant of the external forces is

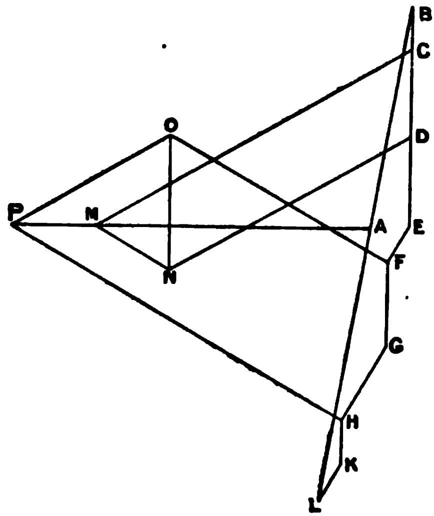


FIG. 21.—STRESS DIAGRAM FOR KING POST TRUSS.

parallel to the line joining L with B. Therefore, the point A must lie on this line since the reactions L A and A B are parallel to each other and to the line of action of this resultant. Consequently, we find the point A by drawing through the point Z a line parallel to the bar Z A so as to intersect L B in the point A. This determines all the remaining elements of the reactions, viz.:—

(1) Their lines of action parallel to L A and A B.
(2) Their ways from L towards A, and from A towards B.
(3) Their magnitudes by the number of units of length in the lines L A and A B.

We can now draw the Stress Diagram for the king post truss,

as shown in Fig. 21, from which the particulars for the various members may be determined. Comparing Fig. 21 with Fig. 20, we see that nearly all the lines of Fig. 21 lie along the lines of Fig. 20. In practice we simply draw Fig. 21 on the top of Fig. 20.

This method of a substituted frame introduces fewer errors due to drawing, than the usual method of the funicular polygon (which will be illustrated further on), because we make use of the same joints of the frame for the two Figs. 20 and 21, and the same line of loads.

There is one line in Fig. 21 which will check the accuracy of the Stress Diagram. In drawing the diagram we begin with the point M, and then find the points N, O, and P. The line joining P to H will then be parallel to the rafter P H, if the Stress Diagram is correct.

Right-Angled Strut Truss.—In this frame we have introduced loads at the lower joints as well as roofing weights and wind

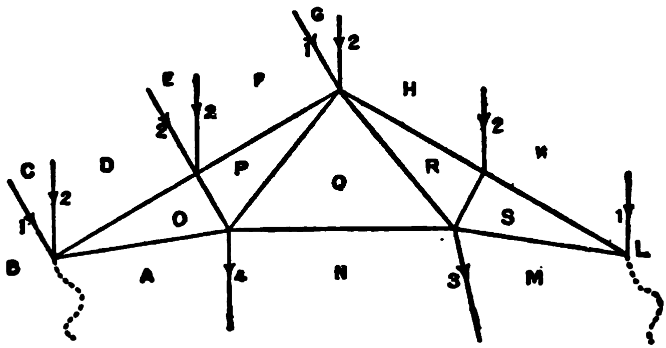


FIG. 22.—FRAME DIAGRAM FOR RIGHT-ANGLED STRUT TRUSS.

pressure. We also assume the two rafters to be anchored to the walls, as indicated by the two dotted curved lines L M and A B.

We must first find the reactions before we can draw the Stress Diagram. In finding the reactions we will substitute the frame which is shown in Fig. 23.

FIRST METHOD OF OBTAINING STRESS DIAGRAM FOR ORIGINAL FRAME.—Produce the lines of action of the loads at the lower joints until they intersect the rafters. These points of intersection are considered as joints in arranging the substituted frame and the lower loads assumed to be acting at these joints

as shown in Fig. 23. The forces fF and H h in Fig. 23 are the loads N A and M N in Fig. 22 transferred as explained.

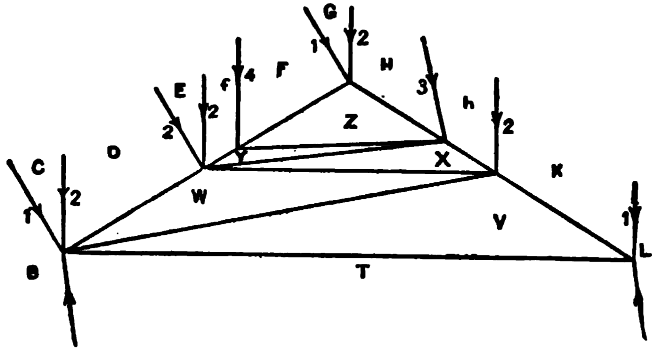


FIG. 23.—SUBSTITUTED FRAME.

The Stress Diagram for the substituted frame is illustrated in Fig. 24 and presents no difficulty requiring explanation. This diagram gives the reactions L T and T B. (*See note at the end of this Lecture.*)

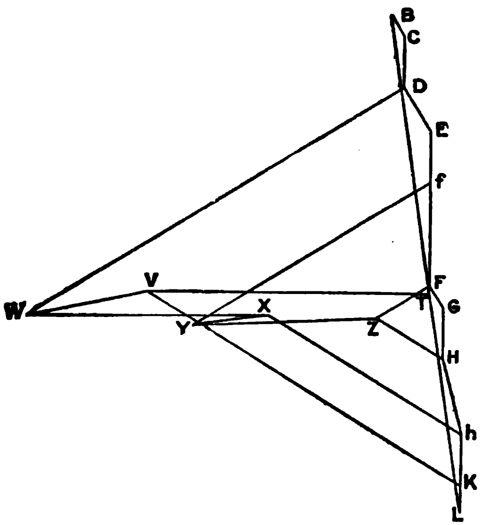


FIG. 24.—STRESS DIAGRAM FOR SUBSTITUTED FRAME.

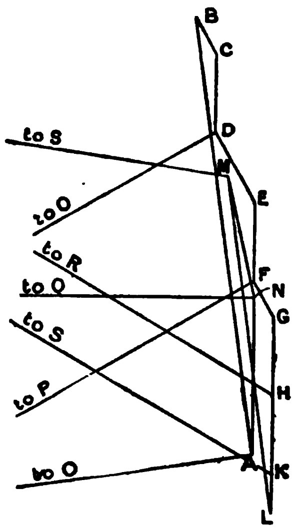


FIG. 25.—STRESS DIAGRAM FOR ORIGINAL FRAME.

In order to draw the Stress Diagram for the original frame which is illustrated in Fig. 25, it is necessary to redraw the line of loads taking them in their order as in Fig. 22. That is, B C,

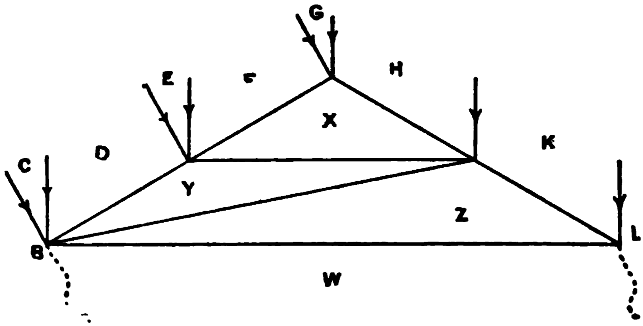


FIG. 26a.—FRAME DIAGRAM.
SUBSTITUTED FRAME FOR TOP JOINT LOADS.

C D, D E, E F, F G, G H, H K, K L, reaction L M, M N, N A and then reaction A B. The drawing of the remaining part of the Stress Diagram calls for no special remark. (The above Stress Diagram is not completed for want of space.)

SECOND METHOD.—Take the top joint and the lower joint loads separately. In Fig. 26a we have the substituted frame for the top joint loads and its Stress Diagram. The Stress Diagram, Fig. 26b, determines the reactions L W and W B due to the loading on the rafters.

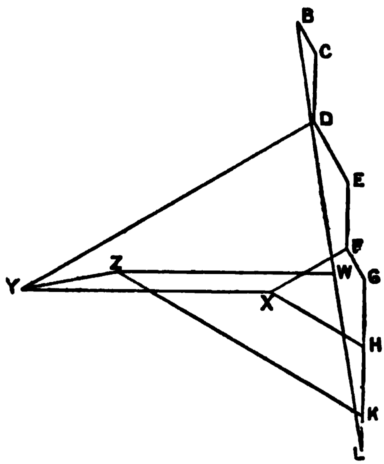


FIG. 26b.—STRESS DIAGRAM.

In Fig. 27 we have a frame similar to the one illustrated in Fig. 14a. The joint A N S is any point in the line of action of the load N A in Fig. 22, and the joint N M R S any point in the line of action of the load M N in Fig. 22. The left and right hand lower joints of Fig. 27 are the rafter ends in Fig. 22

In Fig. 27 the load T A is the reaction W B found in Fig. 26*b* reversed. The loads A N and N M are the loads at the lower

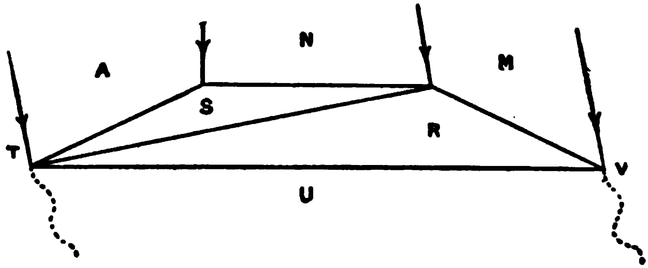


FIG. 27.—SUBSTITUTED FRAME DIAGRAM FOR LOWER JOINT LOADS.

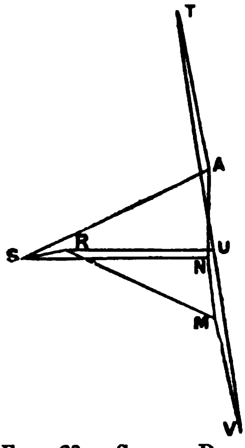


FIG. 28.—STRESS DIAGRAM FOR FRAME IN FIG. 27.

joints in Fig. 22, and the load M V is the reaction L W of Fig. 26*b* reversed. Therefore, if we draw the Stress Diagram for the frame of Fig. 27 we determine the reactions due to all the loads of the original frame of Fig. 22.

In Fig. 28 we have the Stress Diagram for the frame of Fig. 27. The reactions are represented by the lines V U and U T. This figure has been drawn to a smaller scale than Fig. 24, but the lines V U and U T of Fig. 28 contain the same number of units as L T and T B of Fig. 24.

Roof Truss.—In Fig. 29 we have a frame of a type that will not allow of the Stress Diagram being drawn in a regular manner, but only in a step by step process.

Fig. 30 shows the substituted frame used in order to determine the reactions L A and A B, and in Fig. 31 we have the Stress Diagram for both Figs. 29 and 30.

In Fig. 31 we draw first the line of loads, second we find the point O, then O X and D X fix the point X, and X Y and K Y fix the point Y. On drawing Y A parallel to the bar Y A, and L A parallel to the line of action of the reaction at the right-hand joint, we determine the reactions L A and A B.

The point O is the same in both Stress Diagrams, but we

cannot determine the points N and Q until we have found the points M and R. The point M is found by drawing D M

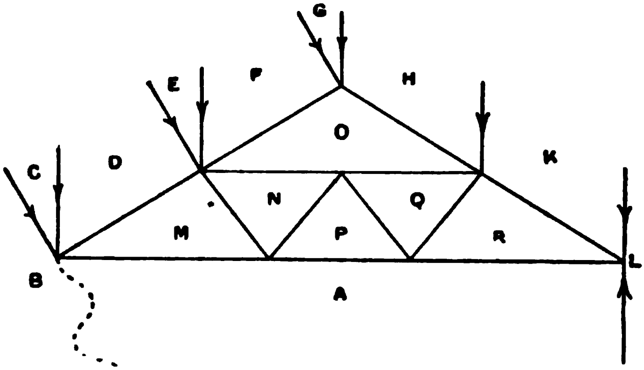


FIG. 29.—FRAME DIAGRAM OF ROOF TRUSS.

parallel to the rafter D M, and A M parallel to the tie-bar A M; then M N and N O fix the point N, and similarly for the point

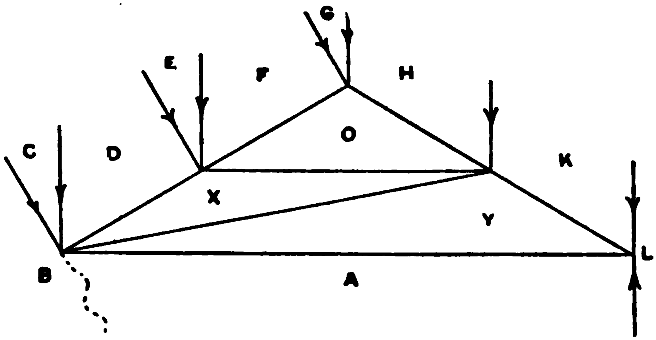


FIG. 30.—SUBSTITUTED FRAME FOR FIG. 29.

Q. Again, although N may be determined, P cannot be fixed until we have found Q.

The line N P or P Q forms a check line.

Note.—*The dotted lines of Fig.* 31 *are the only part of the Stress Diagram for the substituted frame that has not been required for the original frame.*

Load at an Internal Joint of a Frame.—In Fig. 32*a* we have a frame the same as the last one, but with a load applied at a joint inside the frame. This load is represented in the Frame Diagram by the small arrow near the letter P, and is applied at the joint O P Q T.

Note.—If the force had been applied at a point in the bar, then **equivalent parallel forces** *applied to the joints at the end of the bar will allow a solution to be determined.*

To obtain a solution for this frame loaded as shown, introduce a bar with its centre line lying along the line of action of

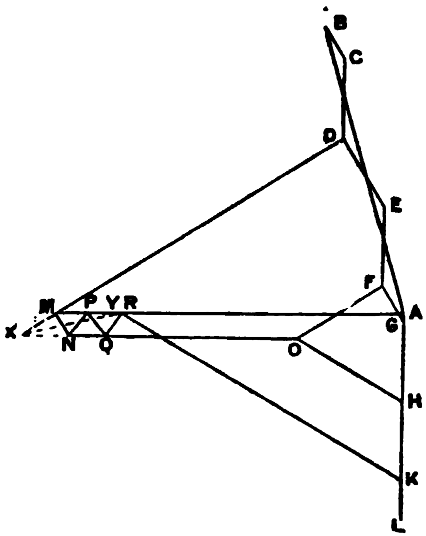


FIG. 31.—STRESS DIAGRAM FOR BOTH ROOF TRUSS AND SUBSTITUTED FRAME.

the given force. Then, where this bar cuts an outside member of the frame, apply a force having all its elements (excepting the point of application) the same as those of the given load.

The introduced bar is represented in Fig. 32*a* by the dotted line S T, and the applied force by the chain dotted line M A. The force M A will have the same action on the members of the frame through the bar S T, that the load at the joint O P Q T has, therefore the load must be left out after M A is applied.

Note.—The introduced bar might have been placed between the

*joint where the load is acting and the rafter, and the equivalent force applied at the rafter end of the bar.**

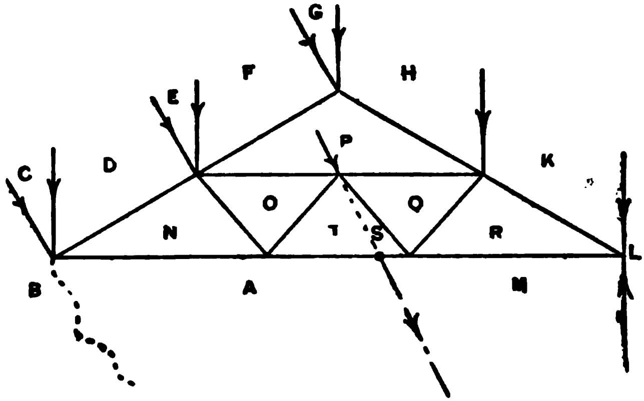


FIG. 32*a*.—FRAME DIAGRAM.
ROOF TRUSS, WITH LOAD AT INTERNAL JOINT.

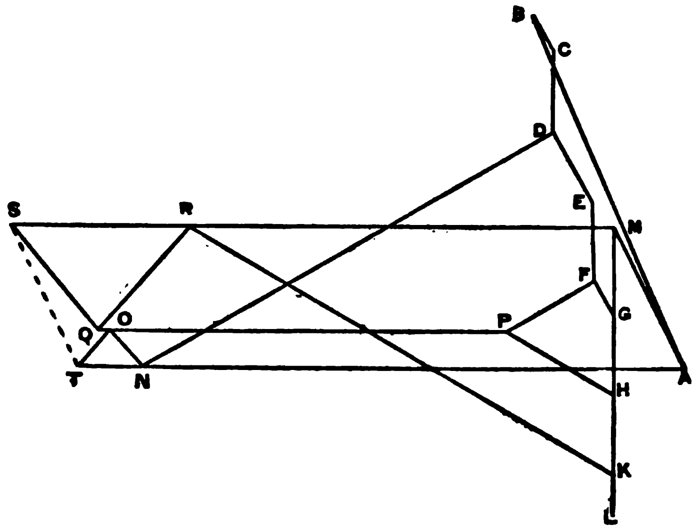


FIG. 32*b*.—STRESS DIAGRAM.

* The student should work this method as an exercise. Some of the lines of the Stress Diagram will be lowered but the stresses will remain unchanged.

The Frame Diagram, Fig. 32*a*, is loaded similarly to the frame of Fig. 22. The reactions are therefore found in the same way. After the external force polygon has been drawn, the Stress

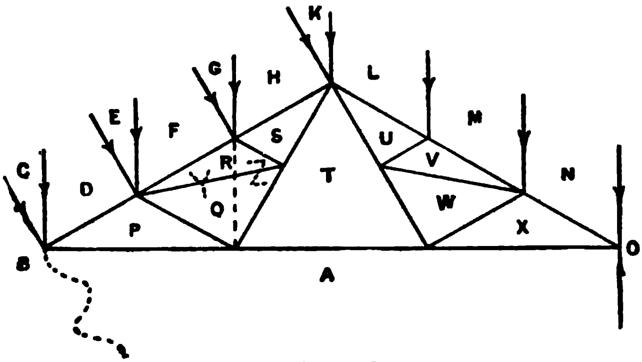


FIG. 33*a*.—FRAME DIAGRAM.

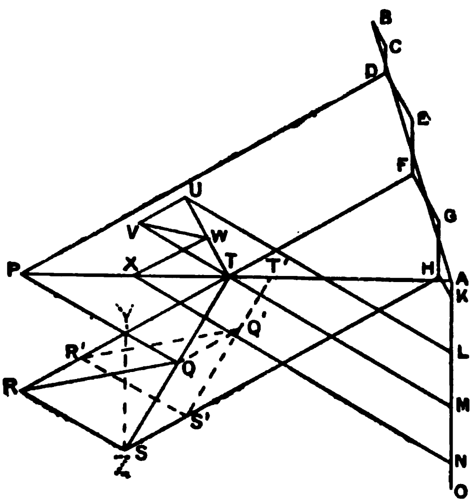


FIG. 33*b*.—STRESS DIAGRAM.
FRAME REQUIRING SPECIAL METHODS FOR SOLUTION.

Diagram may be completed by the same method as that used for the frame of Fig. 29.

Referring to the Stress Diagram of Fig. 32*a*, we see that the

bar T S exerts the same pull at the joint T S M A as the force M A. These two therefore balance each other, and we are left with the pull which the bar S T exerts on the joint O P Q T. This pull S T, on the joint O P Q T, is identical in all its elements with the load; and therefore, the stresses in the members of the frame will be identical with those due to the original load.

Modified French Truss.—This Truss is illustrated in Fig. 33*a*, and presents some difficulties in its solution. We first determine the reactions as already explained and draw the external force polygon as in Fig. 33*b*. Secondly, we draw D P and A P parallel to the bars D P and A P respectively. This fixes the point P. But, although we know the point P, we can get neither Q nor R, nor any other point but X. This point X, however, does not help us, because we can proceed no further by aid thereof with the Stress Diagram.

First Method of Obtaining the Stress Diagram.—We know that the point R lies on a line drawn through F parallel to the bar F R and that S lies on a line drawn through H parallel to the bar H S. Now, assume a point R′ anywhere on the line F R and draw R′ S′ parallel to the bar R S and H S′ parallel to the bar H S. This fixes the point S′. Then S′ T′ and A T′ fix the point T′, and R′ Q′ and T′ Q′ fix the point Q′. Next move the figure R′ S′ Q′ parallel to itself keeping R′ on the line F R until Q′ lies on a line drawn through P parallel to the bar P Q. This is done by drawing Q′ Q parallel to F R so as to intersect P Q in Q. This determines the point Q. Then Q S and H S fix S and Q R and F R fix R and so on for the other points.

Second Method.—Substitute the bar Y Z (as shown by the dotted line in the Frame Diagram, Fig. 33*a*) for the two bars Q R and R S. This bar transfers the action of the loads at the joint G H S R F to the joint P Q T A; and therefore, the stress in T A will be unaffected. If the bar H S had been divided and similarly braced, then a bar from that joint to the joint P Q T A would enable a solution to be found.

In the Stress Diagram, Fig. 33*b*, we begin by finding the point P, then the points Y, Z, and T respectively. Having found the point T we can then proceed to find the other points in the same way in the previous cases.

Third Method.—First, find the stress in the bar T A, by taking one of the sections of the truss and thus obtain the resultant of the loads and the reaction of the wall on that section. Second, ascertain what stress in T A combined with the reaction of the other section of the truss on the apex will

balance this resultant. Fig. 34 represents the right-hand section of the Truss of Fig. 33*a*. There is no wind pressure on this side. The loading consists of ½ K L, L M, M N and N O in Fig. 33*a*. The resultant K′ O, Fig. 34, of these four loads passes through the centre of the rafter since N O is equal to ½ K L and L M is equal to M N.

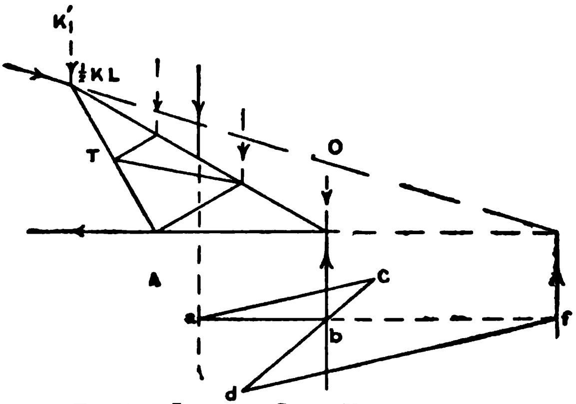


FIG. 34.—LOADS ON RIGHT-HAND SECTION.

We have now to find the resultant of K′ O and the reaction O A. Draw the line *a b* anywhere cutting the lines of action of the forces K′ O and O A as shown in Fig. 34. Through *b* draw *c d* at any angle to *a b*. Make *b c* represent to scale the force K′ O and *c d* to the same scale the reaction O A. Then join *a* with *c* and through *d* draw *d f* parallel to *a c*, so as to intersect *a b* produced in *f*. Then *f* is a point in the line of action of the resultant of the forces K′ O and O A. Its line of action is parallel to the lines of action of the forces K′ O and O A. and its magnitude is equal to their difference—that is, K′ A in Fig. 35.

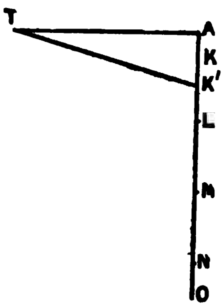


FIG. 35.—TRIANGLE OF FORCES ON SECTION.

Next produce the centre line of the bar T A in Fig. 34 to intersect the line of action of K′ A, the resultant of the two forces K′ O and O A. Then since the resultant K′ A, the action of the stress in the bar T A, and the action T K′ (*i.e.*, the reaction of the left-hand section on the apex of the frame) form a system of

forces in equilibrium. The line of action of the force T K′ must pass through the apex and the intersection of T A and

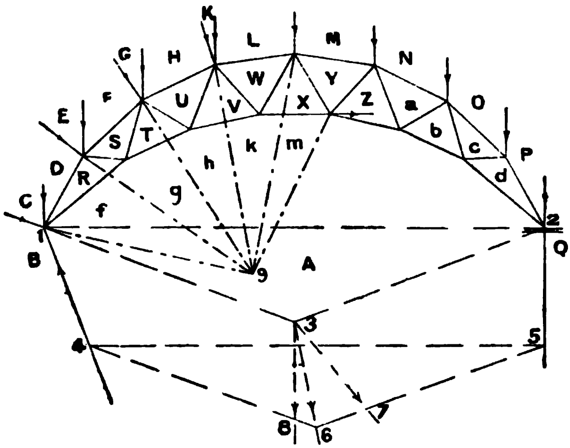


FIG. 36*a*.—FRAME DIAGRAM. BOWSTRING TRUSS.

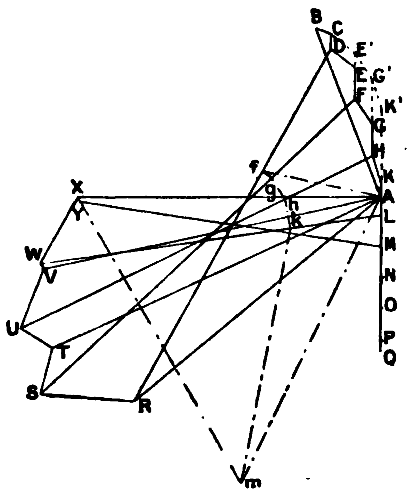


FIG. 36*b*.—STRESS DIAGRAM.

K′ A, consequently an application of the triangle of forces will give the value of the stress in T A. This is shown in Fig. 35, where K′ is the centre of K L and the other points are points in the line of loads as in Fig. 33*b*. When this is known, the Stress Diagram can be completed.

Bowstring Truss.—There is no difficulty in drawing the Stress Diagram for this truss, but if we commence in the usual manner by drawing D R and A R, by the time we get to the other side, the finishing line would most probably not be parallel to its corresponding bar in the Frame Diagram. This is due to the short

length of the bars R S, S T, &c., in the Frame Diagram and to the stresses in them being large compared to the loading on the roof as shown by the Stress Diagram.

We get a very much better diagram by determining the stress in one of the centre ties—*e.g.*, A X, by aid of a supplementary frame. Therefore, in order to find the reactions, the wind pressure may be supposed to act on a surface tangential to the curve of the roof at the joints; the length of the surface being equal to the sum of the two half bays on each side of the joint. The wind pressure at each joint will act along the radial line at the joint, and therefore the resultant of the wind pressures must pass through the centre of the outer curved flange.

The point 3 in the Frame Diagram, Fig. 36*a*, is the centre of the outer curved flange. This is a point in the line of action of the resultant wind pressure. This line is parallel to the line joining B with K′ in the Stress Diagram, and is represented by the line 3—7 in the Frame Diagram. B C, C E′, E′ G′, and G′ K′ represent the wind pressures B C, D E, F G and H K and therefore B K′ is the resultant in magnitude and is parallel to its line of action.

If the roofing is uniform, the centre of the curve of the outer flange will be a point in the line of action of the resultant load. Therefore, the resultant of the wind pressure and of the roofing weight will also pass through the point 3. The line of action of this resultant will be parallel to the line joining B with Q in the Stress Diagram—*i e.*, along the line 3—6 in the Frame Diagram. The Truss is in equilibrium under the resultant load acting along the line 3—6. The line of action of the right-hand reaction is known, and the point of application of the left-hand reaction is also known. These three forces must pass through one point. Therefore the line joining the point 1 with the point where the line 3—6 cuts the line 2—5 will give the line of action of the left-hand reaction. But as the point of intersection of the lines 3 – 6 and 2—5 would be far off the paper we use the following construction:—Join the point 1 with 2, 2 with 3, and 3 with 1; then take any point 5 in the line of action of the right-hand reaction and draw 5—6 parallel to 2—3. Then draw 6—4 parallel to 3—1 and 5—4 parallel to 2—1 so as to intersect 6—4 in the point 4. If the point 1 be joined with the point 4 the the line 1—4 will pass through the point of intersection of the line 3—6 with the line 2—5. The line 1—4 is therefore the line of action of the left-hand reaction.

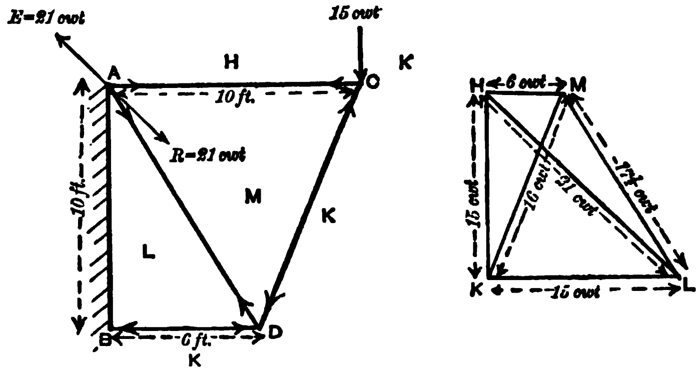
In the Stress Diagram, Fig. 36*b*, draw a line through the point B parallel to the line 1—4, so as to intersect the line Q A in the point A. This determines the reactions and enables us to proceed with the Stress Diagram.

We shall first determine the stress in X A. Assume a point 9 anywhere and connect it as indicated by the chain dotted lines in the Frame Diagram. This point 9, when connected with the joints of the outer flange, forms a supplementary frame in equilibrium under the following forces:—

(1) The wind pressure.
(2) The loads C D, E F, G H, K L and L M.
(3) The reaction A B.
(4) The action of the stress M Y on the joint L M Y X W.
(5) The action of the stresses in Y Z and Z A on the joint X Y Z A.

In the meantime, the internal bars R A, R S, S T, T A, on to W X and X A are left out of account.

In the Stress Diagram, Fig. 36*b*, we begin by drawing A*f* parallel to the supplementary bar A*f* and D*f* parallel to the bar D*f*. This fixes the point *f*. Then *fg* and F*g* give the point *g* and so on until the point *m* is obtained. Now draw *m* Y parallel to the bar *m* Y (which is coincident with the bar X Y) and M Y parallel to the bar M Y this fixes the point Y. In a similar way Y X and A X fix the point X, when the Stress Diagram may then be finished in the usual way. This method gives



To Illustrate Example I.

a more accurate estimate of the several stresses than by following the usual direct plan, as explained at the beginning of this example. Moreover, this construction is perfectly general in its application and may be used to determine the stress in any one bar of a frame.

Example I.—A is a point in a wall 10 feet vertically over another point B. From A and B there project two horizontal

bars, A C, B D (the former being 10 feet and the latter 6 feet long), and D is joined by two bars with A and C. If a weight of 15 cwts. be hung from C, find the stresses on all the bars, and show which are in tension. Find also the resultant stress on the point A. You may neglect the weights of the bars.

ANSWER.—In the figure we have denoted the spaces by letters according to Bow's notation. To obtain the Stress Diagram we must draw H K parallel to the force H K and 15 units long. Then make H M parallel to the bar H M and K M to the bar K M. This gives us the point M. M L parallel to the bar M L, and K L to the bar K L, fix the point L. H L, when joined, gives the reaction at the joint A, and the other lines the stresses in the bars. Their values are marked on the figure. A C and A D are in tension, while C D and B D are in compression.

Note to Fig. 24.—Mr. Vowell says—"In regard to the general method of the 'Substituted Truss or Frame,' the Funicular Polygon appears simpler. (See Index for page where this is explained.) In the 'Substituted Frame' method, one is obliged to get the point T through several short lines, thus making accuracy difficult, whereas, by the Funicular Polygon Method you can always deal with the resultant wind pressure *only*, and split it up by the shorter 'Coulman's Method.'"

Lecture XXV.—Questions.

1. A king post truss, whose height is one-fourth of its span, is loaded at the joints with vertical loads of 15, 30, and 45 units respectively. Determine the nature and amount of the stresses in each member of the frame. (S. & A. Adv. Exam., 1896.)

2. A roof of 28 feet span, height 7 feet, rests on king-post trusses, 10 feet apart. The weight of the roof is 20 lbs. per square foot. Find the stresses on each part.

3. If the above roof has a wind pressure of 40 lbs per square foot on one side, find the stresses on each part.

4. A roof of the form shown in Fig. 22, is 40 feet span and 10 feet high. The horizontal tie-bar is 8 feet below the vertex. Find the stresses in each part when loaded with 2 tons at each joint.

5. If, in the previous question, the maximum wind pressure on one side be 2 tons on each bay, find the stresses on all the bars.

6. Suppose both ends of the roof truss in Fig. 29 are anchored, and that in the substituted frame the bar X Y slopes in the opposite direction to that in Fig. 30; find the reactions and the stresses in the roof truss.

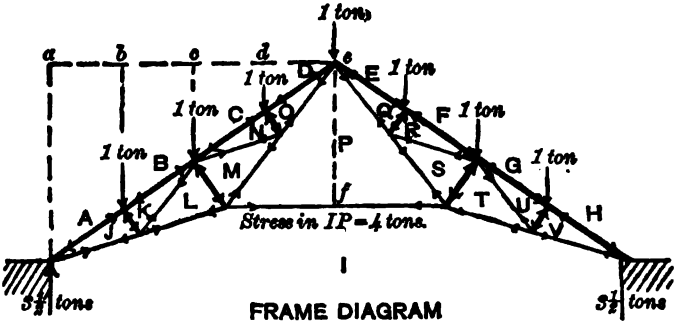
7. Work out the stresses for Fig. 32*a* by the method referred to in the second note.

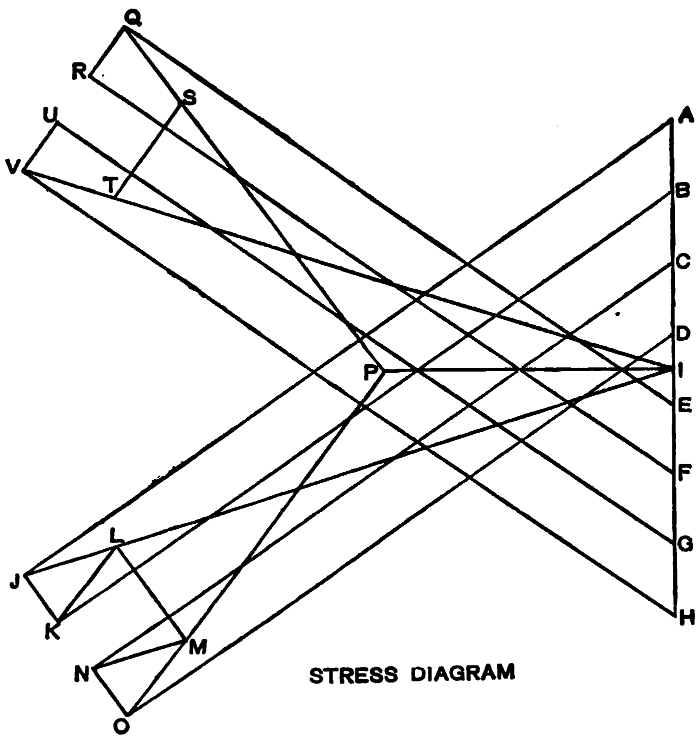
8. Suppose both ends of the modified French struss in Fig. 33*a* are anchored, and that the substituted bar Y Z lies across the spaces V and W; find the reactions and stress (1) neglecting wind pressure, (2) when wind pressure is taken into account.

9. Find the stresses in the bowstring truss, shown in Fig. 36*a*, when both the ends are anchored (1) without wind pressure, (2) when wind pressure is taken into account.

10. The following figures give the Frame and Stress Diagrams for a French Truss. Verify the Stress Diagram and redraw it in the manner explained in the text.

11. Draw the Stress Diagram when there is a wind pressure of 4 tons on the left-hand slope, assuming both sides of the roof to be fixed to the walls.





ILLUSTRATIONS FOR QUESTIONS 10 AND 11.

LECTURE XXVI.

CONTENTS.—Deficient Frames—Iron King Post Truss—Queen Post Frame—Solution of the Second Method—Solution of the Third Method—Solution of the Fifth Method—Yieldingness—Questions.

Deficient Frames.—Iron King Post Truss.—In practice the foot of the king rod N O is made virtually solid, as shown by the full lines in Fig. 37*a*. If the short bars N A and O A are made freely

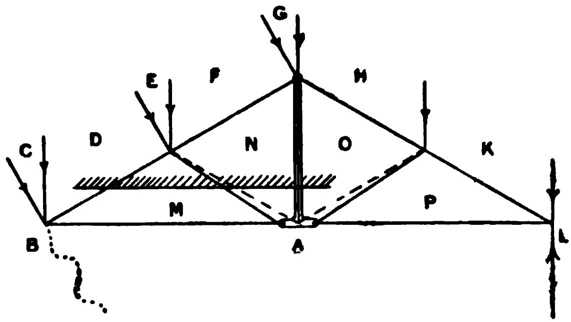


FIG. 37*a*.—FRAME DIAGRAM FOR DEFICIENT KING POST TRUSS.

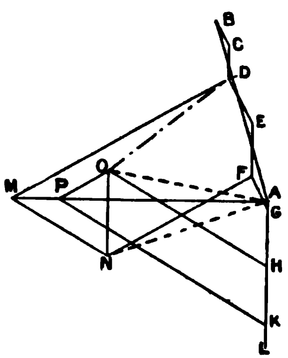


FIG. 37*b*.—STRESS DIAGRAM FOR DEFICIENT KING POST TRUSS.

jointed at N O A, it will be evident that the spaces N and O would change their shape if the loads at the centre of the rafters were unequal. This change of shape is resisted by making the joint N O A rigid.

Since the bars N A and O A are very short compared with the other members, we can draw the Stress Diagram as if the frame were made as shown by the dotted lines in Fig. 37*a*. The full lines in Fig. 37*b*, which is the Stress Diagram of Fig. 37*a*, are drawn on the above assumption.

If a line of section be drawn, beginning in one space of a Frame Diagram and ending in another, so as to pass through a joint or cross two or more bars, then the line joining the points in the Stress Diagram named after the beginning and end spaces gives the resultant of the stresses in all the bars meeting in or crossing that line.

The student can easily verify this by referring to the Stress Diagram.

In Fig. 37*a* a shaded line is shown beginning in the space D and ending in the space O. Then in Fig. 37*b* the chain dotted line D O is parallel to the line of action of the resultant of the stresses in the bars D M, M N, and N O. The length of the line D O gives the magnitude, and from D to O the way of the resultant with respect to the top side of this section. A point in the line of action of this resultant may be found by drawing a line through the joint D E F N M parallel to the line joining D with N in the stress diagram to cut the bar N O produced. The line D N is the resultant of the stresses D M and M N, which must pass through the joint D E F N M. The dotted line O A gives the resultant of the stress actions in the bars O P and P A on the imaginary joint N O A. Therefore, since the bar O A is very short, the force acting on the pin at the end of this bar will be approximately represented by the elements found from the line O A. Similarly, the dotted line N A gives the elements of the force acting at the end of the short bar N A.

The forces acting at the foot of the King Rod are represented by Fig. 38. These forces produce bending and tension in the parts O A, A N, and N O.

FIG. 38.—FOOT OF KING ROD.

Queen Post Frame.—A Queen Post Frame is represented in its normal position by the solid lines in Fig. 39. If the frame be

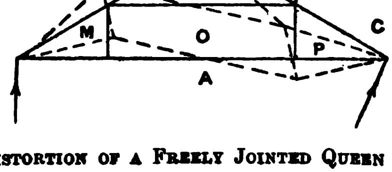


FIG. 39.—DISTORTION OF A FREELY JOINTED QUEEN POST FRAME.

freely jointed, it would be deformed into the shape represented by the dotted lines by a single force B C applied as shown at the joint N B C P O.

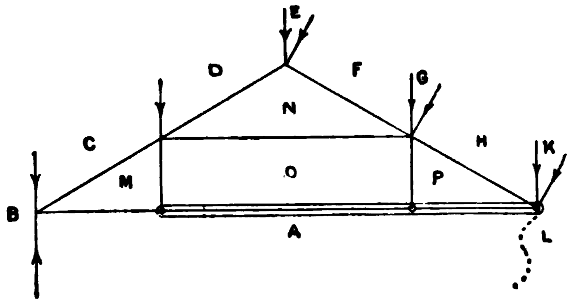


Fig. 40a.—Frame Diagram.

Queen Post Frame, with Part of Tie-Rod Made Continuous.

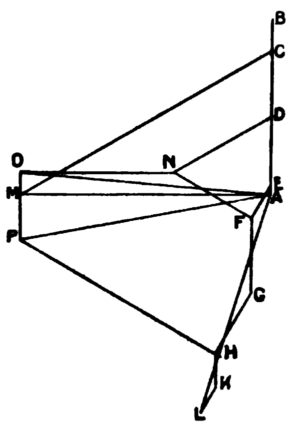


Fig. 40b.—Stress Diagram.

This change of shape may be resisted in several ways, such as the following :—

(1) By a diagonal in the central parallelogram. This diagonal would have to stand push if the wind caught the frame on one rafter, and pull if the wind pressure were on the other; or the stresses might be due to snow. It is the usual practice to put two diagonal ties in the parallelogram, so that when a push comes on one diagonal the other receives it as a pull. In drawing the Stress Diagram for such a frame, if a push comes on one of the ties, we omit that bar and take the other.

(2) By making the bar continuous between the joints M O A and P H K L A, and therefore able to resist being bent into the dotted form shown in Fig. 39.

(3) By making the whole tie-beam continuous. This causes the frame to become redundant; *i.e.*, it may be self stressed, by having the bars M O and O P of unequal length, or badly fitted.
(4) By making one rafter continuous.
(5) By making the rafters and tie-beam continuous. This is the usual form in actual practice and causes the frame to become redundant.

Solution of the Second Method.—The reactions are ascertained by a Substituted Frame as already explained. In the Stress Diagram, Fig. 40*b*, we begin by drawing D N and F N; C M and A M; N O and M O; O P and H P all parallel to their respective bars. Then P and O joined with A give the finishing lines of the Stress Diagram. The lines A P and A O are not parallel to the bars A P and A O. This indicates that there is bending in the continuous part of the tie-rod.

In Fig. 41, the forces are shown acting on the part of the tie-beam which is continuous. The vertical components of the forces A O and A P produce bending in the bar, while the horizontal components produce tension.

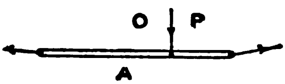


FIG. 41.—THE CONTINUOUS PART OF TIE-BEAM.

Solution of the Third Method.—Having found the reactions and drawn the External Force Polygon, as in Fig. 42*b*, we can then find the point N. We observe that O must lie on the line N O, which is drawn parallel to the bar N O; M must lie on the line D M drawn parallel to the bar D M, and P on the line K P drawn parallel to the bar K P.

On reference to Fig. 39 we see, that so long as the rafter ends always remain in the same horizontal line, the joint O P A must go down as much below the horizontal line as the joint M O A goes above it. Therefore, if the tie-beam is equally rigid along its length, the push required to distort it at the joint O P A must be equal to the pull distorting it at the joint M O A—that is, O P must be equal in length to M O in the Stress Diagram, Fig. 42*b*. If the tie-beam be unequally rigid, then the push and pull will be in proportion to the rigidity at the joints O P A and M O A in Fig. 39. In Fig. 42*a* the distorting force is on the left-hand rafter, and therefore the joint M O A will go down; consequently M O is subjected to push stress.

We can now proceed with the Stress Diagram in Fig. 42*b*.

Since M O is equal in length to O P, P and M must lie where the line D M intersects the line K P. Again, O P and M O are parallel to the bars O P and M O respectively, and N O is

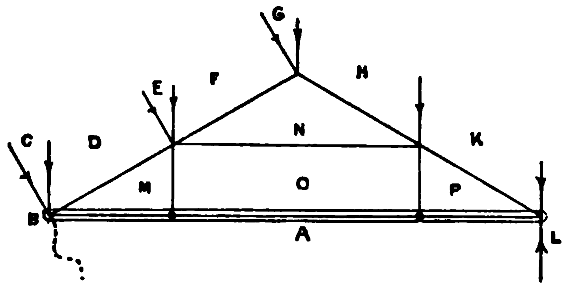


FIG. 42a.—FRAME DIAGRAM.

QUEEN POST FRAME, WITH CONTINUOUS TIE-BEAM.

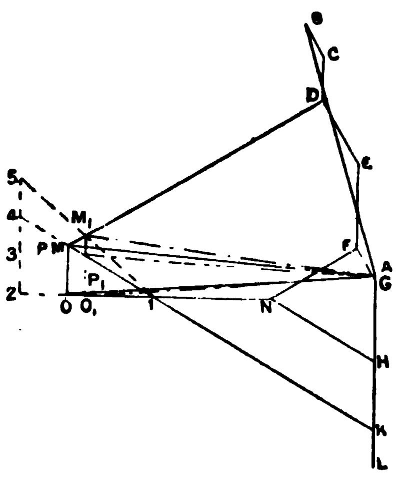


FIG 42b.—STRESS DIAGRAM.

parallel to the bar N O. This fixes the point O. Joining the point P M and the point O with A we complete the stress diagram.

The forces acting on the tie-beam are illustrated by Fig. 43. The force O P and the vertical component of P A constitute a couple tending to produce clockwise rotation. The force M O and the vertical component of A M form another couple of equal moment, and also produce clockwise rotation. These two couples bend the beam, as indicated in Fig. 39. The horizontal components of the forces A M and P A produce tension in the tie-beam.

Now, suppose the rigidity of the tie-beam at the joint P O A to be $\frac{2}{3}$ of its rigidity at the joint M O A, then O_1 P_1 must equal $\frac{2}{3}$ of O_1 M_1. We must remember that the joint O P A is always as

much above as M O A is below the horizontal line. A construction to determine M_1, P_1, and O_1 is shown by the dotted lines in Fig. 42*b*. The point 2 is taken anywhere in the line N O. The line 2—5 is drawn perpendicular to line N O, and the line K P is produced to cut the line 2—5 in the point 4. Then the length 2—4 must be to the length 2—5 as the rigidities at the joints. In other words, the line 2—5 is three when 2—4 is two, and therefore 4—5 is equal in length to half of 2—4. Now join point 5 with point 1. This line cuts the line D M in the point M_1, and by drawing $M_1 O_1$ parallel to the bar M O, and N O_1 parallel to the bar N O, we obtain the point O_1.

The finishing lines of the Stress Diagram, Fig. 42*b*, are obtained by joining M_1, P_1, and O_1 with the point A, and are shown by the chain dotted lines.

FIG 43.—FORCES ACTING ON CONTINUOUS TIE-BEAM.

Solution of the Fifth Method.—In this arrangement of bars (Fig. 43*a*) if the joint F G H P O N descends through a small distance (say 1 inch) then the joint O P A of the tie-beam will descend 1 inch, the joint A M O will go up 1 inch and the joint M C D N O will rise 1 inch. Now, all this will take place irrespective of the rafters and tie-beam being of equal or of unequal yieldingness.

Yieldingness.—Two springs are of equal yieldingness, when they stretch through the same amount under equal loads.

One spring would have a yieldingness of three times another, if the first extended three times the amount that the second stretched under the same load.

Further, if two springs of equal yieldingness are attached to the same load, so that they each extend through the same amount; then each spring will carry one half of that load. But, if two springs of unequal yieldingness are attached to the same load, so that they each extend through the same amount; they will each carry a share of the load *inversely* proportional to their yieldingness. Suppose we have two springs, the first one stretches say 1 inch under a load of 3 lbs., while the second one extends 1 inch under 1 lb.; then, if these two springs are set to carry a load of 4 lbs., they will each extend 1 inch and the first spring will carry 3 out of the 4 lbs., while the second will carry the remaining 1 lb.

The above remarks apply equally to bars supporting a load between them, whether they are under a similar kind of stress or not. For example, suppose a beam is jointed to a rod attached

to a rigid point above it; then, their yieldingness would be measured by the amounts they would each come down under

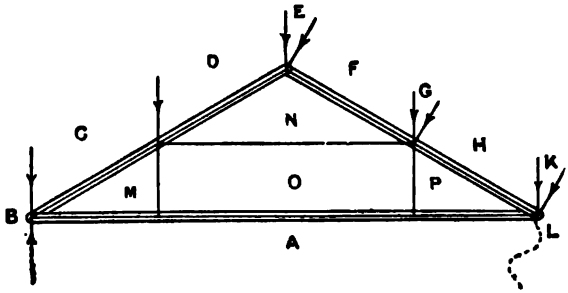


FIG. 43a.—FRAME DIAGRAM.

QUEEN POST FRAME, WITH RAFTERS AND TIE-BEAM CONTINUOUS.

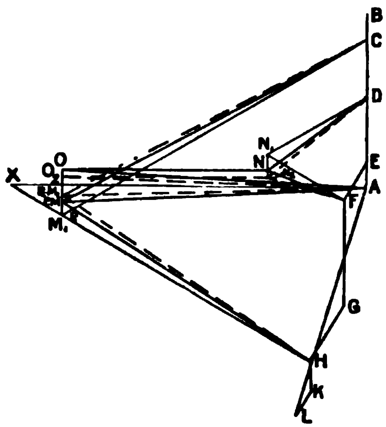


FIG. 43b.—STRESS DIAGRAM.

the same load, as applied to each separately at the point where they are jointed to each other.

Referring to the Frame Diagram, Fig. 43a, we shall assume in the first place, that the yieldingness of the rafter at the centre in a vertical direction, is the same as the yieldingness of the tie-beam at the joint O P A, also in a vertical direction.* Therefore, whatever is the amount of the vertical component of the distorting force, they will each be subjected to the same stress.

Before we can do anything to the Stress Diagram, Fig. 43b, we must first find what amount of the distorting force passes into M O and O P in Fig. 43a, on the assumption that the rafters are freely jointed at their centres.

Figs. 44a and 44b show how this is done. In the Frame Diagram, the force Q G is the difference between the loads C D

* This does not mean that the rafter and the beam have equal rigidity.

and F G in Fig. 43*a*, and G H is the same as in that figure. Now draw the Stress Diagram, Fig. 44*b*, in a similar manner to Fig. 42*b*. That is, Q G and G H are drawn to the same scale as the line of loads in Fig. 43*b*, when T will coincide with Q. Draw T U parallel to the bar T U, Q S to Q S and H V to H V. The pull S U is equal to the push U V; therefore S and V are at the intersection of Q S and H V. Then S U drawn parallel to the bar S U completes the Stress Diagram, Fig. 44*b*.

The lengths of S U and U V, give the stresses in the Queen Rods S U and U V, on the assumption that the yieldingness of the rafters is infinitely large. But, the rafters have the same yieldingness as the tie-beam and therefore only half of the distorting forces will pass to the tie-beam. This means, that the pull in the Queen Rod M O and the push in the Queen Rod O P, are equal to one-half of S U and U V respectively.

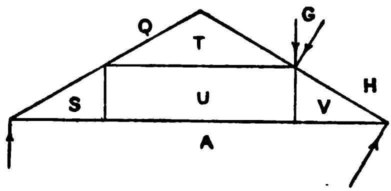


FIG. 44*a*.—FRAME DIAGRAM.

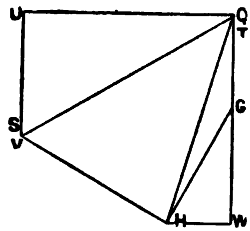


FIG. 44*b*.—STRESS DIAGRAM.

FRAME CARRYING DISTORTING FORCES ONLY.

In the Stress Diagram, Fig. 43*b*, H P_1 and C M_1, are drawn parallel to H P and C M of Fig. 43*a* until they intersect. M_1O is drawn parallel to M O and A X is a horizontal line through A. On the line M_1O mark off two points O and M, where O is as much above A X as M is below it and the distance M O is equal to one-half of S U. This fixes the points M, O and P of the Stress Diagram, because P coincides with M.

Now, draw D N_1 and F N_1 parallel to the bars D N and F N until they intersect at N_1. Through N_1 draw N_1N parallel to M_1O. Then draw O N parallel to the bar O N, and we shall have found all the points in the Stress Diagram, Fig. 43*b*. On joining C with M; H with P; D with N; F with N; O with A; M with A; and P with A we finish the Stress Diagram. The dotted lines represent the Stress Diagram when the yieldingness of the rafters is less than that of the tie-beam.

Divide S U into two parts, having the ratio to each other that the yieldingness of the rafter bears to the yieldingness of

the tie-beam. Then $O_2 M_2$ will have the smaller length as its value, if the yieldingness of the rafter is the smaller; and $O_2 M_2$ will have the larger length of S U if the yieldingness of the tie-beam is the smaller. For example, let the tie-beam be twice as yielding as the rafter. Then divide S U in the proportion of 2 to 1—*i.e.*, into three equal parts—and make $O_2 M_2$ equal to one of the three parts; keeping in mind, that O_2 is as much above A X as M_2 is below it.

LECTURE XXVI.—QUESTIONS.

1. Explain and prove the rule for obtaining the resultant of the stresses in all the bars of a frame crossing any given section. In Fig. 37*a* what is the resultant of the stresses in the bars A P, P K, and also in the bars A M, M N, and N F?

2. The dimensions of an iron king post truss for a roof are:—Span, 20 feet; height, 7 feet; distance between trusses, 8 feet. The roof weighs 12 lbs. per square foot. Find the stresses in each part.

3. In the above question find the stresses when the wind causes a pressure of 30 lbs. per square foot on one slope.

4. Explain the differences caused in the stresses by the different methods of completing a queen post frame. Mention some of the advantages and disadvantages of each.

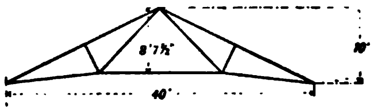
5. A queen post roof has a span of 30 feet, and is 10 feet high. The roof weighs 10 lbs. per square foot, and the principals are 10 feet apart. Find the several stresses if the rafters and tie-beam are continuous.

6. If there is a wind pressure of 25 lbs. per square foot on the roof in Question 5, find the stresses in the bars.

Lectures XXV. and XXVI.

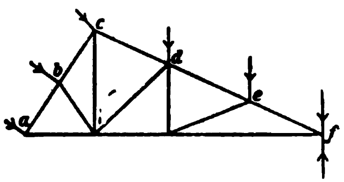
A.M.Inst.C.E. Exam. Questions.

1. Give a reciprocal diagram of the stresses in the bars of the roof shown in the sketch, loaded with 2 tons at each joint of the rafters. (I.C.E., *Oct.*, 1897.)



2. Show how to find, graphically or otherwise, the loads borne by the supports of a roof when the weights and wind forces are specified. Give an example. (I.C.E., *Oct.*, 1897).

3. Draw a diagram of stresses for the roof frame shown in the fig., when carrying a vertical load of 2,000 lbs. at each of the joints *b*, *c*, *d*, and *e*, and 1,000 lbs. at *a* and *f*, and a uniformly distributed normal wind-pressure on the left side 6,000 lbs.—assuming all the joints to be flexible. Show for each bar whether the stress is push or pull. (I.C.E., *Feb.*, 1898.)



4. In the "queen" truss used for iron roofs of large span, the straight rafters and horizontal tie-rod are connected by a triangulation of equidistant vertical suspending rods and diagonal struts; find the stress on each member of the truss when the load on the roof is uniformly distributed. (I.C.E., *Oct.*, 1899.)

5. Sketch the common king-post truss employed in roofs of small span, and explain how the stress on each member is calculated so far as is due to roofing material of given weight per square foot. (I.C.E., *Feb.*, 1900.)

6. Describe the action of wind-pressure on a roof. In the last question, assuming the upward reaction of one of the walls on which the roof rests to be vertical, find the stress on each member due to wind-pressure of known amount. (I.C.E., *Feb.*, 1900.)

7. In a bowstring bridge the platform is suspended by vertical suspending rods, without diagonal bracing, from a pair of parabolic arched ribs which may be assumed to be jointed at crown and springing. Describe the straining actions produced in the ribs by a load at the centre of the bridge and calculate their maximum values. Explain the object of the diagonal bracing introduced in practice. (I.C.E., *Feb.*, 1900.)

8. A load of 1200 lbs. hangs suspended at a point, B, by a pair of ropes, B A and B C, to the two points of attachment, A and C, on the underside of a sloping rafter. The ropes, B A and B C, are each 16 feet long, and the rope, B A, is horizontal, while the point, C, is 8 feet higher (measured vertically) than the points, A and B. Find the direct pull in each of the ropes, B A and B C. (I.C.E., *Oct.*, 1900.)

9. Show with the help of an example how you would proceed to determine the stresses in the members of a simple roof-truss, produced by a given horizontal wind-pressure, and also the reactions on the points of support (the horizontal reaction is supplied by the right-hand wall). (I.C.E., *Oct.*, 1901.)

10. If you were supplied, for a given roof-truss, with a table of the maximum stresses due to the dead load, and also those due to the wind-pressure, how would you proceed to select the working stresses per square inch which it would be safe to allow in each bar? Explain fully the reasons for any method you adopt. (I.C.E., *Oct.*, 1901.)

LECTURE XXVII.

CONTENTS.—Wharf Crane—Example I.—Common Jib Crane—Balanced Jib Crane—Derrick or Scotch Crane—Foundry Crane—Sheer Legs—Example II.—130-Ton Steam Crane—Tables of Dimensions and Weights of 130-Ton Crane—Example III.—Questions.

Wharf Crane.—Suppose, as in Fig. 1*a*, that a single movable pulley carries the load W. Then, neglecting friction, the pull throughout the chain will be one half of W. Again, assume that the pull of the chain acts at the centre of the pulley or barrel round which it may be passing. In the Frame Diagram, Fig. 1*a*, the external forces acting on the frame are all duly indicated. At the Jib end, there are two forces—viz., the pull of gravity D E, on the supported mass, acting vertically downwards and the force C D, due to the pull in the chain which is assumed to be parallel to the tie-rod C H. At the top end of the vertical post, there are the forces B C and A B acting as shown, which are both due to the pull in the chain on the pulley at the post head. There is also a force acting at the centre of the barrel along the crane

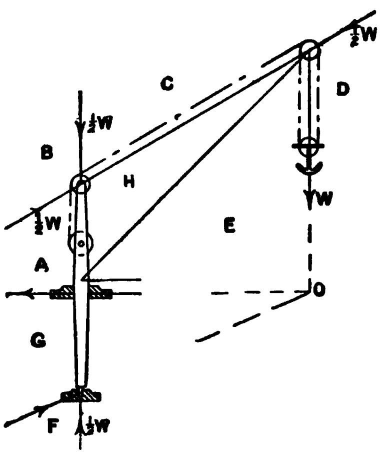


FIG. 1*a*.—FRAME DIAGRAM.

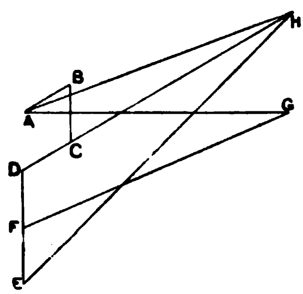


FIG. 1*b*.—STRESS DIAGRAM.

WHARF CRANE.

post. This force must be transferred to the footstep as shown and is called E F in the diagram. There is also a pressure transmitted by the sole plate to the vertical post. This pressure is

assumed to have its line of action horizontal and is lettered G A. Finally, we have the action of the footstep on the lower end of the vertical post.

The three forces D E, F G and G A, must form a system in equilibrium. Therefore, since the lines of action of D E and G A are known, if we produce them to meet in O, then the line of action of F G is known because it must also pass through O.

The Stress Diagram, Fig. 1*b*, may now be drawn. Draw the line of loads A B, B C, C D, D E, and E F. Now, draw F G parallel to the line of action of the force F G and A G parallel to the line of action of the force A G. These close the External Force Polygon. Then if C H and E H be drawn parallel to the bars C H and E H respectively they fix the point H. The line joining the point A with the point H is the finishing line of the Stress Diagram. This line is not parallel to the bar A H because the bar A H is subject to bending.

In Fig. 2, we have a representation of the forces acting on the vertical post; from which, we can determine the bending, tension, and compression stresses in the crane post.

The lengths of the lines C H and E H in the Stress Diagram, Fig. 1*b*, give the stresses in the tie-rod and jib respectively. The horizontal component of G F gives the shear on the bolts of the footstep and G A the shear on the bolts of the sole plate.

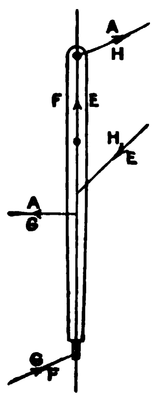


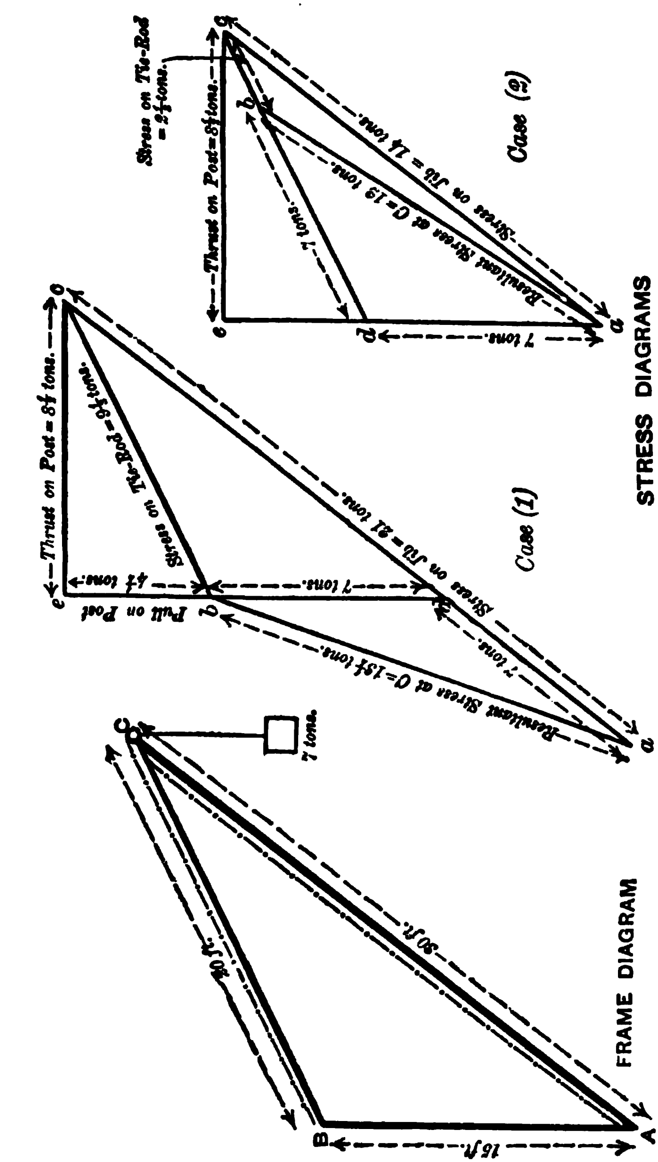
FIG. 2.—FORCES ACTING ON VERTICAL POST OF CRANE.

EXAMPLE I.—In a wharf crane the post, tie-rod, and jib measure 15, 20, and 30 feet respectively, what would be the nature and amount of the stresses in each of the three members when a load of 7 tons is suspended over the pulley at the jib head, (1) when the lifting chain passes from the pulley to the drum or barrel parallel with the jib, (2) when the drum is placed so that the chain passes from the jib head parallel with the tie-rod? (S. and A. Exam., 1890.)

ANSWER.—First, draw to scale a Frame Diagram A B C, as shown. This will be coincident with the centre lines of the different members of the crane.

Case (1).—Here the lifting chain passes from the pulley at the jib head parallel to the jib, and, neglecting the friction of the pulley, we shall have two equal external forces at the joint C due to the tension in the two parts of the chain.

In order to draw the Stress Diagram, we may first proceed to



STRESSES IN A JIB CRANE

determine the resultant of these two forces and consider it as a single external force applied to the joint C, and then draw the triangle of forces. Or we may at once draw the polygon of forces for the joint. Thus, draw *b d* to represent the load of 7 tons, and *d a* equal to *b d*, and parallel to C A, to represent the tension in that part of the chain over the pulley; then drawing *a c* and *b c* respectively parallel to A C and B C, we determine the point *c*, and therefore the magnitude of the stresses in the jib and tie-rods. These will evidently be compression and tension respectively. If we join *b a* we obtain the resultant external force at the jib head, and *b a c* will be the triangle of forces determining the same stresses as above.

The nature and amount of the stress in the post will depend on the mode of fixing it. It is evident that the pull *b c* in the tie-rods may be resolved into a vertical component *b e*, producing tension in the post, while the horizontal component *e c* represents the force tending to bend the post round A.

Case (2).—Here the chain passes from the pulley parallel to the tie-rods. We proceed as before, and draw *b d* to represent the pull in the part of the chain above the pulley, and *d a* the pull in the vertical part of it, then *a c* and *b c* drawn parallel to A C and B C respectively, determine the point *c*, and represent the stresses in the jib and tie-rod. Again joining *b a*, we see that this represents the resultant external force at the pulley. The remainder of the diagram is the same as in Case (1).

The magnitudes of the different stresses are shown on the diagrams, and enable us to compare the relative merits of the two arrangements. Thus, if we suppose the load to be suspended from the end of the jib without the intervention of a pulley, we get *b d c* in Case (1), or *d a c* in Case (2) as the corresponding Stress Diagram. The effect of introducing the chain and pulley is in Case (1) to *increase* the thrust in the jib by 7 tons—*i.e.*, the pull in the chain—without affecting the pull in the tie-rods, while in Case (2) the effect is to *diminish* the pull in the tie-rods by the same amount—7 tons—without increasing the thrust in the jib. Thus, other things being equal, Case (2) is the better arrangement.

Common Jib Crane.—In the common Jib Crane represented in Fig. 3, the movable pulley has one sheave, and the chain passes direct to the barrel from the Jib-head. The barrel is carried by the cast-iron framing. There are two tie-rods inclined at an angle θ degrees to the centre line of the crane as shown by the plan of the tie-rods. We assume the cast-iron frame to be freely jointed where the tie-rods and the jib meet it, and also where the horizontal part meets the upright post.

The forces acting on the frame of the crane are indicated in Fig. 4*a*. At the Jib-head there are the two forces W and $\frac{1}{2}$ W. At the point in the bar G H, representing the centre of the barrel, there is a force $\frac{1}{2}$ W, indicated by the dotted line and arrow head. The lines of action of the dotted force and the

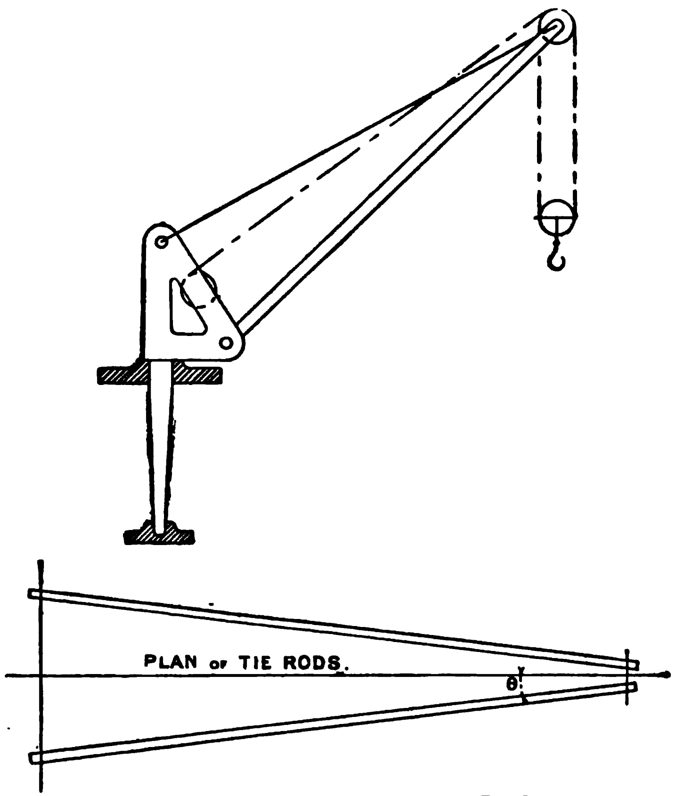


FIG. 3.*—OUTLINE DIAGRAM OF COMMON JIB CRANE.

force B C are coincident. They lie along the line joining the centre of the Jib-head pulley and the chain barrel.

We have here an example of a force acting at a point in a bar. The force acting at a point in the bar G H as represented

* The Plan of the Tie-Rods of this crane has been drawn to a larger scale than the crane itself.

by the chain dotted line, is replaced by the two equivalent parallel forces D E and A B applied as shown. Their magnitudes will be inversely as the lengths into which the bar G H is divided while the sum of their magnitudes is $\frac{1}{2}$ W.

Before beginning the Stress Diagram, we must first determine the values of A B and D E. Lay down a line to measure $\frac{1}{2}$ W and divide this line in the same proportion as the bar G H is divided by the line of action of the dotted force. Then, these two parts will measure to the same scale as the whole line, the respective values of the forces A B and D E. The greater force is placed at the end of the shorter division of G H.

As in the last crane, the pull of gravity C D on the supported mass, the pressure of the sole-plate E F on the upright and the reaction of the footstep F A on the upright, are in equilibrium and therefore their lines of action all pass through one point O.

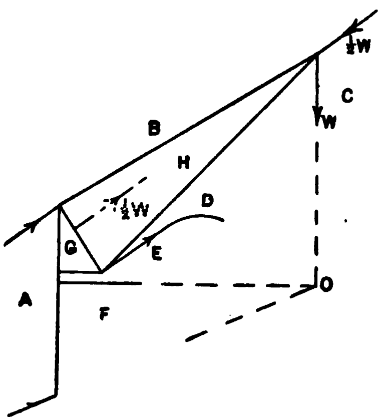


FIG. 4a.—FRAME DIAGRAM.

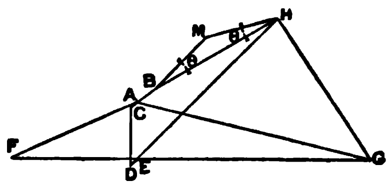


FIG. 4b.—STRESS DIAGRAM.

COMMON JIB CRANE.

The line of action of E F is assumed to be horizontal and the line of action of C D to be vertical. In the Stress Diagram, Fig. 4b, we first draw A B, then B C, C D, and D E. This completes the line of loads. Now, from E draw E F parallel to the line of action of the pressure E F; and, from A draw A F parallel to the line of action of the reaction A F. These two lines determine the point F and complete the External Force Polygon. Then, draw B H and D H parallel to the tie-rod B H and the jib D H respectively. These fix the point H. Draw H G and E G parallel to the bars H G and E G. This fixes the point G. On joining C with G the Stress Diagram is completed.

B H in Fig. 4b represents the stress on the tie-rod, on the assumption that there is only one tie-rod lying along the centre line of the crane. We require, therefore, to resolve this stress

into two components, one along each tie-rod. This is done in the Stress Diagram by drawing from H and from B lines H M and B M each making an angle with B H of θ degrees. Then the lengths of H M and B M measure to scale the stresses in the tie-rods.

Balanced Jib Crane.—The balance weight B W acting at G, is usually mounted on rollers in order that it may be moved nearer to the central post A when the load W is reduced. In this way the moments of the load and balance weight may be kept in equilibrium and thus prevent any undue bending action on the

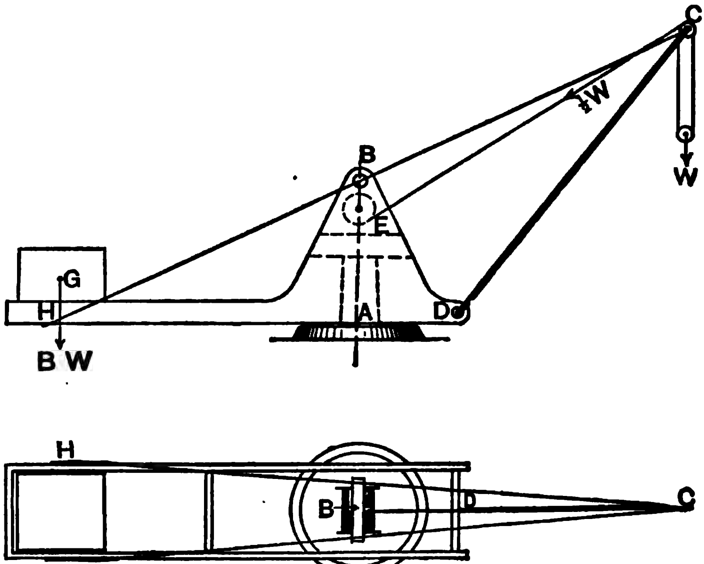


FIG. 5.—BALANCED JIB CRANE.

post at A. We may here remark that the balance weight B W at G and the load W at C are not necessarily equal.

A single movable pulley carries the load W and therefore the tension throughout the chain is ½ W. We assume that bars join B with D; D with A; A with B; and A with H. These are all indicated in the Frame Diagram of Fig. 5. The line joining the centre of the pulley C with the centre of the chain barrel E is considered as the line of action of the stress in the chain. From the plan it will be seen that there are two tie-rods

inclined to each other; these rods are often made continuous from C to H.

The load of ½ W acting on the bar F G, Fig. 6*a*, is divided as explained in the previous example into forces B C and K A. If W is known, we can find on completing the Stress Diagram, the magnitude of the balance weight A B required to balance the moment of the load W about the joint F G E K A. Or, if the balance weight be moved nearer the crane post we can find what weight may be placed in the crane hook.

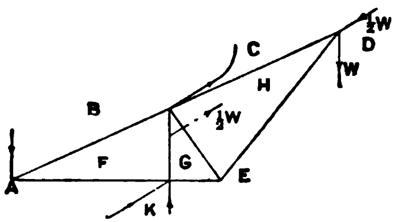


FIG. 6*a*.—FRAME DIAGRAM.

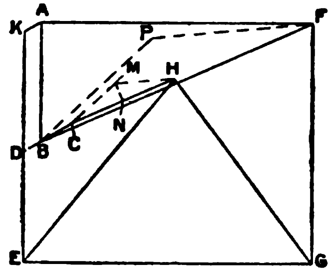


FIG. 6*b*.—STRESS DIAGRAM.

BALANCED JIB CRANE.

We begin the Stress Diagram, Fig 6*b*, with the line B C, then C D, and D E. We next find the point H, then, G, F, A, and finally K.

The actual stress in the tie-rods is found, by drawing H M and C M at an angle with H C, equal to half of the real angle between the tie-rods, as previously explained. From F and B draw F P and B P inclined to F B at half the angle between the parts of the tie-rods carrying the balance weight. If the tie-rods are continuous, then H M and F P are parallel.

Derrick or Scotch Crane.*—In Fig. 7, A B is the central upright post, capable of turning round A and B. B C is the jib and A C the tie-rod, which is usually a chain for raising or lowering the jib. The vertical post A B is kept upright by the back stays A E and A E_1. These stays are sometimes anchored to the ground, but are generally attached to the bars B E and B E_1. Boards are placed over these bars and stones or pig iron are placed thereon to act as counterweight to the load W. This crane is similar to that in Fig. 5, in

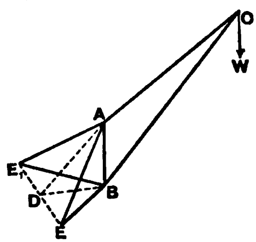


Fig. 7.—DERRICK CRANE.

* See figure at end of this Lecture.

that back balance weights are used. The weights in Fig. 7 are, however, not required to be made movable in order to produce a moment round B equal and opposite to the moment of W round the same point. This is due to the fact, that the under sides of the bars B E and B E_1 rest upon the ground, and, therefore, no matter how much the moments of the balance weights round B may exceed the moment of the load, there is no bending stress produced on the pivot at B.

The chain carrying the load W is usually parallel to the tie-rod A C. It then passes round a pulley at A and down to the barrel on the upright post.

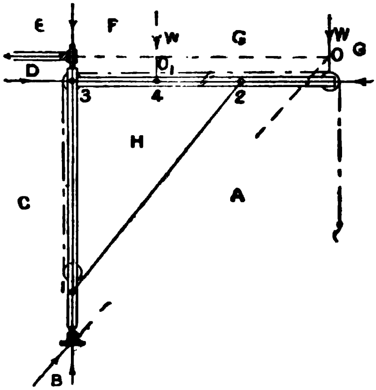
By reference to the Stress Diagrams, Figs. 1*b* and 6*b*, the Stress Diagram for Fig. 7 may be drawn. When the plane of the triangle A B C in Fig. 7, coincides with the plane of the triangle A B E, the stress in A E will be a maximum, the stress in A E_1 will be theoretically zero, and the weight required at E may then be found. Similar considerations will give the stress in A E_1 and the weight required at E_1.

Let the plane of the triangle A B C now occupy any intermediate position between the planes of the triangles A B E and A B E_1. Then the stresses in A E and A E_1, may be found by producing the plane of the triangle A B C to intersect the planes A E_1 E and E_1 B E in A D and B D. Now proceed to find the stress in A D as if D were anchored to the ground, then resolve this stress along the stays A E and A E_1, as explained for the inclined tie-rods of the two previous examples. The angle E_1 B E is usually a right angle.

Foundry Crane.—The Frame Diagram, Fig. 8*a*, illustrates the arrangement of the parts of this type of crane. The pulley carrying the load W is attached to a small bogie running between two parallel horizontal beams. The external forces acting on the crane are F G, G A, A B, C D, E F (each equal to W), B C, and D E. The external force A B balances E F, and C D balances G A, therefore the remaining external forces B C, D E, and F G form a system in equilibrium. Their lines of action will pass through one point O, and their magnitudes may be determined by the triangle of forces. These results may be made use of in drawing the Stress Diagram, Fig. 8*b*. Commence by drawing E F, F G, G A, and A B all equal in magnitude to W, and parallel to their respective lines of action. If we draw B C parallel to the line of action of the force B C, so as to intersect the line through E parallel to the line of action of the forces C D and D E in the point C, then by marking off C D equal to W we complete the external force polygon.

The stress in A H will be unaffected by raising or lowering

either the footstep or the bearing at the top, so long as the line of action of the load and the position of the joints 1, 2, and 3 remain unchanged. Let us suppose that the footstep coincides



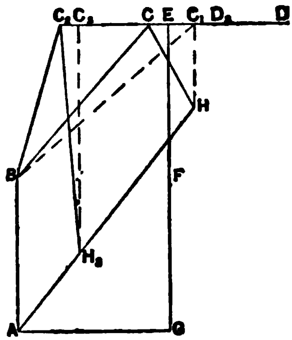
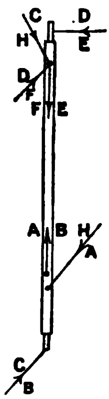


FIG. 8*a*.—FRAME DIAGRAM. FIG. 8*b*.—STRESS DIAGRAM.

FOUNDRY CRANE.

with the point 1, and the bearing at the top with the point 3, then the line of action of the stress in C H will be along the



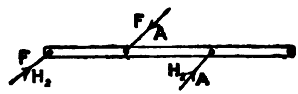


FIG. 10.—FORCES ACTING ON JIB WITH PULLEY AT POINT 4 ON FIG. 8*a*.

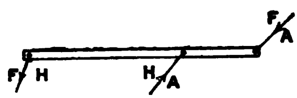


FIG. 9.—FORCES ACTING ON UPRIGHT OF FOUNDRY CRANE.

FIG. 11.—FORCES ACTING ON JIB IN FIG. 8*a* WITH PULLEY AT END OF SAME.

centre line of the bar C H, while the line of action of the footstep reaction B C_1 will pass through the point 1 and the centre of the pulley carrying the load.

In the Stress Diagram, Fig. 8*b*, draw B C_1 through the point B parallel to the line joining the point 1 with the centre of the pulley carrying the load, so as to cut the line C D in the point C_1; then by drawing C_1 H and A H parallel to the bars C H and A H respectively, we determine the point H. Finally, C joined with H finishes the Stress Diagram. When the pulley carrying the load occupies the position represented by the point 4, then C_2, D_2, and H_2 (found in a similar manner) are the corresponding points in the Stress Diagram.

Fig. 9 represents the forces acting on the upright, which

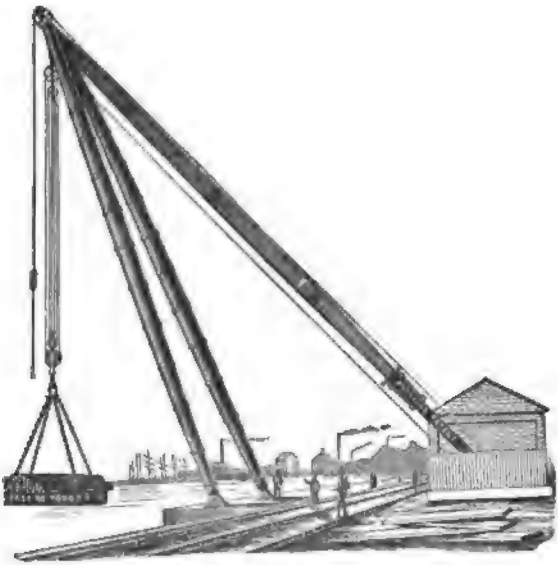


FIG. 12.—RUSSELL'S PATENT SHEER LEGS.

produce bending, tension, and compressive stresses, for the case when the movable pulley is at the end of the horizontal jib.

In Figs. 10 and 11, if the maximum bending moment in each case be the same, then the point 2 in Fig. 8*a* has been so chosen, as to make the bending moment on the horizontal jib have its least value whilst the pulley carrying the load passes from one end of the jib to the other. (See Ques. 41 of Hons. S. & A. Exam. in Machine Constn., 1890.)

Sheer Legs.—A common appliance for lifting engines and boilers into ships is the sheer legs or sheers. The illustration

shows one that has been erected at West Hartlepool by Messrs. George Russell & Co., of Motherwell, for a load of 80 tons overhung 38 feet 6 inches. It consists of two tubular front legs, each 105 feet long, swinging upon pins at their lower ends, and connected together at the top, which is supported by a hollow stay or back leg. This stay is fixed to the gunmetal nut of a forged steel screw, which rotates inside the back leg. The screw is anchored at its lower end, and can be rotated by a hydraulic engine. As the screw revolves one way or the other, the back leg is shortened or lengthened, and the top is moved in or out, as shown on Fig. 13. The total horizontal travel thus given to the load is 50 feet.

Chains worked by a pair of hydraulic engines are used for lifting, and there are separate chains for light and heavy loads. The latter chain operates a six-purchase pulley block.

In Fig. 13, A_1 B is the line of the front legs hinged at B, and A_1 C that of the back leg. The top can move between A_1 and A_2 by altering the length of A C. The vertical A_1 D represents the load on the sheers (80 tons), and A_1 E the tension in the chain,

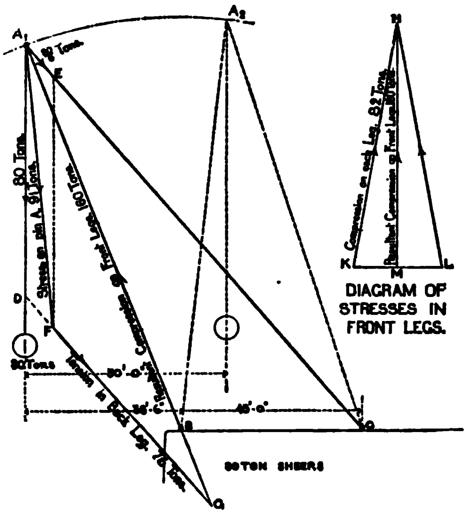
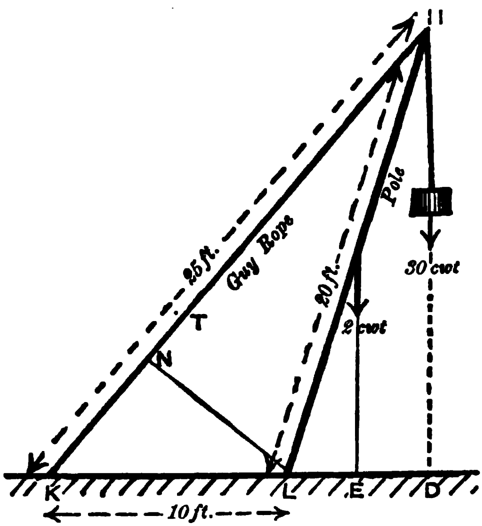


FIG. 13.—COMBINATION OF FRAME AND STRESS DIAGRAMS FOR FIG. 12.

($\frac{1}{6}$ of 80 or $13\frac{1}{3}$-tons since there are six chains supporting the load). Draw E F parallel to A_1 D, and D F parallel to A_1 E. Then A_1 F is the resultant force to be balanced by the stresses in the legs A_1 B, A_1 C. Draw F G parallel to A_1 C, and meeting A_1 B produced, if necessary, in G. Then A_1 G (160 tons) is the compression transmitted to the front legs, and F G (76 tons) the tension in the back leg. As the two front legs are not parallel, we must, in order to determine the actual stress in each, draw a second figure, as shown at the right-hand side. Here H M is equal to half A_1 G, and K H M and L H M are each half the

angle between the front legs. K L being drawn perpendicular to H M, we ob ain H K and H L (82 tons) as the compression in each leg.

EXAMPLE II.—The foot of a uniform derrick pole of weight 2 cwts. rests on the ground, and the pole carries a weight of $1\frac{1}{2}$ tons suspended from its upper extremity. The length of the pole is 20 feet, and it is kept in position by a guy rope fastened to the ground 10 feet to the rear of the foot of the pole and 25 feet in length. Find by construction or otherwise the tension of the guy rope.



DERRICK POLE.

ANSWER. — In the figure, L H is the derrick and K H the guy rope. The weight of the derrick acts at its centre, and may be replaced by 1 cwt. at each end, so that the total load in the upper end may be taken as 31 cwts. This is the force A B on the Frame Diagram. For the Stress Diagram draw A B parallel to the force A B and equal to 31 units. Make B C parallel to the pole and A C to the guy rope. Then, A C is the tension in the guy rope and is equal to 25 cwts. and B C (= 53 cwts.), the compression in the pole.

This problem could have been solved by the Principle of Moments, as follows :—

Let T be the Tension in the guy rope.
„ w „ Weight of the pole.
„ W „ Weight hung from pole.

Drawing L N perpendicular to K H and taking moments about L (in the first figure), we have :—

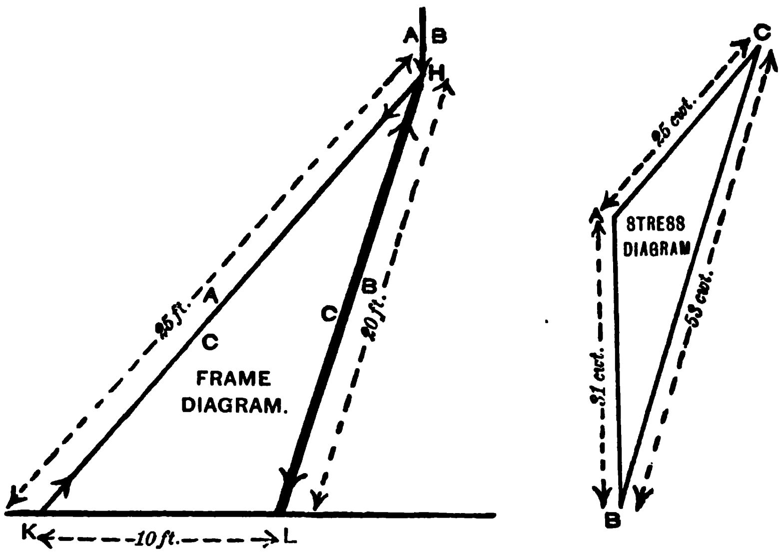
$$T \times LN = W \times LD + w \times LE$$

Or, $$T = W \times \frac{LD}{LN} + w \times \frac{LE}{LN} = (W + \tfrac{1}{2}\, w)\frac{LD}{LN}.$$

From Euclid II., 12, we have :—

$$KH^2 = KL^2 + LH^2 + 2\,KL\,.\,LD$$

$$\therefore \quad LD = \frac{KH^2 - KL^2 - LH^2}{2\,KL} = \frac{625 - 100 - 400}{20} = 6{\cdot}25 \text{ ft.}$$



STRESSES IN A DERRICK POLE.

Also, $LN \times KH = \text{twice triangle } LKH = HD \times KL$

Or, $LN \times KH = (\sqrt{LH^2 - LD^2}) \times KL$

$$\therefore \quad LN = \frac{KL}{KH}\sqrt{LH^2 - LD^2} = \frac{10}{25}\sqrt{400 - 39}$$

Or, $$LN = \frac{2}{5}\sqrt{361} = \frac{38}{5} = 7{\cdot}6 \text{ ft.}$$

Hence, $$T = (30 + 1)\,\frac{6{\cdot}25}{7{\cdot}6} = \frac{31 \times 6{\cdot}25}{7{\cdot}6} = 25{\cdot}5 \text{ cwts.}$$

130-Ton Steam Crane* (*see Frontispiece*).—As an example of a very large crane, we have illustrated in the frontispiece to this volume the 130-ton steam crane erected for the Clyde Trustees at Finnieston Quay, Glasgow, by Messrs. Cowans, Sheldon, & Co., Limited, of Carlisle. A similar crane has also been put up at the new Cessnock Dock, Glasgow. The jib of this crane is made up of two steel tubular girders braced together by diagonal and cross stays. The tension rods have been sawn out of solid steel plates, and were not heated during their manufacture. They are connected to the jib by stays at intervals along their length. The foot of the jib is attached to one of the bottom corners of a large vertical triangular frame, and the tension rods to the upper corner, while the back one supports the balance weight which is placed between the two sides of the main framing. The boilers and engines are also placed within this framing, which is covered in so as to form an engine-house. The whole is fixed on the top of a circular base, which can rotate around a large central pin, and rests on steel rollers running on a steel pathway on the top of the foundation. There is also a roller bearing between the base and top of the centre-pin. The foundation, which is square in plan, is of concrete with granite corners and cope, and is supported on twenty-two concrete cylinders sunk into the sand. The centre piece of the crane is fixed to the foundation by six steel bolts cottered to washer plates in a tunnel inside the foundation.

There are two separate lifting blocks, the one for heavy, and the other for light weights. Each of these can be raised or lowered at two different speeds. Separate engines are provided for each of these blocks, and for rotating the crane. All three sets of engines have two cylinders with cranks at right angles, so as to start from any position. Steel wire ropes are used for hoisting instead of chains. The heavy weights are taken on an eight-purchase pulley block, and the light weights on a double-purchase pulley block. All the gearing, up to 24 inches diameter, is of cast steel, and the remainder of a mixture of cast-iron and steel. Gun-metal bushes are used throughout.

The crane is provided with a 160-ton Duckham hydrostatic weighing machine, and was tested by loading it with 150 tons of steel rails. Its radius of action is 65 feet, and the total lift is 100 feet.

The following tables show some of the leading dimensions and weights:—

* For a complete description of this crane see *Engineering*, June 9, 1893.

PARTICULARS OF 130-TON CRANE AT FINNIESTON QUAY, GLASGOW.

Part.	Number.	Length.	Breadth.	Diameter.	Thickness or Height.	Height above or below Quay.	Total Weight in Tons.	Projects beyond Cope of Quay Wall.
Foundation above front cylinders,	...	40′ 0″	40′ 0″	...	45′ 0″	Top 20′ up	4,300	5′ 6″
Foundation, Concrete cylinders,	22	36′ 6″	...	9′ $7\frac{1}{2}$″ and 5′ $9\frac{1}{2}$″	...	...	...	...
Tunnel and Passage,	...	...	2′ 0″	Centre 10′ 9″	6′ 0″	Top 10′ down	...	...
Foundation bolts,	6	38′ 9″	...	0′ 5″	...	...	8	...
Washer Plates,	6	6′ 0″	6′ 0″	..	...	10′ down.	13	...
Centre Piece for Crane,	...	...	...	...	...	...	9	...
Centre Pin,	...	...	...	0′ 17″	...	...	6	...
Roller Path,	...	...	...	33′ 0″	...	...	12	...
Rollers,	75	...	...	0′ 14″	...	...	10·5	...
Framing,	...	...	...	...	27′ 0″	...	50	...
Boiler,	...	...	...	6′ 0″	14′ 0″	...	6	...
Back Balance,	...	...	...	...	...	...	100	...
Jib,	...	90′ 0″	...	Tubes 3′ 3″ at centre to 2′ 6″ at ends.	$\frac{7}{16}$″ at centre to $\frac{1}{2}$″ at ends.	Top 110′ up.	45	...
Single Tension Rods,	...	...	0′ 10″	...	0′ $2\frac{3}{4}$″	...		
Double ,,	...	...	0′ 10″	...	0′ $1\frac{1}{8}$″	...	15	...
Pins for ,,	...	...	...	0′ 8″	...	...		
Pulley for light weights,	...	...	...	2′ 6″	...	107′ 6″	...	44′ 3″
Pulley for heavy weights,	...	...	...	5′ 3″	...	100′ 0″	...	39′ 6″
Gin block for light weights,	...	...	...	...	...	...	...	44′ 3″
Gin block for heavy weights,	4 pulleys	12′ 0″	7′ 0″	Pulleys 5′ 3″	3′ 0″	...	7	39′ 6″
Hoisting Drum for light weights,	...	...	...	2′ 6″	...	...	...	...
Hoisting Drum for heavy weights,	...	10′ 0″	...	5′ 2″	...	...	10·5	...
Gearing,	...	...	...	...	...	...	8	...
All Castings,	...	...	...	...	...	...	120	...
Crane in working order, exclusive of back balance,	...	...	...	...	...	...	270	...
Radius of sweep for light loads,	...	69′ 9″	...	...	...	...	...	...
Radius of sweep for heavy loads,	...	65′ 0″	...	...	...	...	...	...

Particulars of Engines for 130-Ton Crane.

For Load.	Engines.			Distance Raised in One Minute.	Time to make Complete Revolution of Crane.
	No. of Cylinders.	Diameter.	Stroke.		
130 tons, . .	2	12″	16″	4′	5 mins.
60 " . .				8′ $10\frac{1}{2}$″	2 mins. 17 secs.
20 " . .	2	8″	12″	28′	...
8 " . .				60′	...
Revolving, . .	2	8″	12″	...	...

Particulars of Steel-Wire Ropes for 130-Ton Crane.

Rope.	Inner Core.		Second Layer.		Third Layer.		Outer Layer.		Total No. of Wires in each Strand.	No. of Strands.	Total No. of Wires.	Remarks.
	No. of Wires.	Diam. of Wires.	No. of Wires.	Diam. of Wires.	No. of Wires.	Diam. of Wires.	No. of Wires.	Diam. of Wires.				
Heavy Lifts, .	3	·078″	9	·084″	15	·084″	18	·101″	45	7	315	Six strands are of patent steel, and are wound round a central strand of soft steel.
Light Lifts, .	1	·060″	6	·060″	12	·060″	18	·060″	37	7	259	

The factors of safety allowed in the different parts are :—

Main framing, jib, tension rods, &c., ...	6
Wire ropes, ...	8
Centre holding down bolts, to allow for rusting, .	12

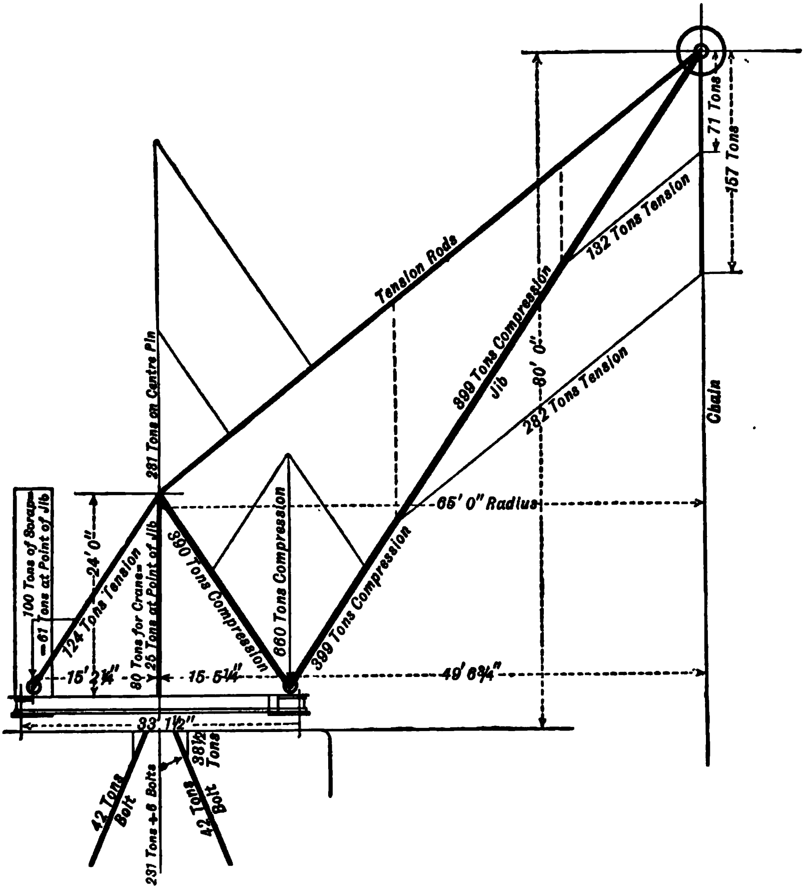


FIG. 14.—STRESS DIAGRAM FOR 130-TON CRANE. (TEST LOAD, 150 TONS.)

The accompanying figures show the Stress Diagram for the crane when loaded with 150 tons, or, including the weight of the gin block, 157 tons in all, and also for a gross load of 71 tons.

Fig. 14 is the Stress Diagram as worked out by the makers. Fig. 15*a* represents the Frame Diagram and Fig. 15*b* the Stress Diagram worked out for the Test Load as we have treated the previous cranes—viz., all in one figure.

First, find the values of the Forces D E and A B in Fig. 15*a* as already explained. Then begin the Stress Diagram with the force A B and follow with the forces B C, C D, D E, E F and F G. Now E L and G L fix the point L; B H and C H the point H; H K and L K the point K; K N and G N the point N; and N M and A M the point M.

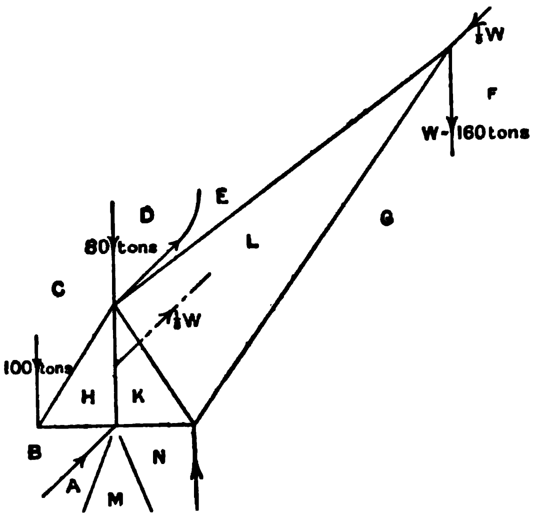


FIG. 15*a*.—FRAME DIAGRAM.

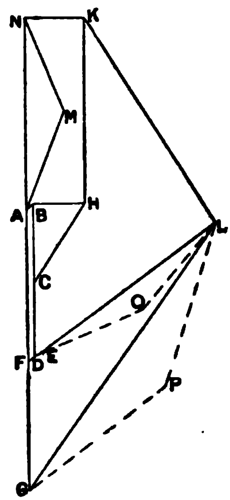


FIG. 15*b*.—STRESS DIAGRAM.

130-TON CRANE WORKED OUT AS IN THE PREVIOUS EXAMPLES.

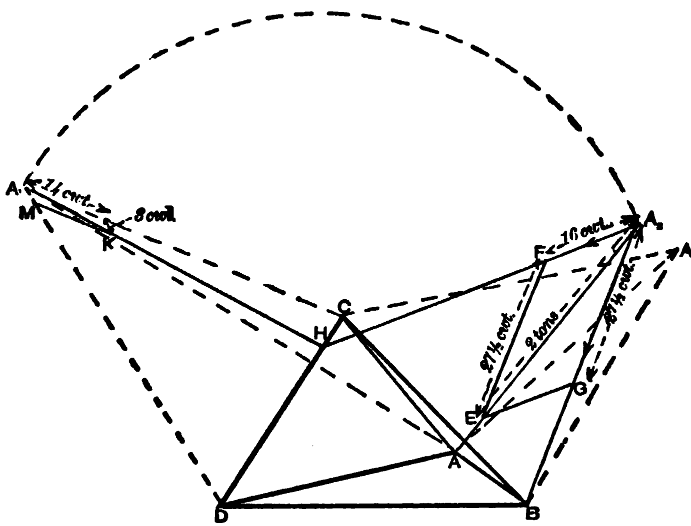
The stress E L is borne by the two tie-rods and L O and E O represent the stresses in each tie-rod. The stress G L is divided over the two jibs, consequently L P and G P represent the stresses in each jib, as already explained.

The stress A M is divided over half of the holding down bolts and N M over the other half. The point about which the crane tends to topple over is the joint K L G N.

EXAMPLE III.—A tripod whose vertex is A, and whose legs are A B, A C, A D, of lengths 8, 9, and 10 feet respectively, sustains a load of 2 tons. The ends B, C, D form a triangle, whose sides are B C = 7 feet, C D = 6 feet, B D = 8 feet, find

by graphical construction the compressive stress in each leg. (S. and A. Exam., 1889.)

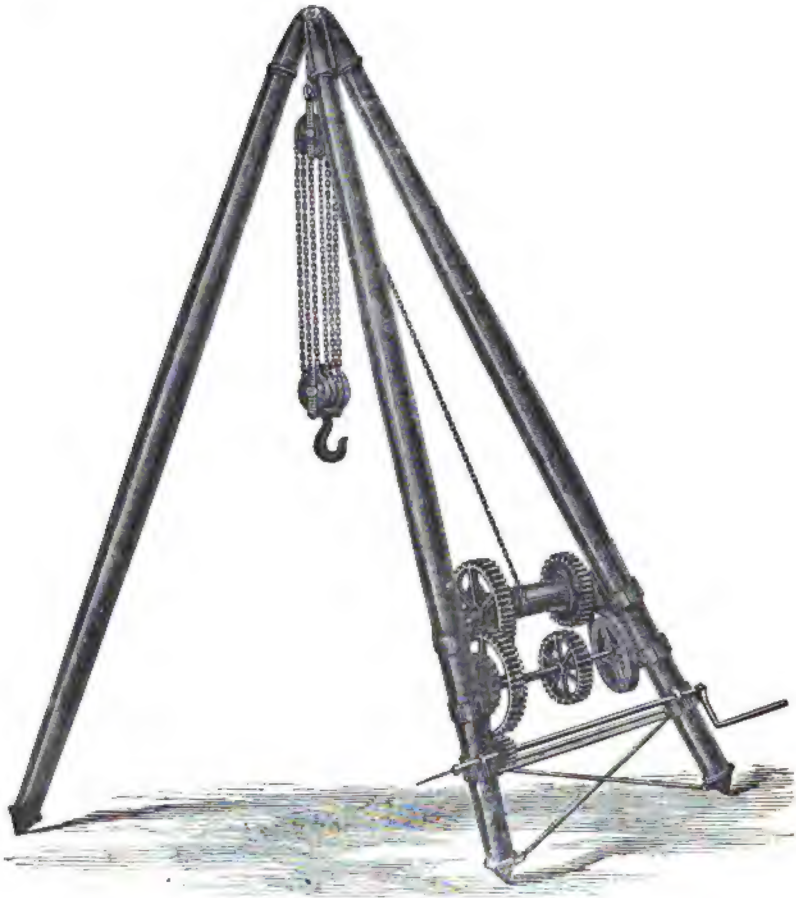
Answer.—Hitherto we have dealt only with forces in one plane, or symmetrical with respect to one plane. Thus, in the case of the sheer legs, in determining the compression on the front legs, we first found what would be the compression on an intermediate leg in the same plane as the two replaced, and equally inclined to both. This hypothetical leg would be in a vertical plane containing the back leg and the externally applied force, and evidently the stress in the back leg would not be affected by the substitution. In the present example we must find the



Stresses in a Tripod.

corresponding intermediate leg, so that if the load be supported by it and the third leg, the stress in the latter will not be altered. It will be in a vertical plane containing the third leg, and will be represented by the line of intersection of this plane with that of the other two legs, hence the following solution :—Draw B C D the triangle formed by the feet of the tripod; to find the plan of the vertex, draw the triangle D C A_1, making C A_1 and D A_1 equal to C A and D A respectively. Similarly, draw the triangle A_2 C B; then A_1 A drawn perpendicular to C D, and A_2 A drawn perpendicular to C B, will give by their

intersection the plan of the vertex A, and we may now complete the plan. The vertical plane containing the leg A B will cut the line D C in a point H lying on the line B A produced;



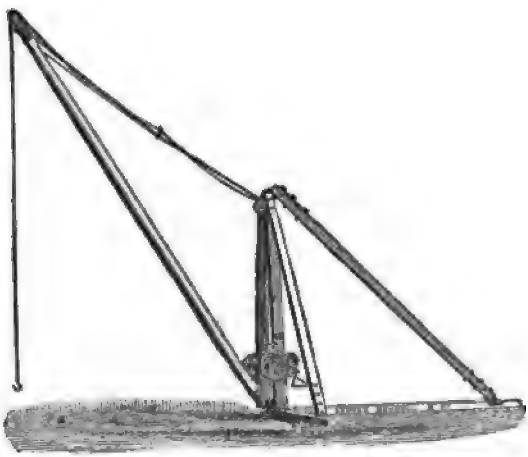
TRIPOD FOR LAYING PIPES.

then A_1 H will be the length of the required intermediate leg. Draw A A_3 at right angles to B A H, and make H A_3 = H A_1. Thus we get A_3 H, A_3 B, and the applied force A_3 E in one

plane. The stress in the leg A B will be represented by A_3 G, and the resultant stress in the legs A C and A D—*i.e.*, the stress in the hypothetical leg A H by A F. Dividing this between the actual legs by the triangle of forces A_1 M K, where A_1 K = A F, we get the stress in A C represented by · K, and that in A D by A_1 M. The resp ctive values are marked on the diagram.

As a practical example of the use of a tripod, we illustrate a form much used—in laying water and other mains—for lowering large pipes into position. Two of the legs are braced together, and carry a winch which may be used in conjunction with a block and tackle. The pipe is rolled on to wooden beams laid across the drain, and the tripod then placed in position over it. The pipe is slightly raised by means of the winch, and the wooden beams removed, when the pipe may be lowered with ease.

We also illustrate a simple hand crane as used by contractors for building, &c. It is of the kind discussed in connection with Fig. 7.



HAND DERRICK CRANE.

Lecture XXVII.—Questions.

1. In a model of a crane the jib is $3\frac{1}{2}$ feet long, the tie-rod is 3 feet long, and is fastened to a point 1 foot vertically above the lower end of the jib. What is the thrust on the jib when a weight of 20 lbs. is hung at the upper end of it? *Ans.* 70 lbs.

2. In a wharf crane the post, tie-rod, and jib measure 15, 20, and 30 feet respectively, what would be the nature and amount of the stresses in each of the three members when a load of 7 tons is suspended over the pulley at the jib head, (1) when the lifting chain passes from the pulley to the drum or barrel parallel with the jib, (2) when the drum is placed so that the chain passes from the jib head parallel with the tie-rod?

3. In a hydraulic wharf crane the height of the post is 6 feet, the jib is 22 feet, and the tie-rod is 18 feet; find the horizontal thrust on the post when 5 tons are supported. In what way is the friction which opposes the *s'ewing* motion reduced to a minimum? *Ans.* 12·24 tons.

4. Find, either graphically or otherwise, the stresses on the jib and tie-bar respectively of a crane, whose jib measures 20 feet in length, when the tie-bar and post are 16 feet and 6 feet in length respectively, and a weight of 25 cwts. is suspended from the end of the jib. The line of direction of the chain after leaving the barrel or drum runs parallel to the tie-bar. Also calculate the pressure on the end of the handle of 16 inches radius when the weight is lifted, supposing the drum of the crane to be 15 inches in diameter, and the gearing to consist of a pinion of 12 teeth, gearing into a wheel with 72 teeth, while a second pinion of 18 teeth gears with a wheel of 56 teeth.

5. A contractor's portable hand crane has a vertical post A B, to which the jib A C is inclined at 45°, and the tension rod B C makes with A B an angle A B C of 120°. The back stay from the head of the post B to the extremity D of the horizontal strut A D is inclined at an angle of 45° to A D. Find the weight of the counterbalance required at D to balance a load of 10 tons suspended from the end C of the jib. Determine also the nature and amount of the stress on the jib A C, and in the rods B C and B D? (The tension in the chain may be neglected.) (S. and A. Adv. Exam., 1896.)

6. A jib foundry crane consists of a vertical post A B, 16 feet long, fitted with pins working in sockets at both A and B. From the upper end A of the post extends a horizontal member A D, 28 feet in length, and from the foot B is a strut B C, which meets A D at a point 16 feet from A. A load of 20 tons being suspended from D, find the shearing stress on the pin at A, and the stress along the strut B C.

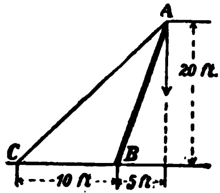
7 A sheer-legs is formed of two sheer-poles B C, D C, each 25 feet in length, and secured to a base-plate in the ground at B and D. The wire guy or tension rope A C is attached to the ground at a point A, which is 60 feet distant from B D. The perpendicular from the top C of the poles meets the ground at a distance of 10 feet from the centre of the line B D, which is 15 feet long. Find by measurement or otherwise the tension in tons on the guy when a weight of 20 tons is suspended. *Ans.* 11·3 tons.

8. The jib of a 10-ton crane is inclined at 45° to the vertical, and the tie rod at an angle of 60°. Find the thrust on the jib and the pull on the rod, the tension in the chain being neglected. If the tension in the chain is by means of tackle, half the load and the chain barrel is so placed that the direction of the chain bisects the crane post, find the forces in the jib and tie rod. Graphical constructions may be used. (C. & G., 1902, H., Sec. A.)

Lecture XXVII.—A.M.Inst.C.E. Exam. Questions.

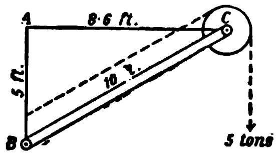
1. A pair of sheer-legs make an angle of 20° with one another and their plane is inclined at 60° to the horizontal. The back-stay is inclined at 30° to the plane of the legs. Find the force on each leg, and on the stay, per ton of load carried. (I.C.E., *Oct.*, 1897.)

2 A set of shear legs consists of 2 struts, A B set 10 feet apart at the



lower ends (B), and a stay rod AC. Determine graphically the stresses in the strutts and in the tie when a load of 20 tons hangs from A. (I.C.E., *Feb.*, 1898.)

3. A weight of 5 tons hangs upon the chain of the bracket or crane here sketched. The jib B C has an inclination of 30° with the horizontal tie-rod A C; so that if B C is 10 feet, A B is 5 feet, and A C is 8·6 feet. At



C the chain is carried over a frictionless pulley, and led down parallel to the jib. Find the stresses in the members A C and B C. (I.C.E., *Feb.*, 1899.)

4. A load is suspended from a pair of sheer legs supported by a back leg, the foot of which is moved by a horizontal screw. Show by diagrams the stress on each leg when the load overhangs by a given amount, and find the resistance overcome by the screw in moving the legs. (I.C.E., *Oct.*, 1899.)

5. In a portable crane stability is obtained by a counterweight placed at the end of a horizontal piece stayed to the crane-post in the usual way. Show by diagrams the stress on each part of the structure, and the magnitude of the counterweight when a load is simply suspended from the end of the jib. (I.C.E., *Feb.*, 1900.)

6. Explain how you would determine graphically whether five given coplanar forces acting at a point are in equilibrium or not. In a portable erecting shop crane the height of the crane post is 8 feet; angles of back stay and jib with crane post, each 45°; angle of front stay with crane post, 120°. Find, by graphical means, the nature and magnitude of the forces in the various parts, and determine the necessary amount of the balance weight applied at the extremity of the back stay when 5 cwts. are being lifted. (I.C.E., *Oct.*, 1900.)

7. In a wall-crane the jib is 10 feet long, and makes an angle of 45° with the wall. Determine graphically the position of the tie-rod, so that the tension in it may be a minimum for a given load on the crane. Then, assuming the tie-rod to be fixed in this position, determine graphically (*a*) the stresses in the tie and jib, when the load of 5 tons is suspended from a chain, which passes over a pulley at the junction of the tie and jib to some point on the wall, so that it bisects the angle between the tie and the jib. (*b*) How those stresses are affected by the position of the chain relative to the tie and jib, taking as extreme positions, (i.) when the chain is parallel to the tie, (ii.) when the chain is parallel to the jib. (The diameter of the pulley may be neglected.) (I.C.E., *Oct.*, 1901.)

8. A weight of 10 tons is suspended from a tripod of equal legs whose feet rest on the ground in an equilateral triangle of side 12 feet, and whose apex is 25 feet from the ground. Calculate the stress in each leg. (I.C.E., *Oct.*, 1902.)

9. A crane has a vertical crane-post, A B, 8 feet long, and a horizontal tie, B C, 6 feet long, A C being the jib. It turns in bearings at A and B, and the chain supporting the load passes over pulleys at C and A, and is then led away at 30° to A B. Find the stresses in the bars and thrusts on the bearings when lifting 1 ton at a uniform rate. (I.C.E., *Feb.*, 1903.)

LECTURE XXVIII.

Reactions on a Beam.—FIRST METHOD.—*First Case.*—In the Frame Diagram, Fig. 1*a*, we have a beam with five concentrated loads upon it. The line of action and point of application of the left-hand reaction, and the point of application of the right-hand reaction are known.

We commence Fig. 1*b* by drawing the line of loads, B C, C D, D E, E F and F G. Then, any point O is taken and joined with all the points in the line of loads as illustrated in the figure. This point O is called a Pole and Fig. 1*b* a Polar Diagram.

Since the point of application of the right-hand reaction (Fig. 1*a*) is the only one of its elements which is known, we must begin at this point 1 when drawing the Funicular Polygon* 1 2 3 4 5. In the Polar Diagram, Fig. 1*b*, the line O F comes between the loads E F and F G. Then, from the point 1, in the Frame Diagram, draw the line 1—2, parallel to O F. The line 1—2 begins in the line of action of the load F G and ends in the line of action of the load E F—viz., the two loads between which the line O F lies in the Polar Diagram. Between the lines of action of the loads E F and D E, draw the line 2—3 parallel to the line O E in the Polar Diagram, which lies between these same two loads. Similarly draw 3—4 parallel to O D and 4—5 parallel to O C. On joining the point 5 with the point 1 we close the Funicular Polygon. This closing line 5—1 has one end 5, in the line of action of the reaction A B, and the other end 1, in the line of action of the reaction A G. Therefore, if a line O A be drawn parallel to this closing line from O, it must lie between the reactions in the line of loads, just as the other lines radiating from O have done.

If we draw B A from B in the line of loads parallel to the line of action of the reaction A B, so as to meet the line O A in the

* See explanation of Funicular Polygon given at the end of this Lecture.

point A, then A joined with G completes the external force polygon.

Second Case.—If the two ends of the beam had been anchored, then the lines of action of the reactions G A and A B would have been parallel to each other and to the line joining B with G in the Load Diagram. In this case the lines of action of the

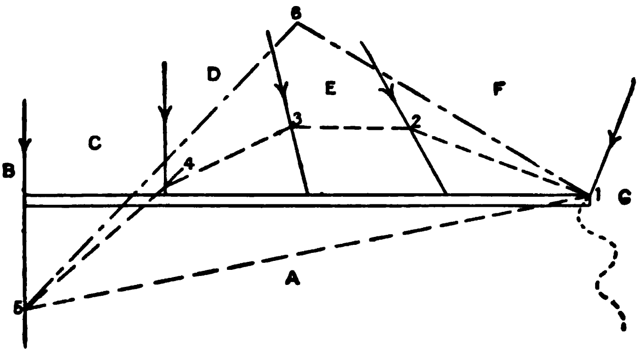


FIG. 1*a*.—FRAME DIAGRAM AND FUNICULAR POLYGON.

reactions would be drawn on Fig. 1*a*, and the point 1 taken anywhere in the line of the reaction G A; or we could begin with the point 5 anywhere in the line of the reaction A B, and draw the Funicular Polygon as stated above.

If the load F G had not existed, we would still have begun at the point 1 for the first case. The line O F would then be between the load E F and the reaction F A. In the second case mentioned above, the solution would still be the same as before.

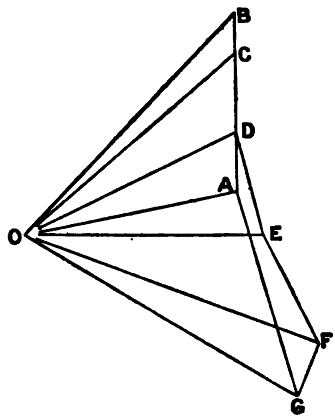


FIG. 1*b*.—LOAD AND POLAR DIAGRAM FOR DETERMINATION OF REACTIONS AND RESULTANT LOAD.

Resultant of the Loads on a Beam.—From the point 1 in the Frame Diagram, Fig. 1*a*, draw the line 1—6 parallel to the line O G in the Polar Diagram. This line O G lies between the load F G and the resultant G B. Now draw the line 5—6

parallel to the line O B, so as to intersect the line 1—6 in the point 6. Then the point 6 is a point in the line of action of the resultant of the loads. If through the point 6 we draw a line parallel to B G, then that line will represent the line of action of the resultant load. This line cuts the beam at the point round which the beam would balance. The magnitude of the resultant is represented by the length of the line B G.

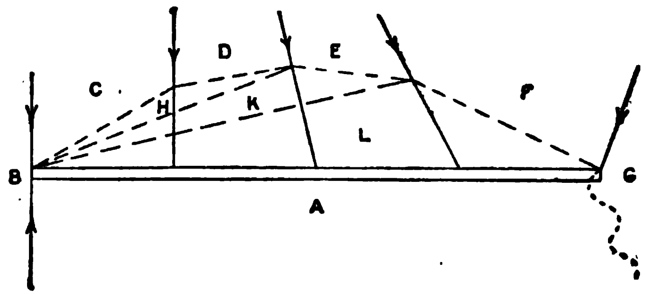


Fig. 2*a*.—Frame Diagram.

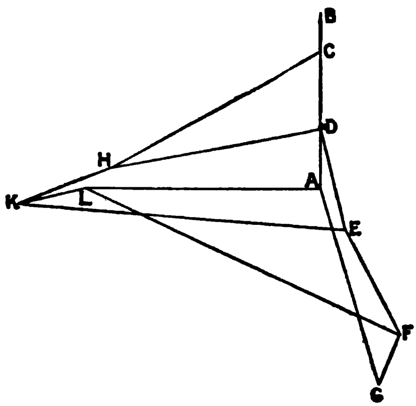


Fig. 2*b*.—Load and Reaction Diagram for Determination of Reactions.

If we wished to find a point in the line of action of the resultant of the loads C D, D E, and E F, then from the point 2, draw a line parallel to the line O F, and from the point 4 a line parallel to the line O C, so as to intersect the first line from the point 2. This point of intersection is a point in the line of action of the resultant of C D, D E, and E F. Its line of action passes through this point and is parallel to the line joining C with F, while its magnitude is represented by the line C F.

Reactions on a Beam.—Second Method.—On the top of the beam draw a Firm Frame of any shape, having its joints in the lines of action of the loads as indicated in the Frame Diagram, Fig. 2*a*. Letter the Frame Diagram according to

Bow's method, and draw the Load and Stress Diagram as already explained. As a graphic solution the second method is the more accurate of the two, and may be applied with greater ease in the case of lattice girders, for we have only to join the points of the girder with one end, or some points with one end, and the remainder with the other end.

Fink Truss.—This truss was largely used in America for wooden bridges and is represented in the Frame Diagram, Fig. 3*a*. It consists in this case, of a primary truss B A G, and two secondary trusses. The divisions of the beam are called panels. The truss in Fig. 3*a* is therefore a Fink Truss of four panels.

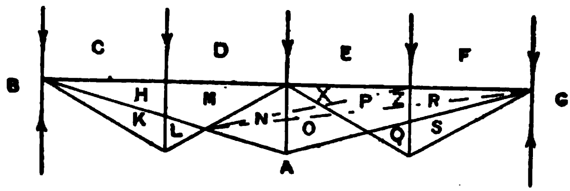


FIG. 3*a*.—FRAME DIAGRAM, FOR A FINK TRUSS.

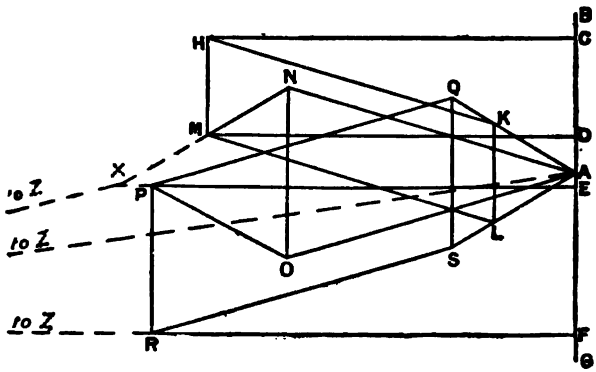


FIG. 3*b*.—STRESS DIAGRAM.

If the foot of every upright had been joined with each end of the beam instead of as shown, the truss would be called the **Bollman Truss**, of which the solution is similar to the following explanation for the Fink Truss. Draw the external force polygon as explained for Figs. 1*a* and 2*a*.

FIRST METHOD FOR STRESS DIAGRAM.—Join the joint M N A L with the joints E F R P and F G A S R, as shown by the dotted lines in the Frame Diagram, Fig. 3*a*. Call the spaces thus formed

X and Z as shown in the Frame Diagram. This will enable us to determine the point M in the Stress Diagram, Fig. 3*b*. Draw F Z and A Z, parallel to the bars F Z and A Z. This gives the point Z. Then Z X and E X fix the point X; and X M and D M fix the point M. The Stress Diagram may now be completed in the usual way, and the closing line will form a check line.

Second Method for Stress Diagram.—In Fig. 4, we have four forces in equilibrium acting at a point and their lines of action lie along two straight lines. The condition for equilibrium is, that H M is equal and opposite to K L and M L is equal and opposite to K H. The Force Polygon will therefore be a parallelogram and may be represented by H M L K in the Stress Diagram, Fig. 3*b*.

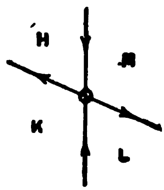


Fig. 4.—Forces Acting in Straight Lines at a Point.

From the above, it is evident that the stress in H M must be equal to the load C D and the stress in K L equal to the stress in H M. Therefore, draw A K and A L parallel to the bars A K and A L respectively and draw K L between them parallel and equal in length to C D. This will determine the points K and L in the Stress Diagram, Fig. 3*b*, which may now be completed. The closing line will form a check as in the first method.

Trapezoidal Truss of Three Panels.—In this Truss, Fig. 5*a*, the space F will always remain a parallelogram, as was the case in the Queen Post Frame which has been already discussed. In fact this is a similar case to the roof considered with a continuous tie-beam.

If the beam is uniformly rigid, then whatever distance the joint C D G F is deflected below the horizontal line, the joint B C F E rises an equal distance above the horizontal line. Therefore, the difference between the vertical component of the load C D and the stress action in G F, must be equal and opposite to the difference between the vertical component of the load B C and the stress action in E F.

In order to obtain the Stress Diagram, Fig. 5*b*, we first of all draw the external force polygon B C D A, and determine the point A, as explained for Figs. 1*b* or 2*b*. Then draw A F parallel to the bar A F and $B_1 C_1 D_1$ perpendicular to A F. $B B_1$, $C C_1$ and $D D_1$ are parallel to A F. $B_1 C_1$ and $C_1 D_1$ will be the vertical components of B C and C D respectively.

Since $C_1 D_1$ is greater than $B_1 C_1$, the joint C D G F will be deflected downwards. Therefore, F G will be less than $C_1 D_1$

and is as much less than $B_1 C_1$ as $C_1 D_1$ is less than E F. This is the same thing as saying that G F and F E are together equal to $B_1 D_1$.

The point E lies on the line A E drawn parallel to the bar

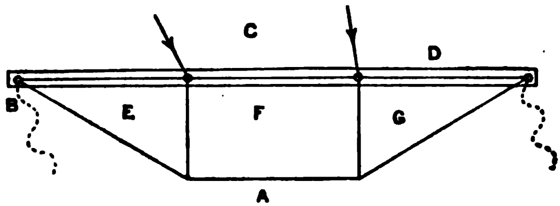


FIG. 5a.—FRAME DIAGRAM FOR A TRAPEZOIDAL TRUSS.

A E and similarly for the point G. Place the line E G between the lines A E and A G, parallel to the bars G F and F E and equal in length to $B_1 D_1$. This determines the points E and G. Where E G cuts the line A F fixes the point F. Then by joining F C, E B and G D we complete the Stress Diagram.

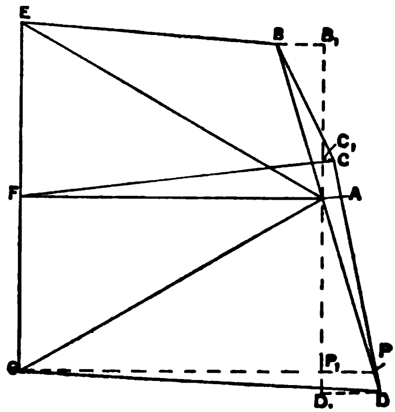


Fig. 5b.—STRESS DIAGRAM.

Another method of determining the point G is to make A P_1 equal to one-half of $B_1 D_1$, or A P equal to one-half of B D, and draw P_1 G or P G parallel to A F, so as to cut the line A G. This determines the point G.

Trapezoidal Truss of Five Panels.—In this truss, Fig. 6a, the space O always remains a parallelogram, but the other spaces cannot change. Similar reasoning to what we used in the previous figure will show that the loads D E and E F are together equal and opposite to the sum of the stress actions P O and O N. The load C D is equal to the stress in L N, and also equal to the stress in K M. Similarly the load F G is equal to the stresses in the bars P R and Q S. It follows from this, that the stresses R P, P O, O N, and N L are together equal and opposite to the loads C D, D E, E F, and F G. The Stress

Diagram, Fig. 6*b*, may be determined by drawing A K parallel to the bar A K, and making K M equal to C D. Then through M draw M N parallel to the bar M N. The point N lies on this

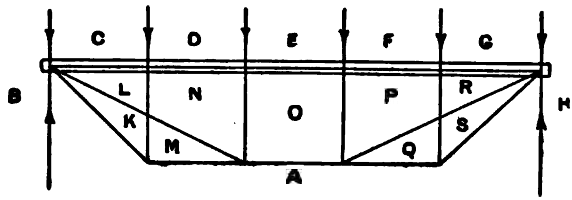


FIG. 6*a*.—FRAME DIAGRAM FOR FIVE PANEL TRAPEZOIDAL TRUSS.

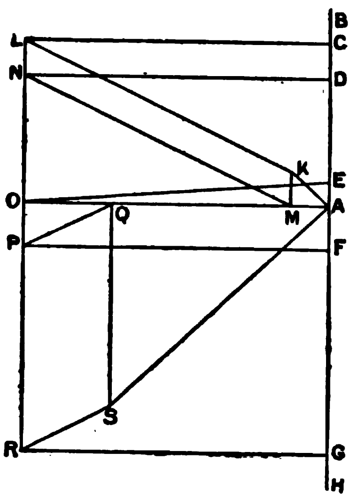


FIG. 6*b*.—STRESS DIAGRAM.

line. Draw A S parallel to the bar A S, and make S Q equal in length to F G. Through Q draw Q P parallel to the bar Q P. The point P lies on this line. Now O P and O N are together equal to F E and E D. Consequently, if we make P N equal to F D and parallel to the bars O P and O N, this will determine the points P and N. Draw A O parallel to the bar A O. This fixes the point O. The points L and R may now be determined, and the finishing lines of the diagram inserted.

In Fig. 7 the forces acting on the left-hand portion of the beam are shown as taken from the Stress Diagram, Fig. 6*b*. They are the actions of the pins on the beam, and of the right-hand half on that end.

In Fig 8 is shown the form which the left-hand end must take from the previous assumptions. The three points shown

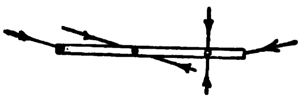


Fig. 7.—Forces acting on the Left-Hand Half of the Beam.

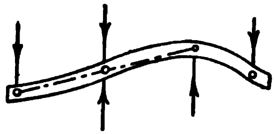
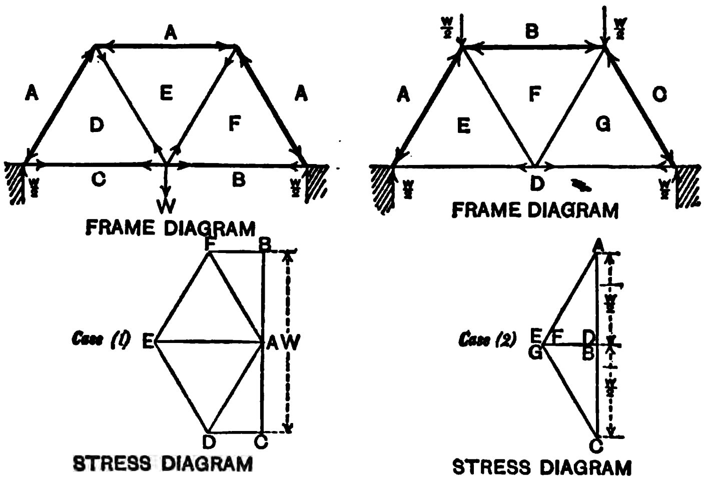


Fig. 8.—Form into which the Forces Bend the Left-Hand Half of the Beam.

in a straight line must always remain in a straight line. There will be three points of contrary flexure in the length of the beam.

These Trapezoidal Trusses are deficient frames made redundant by stiff joints.



Triangular Frame.

Example I.—A triangular frame, consisting of three equilateral triangles, is loaded with a weight W. Find the stresses on the several members of the frame (1) when W is hung at the lower apex of the central triangle, (2) when each of the triangles is loaded at the upper apex with $\frac{W}{2}$.

Answer.—Since the loading is symmetrical in both cases, the reactions are each equal to one-half of the total load—that is, to $\frac{1}{2}$ W.

Case (1).—Draw B C vertical and equal to W units. Bisect it at A, so that C A and A B are the reactions. Make C D parallel to the bar C D, and A D to A D. This gives us D. Then draw D E and A E respectively parallel to the bars D E and A E, so as to obtain E. A F and E F fix F, and B F completes the Stress Diagram. B F should be parallel to the bar B F, and this gives us a check on our work.

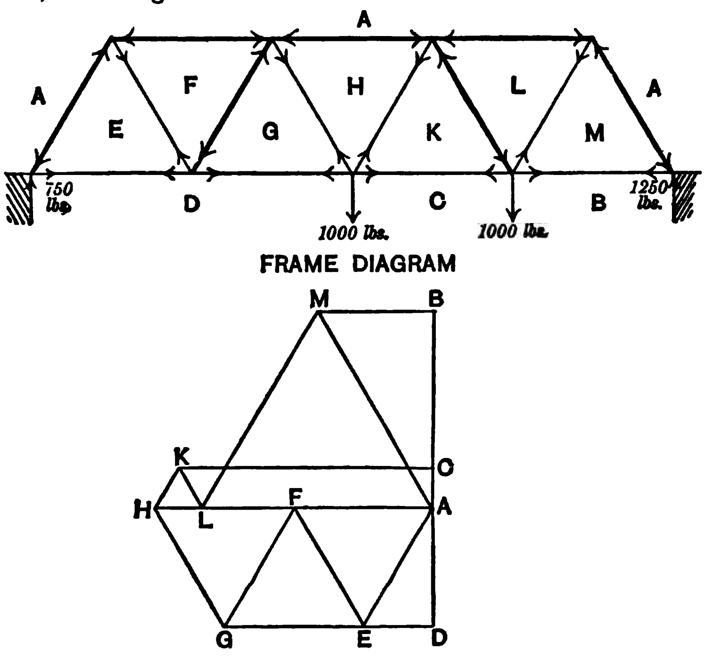


Fig. 9.—Warren Girder.

Case (2).—Here we must make A B and B C each equal to $\frac{1}{2}$ W. D A and C D will represent the reactions, D being the middle point of B C, and therefore coincident with B. A E and D E determine E, and C G and D G give G. E and G will coincide since B and D do, and everything is symmetrical. The point F also coincides with E and G, so that there is no stress in the bars E F and F G.

Warren Girder.—Fig. 9 illustrates a Warren Girder of four bays or panels, which is simply an extension of the above triangular frame. The upper horizontal member is called the **Upper Boom** or **Flange**, and the lower horizontal member is called the **Lower Boom** or **Flange**. The inclined members are called **Lattice Bars** or **Braces**. The joint A F E or E F G D is called an **Apex**. The angle of triangulation is usually 60°, but sometimes the triangles are right-angled isosceles. When loads are applied at the centres of the lower members, tie-rods are put from the centres of the lower members to the opposite apices. These tie-rods transmit the central loads to the upper apices. The Frame Diagram would then show a system of loads at the lower and upper apices. From what has already been said the student should find no difficulty in drawing the Stress Diagram, which is obtained in exactly the same way as those in Example I.

Linville or N Girder.—This girder differs from the Warren Girder, in that the bars connecting the two horizontal booms are placed alternately vertical and oblique, forming a series of right-angled triangles instead of the corresponding equilateral ones in the Warren Girder. It is so arranged that the shorter vertical bars shall be in compression and the oblique ones in tension. This clearly tends to a saving of material and a diminution of weight, since compression members, unless very short, must, other things being equal, be much heavier than tension members. Also, compression members are much strengthened by shortening, while a tension member is not weakened by lengthening.

We may determine the reactions by a "substituted frame" by the Funicular Polygon, or, in this case, by calculation.

In drawing the Stress Diagram we observe that A *a* must be equal to Q A, and *a* Q is zero, since the reaction is vertical. The members *a* Q and *n* K are necessary to give the required stability. These, together with the end vertical members, might be dispensed with by carrying the supports up to the upper boom.

Draw the lines representing the loads and reactions, viz. :—K L, L M, M N, N P, P Q, Q A, A B, B C, C D, D E, E F, F G, G H, and H K. Since A *a* = A Q, and H *n* = H K, so that *a* coincides with Q, and *n* with K, we may draw the Stress Diagram as follows :—

B *b*	and *a b*	fix the point *b*,	and G *m*	and *n m*	the point *m*.
P *c*	,, *b c*	,, *c*,	,, L *l*	,, *m l*	,, *l*.
C *d*	,, *c d*	,, *d*,	,, F *k*	,, *l k*	,, *k*.
N *e*	,, *d e*	,, *e*,	,, M *h*	,, *k h*	,, *h*.
D *f*	,, *e f*	,, *f*,	,, E *g*	,, *h g*	,, *g*.

By then joining fg we complete the Stress Diagram, and this line should be parallel to the central strut.

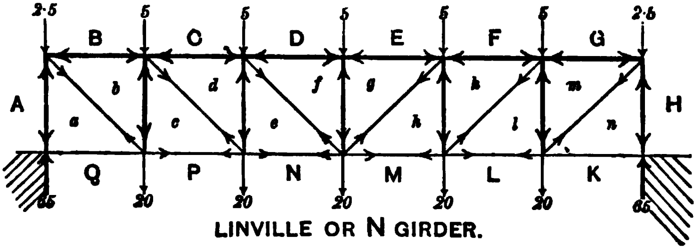


FIG. 10a.

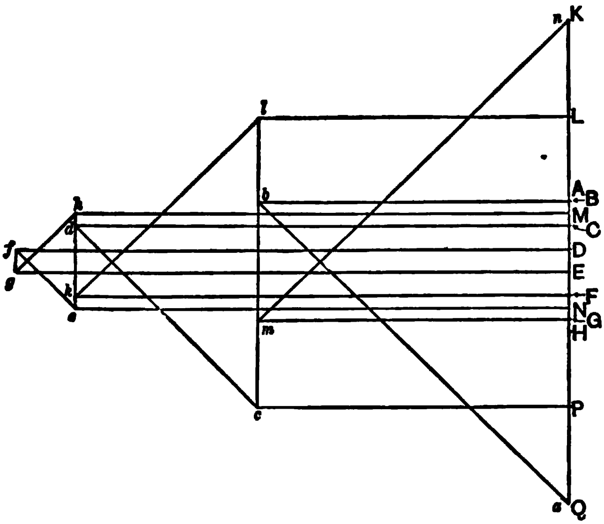


FIG. 10b.—STRESS DIAGRAM.

Lattice Girder.—Although in Fig. 12, we have drawn the external force polygon for Fig. 11, we can go no further with the Stress Diagram for frames of this kind until we have determined in some way the stress in one member, because these are redundant frames.*

* See the beginning of Lecture XXIV. for the properties of redundant frames.

ASSUMPTION.—**We assume, first of all, that all the members have been accurately fitted; that is, the frame is not initially stressed.**

One method of solution which has often been suggested is, to assume that the shear over any section is taken equally by the braces in that section. We shall however prove that this latter assumption is not consistent with the actual conditions. The shear on the right-hand bay of Fig. 11 is evidently one-half of B C, and according to the above method, the vertical component of the stress in the member G H would be equal to one half of the shear, that is one quarter of B C,

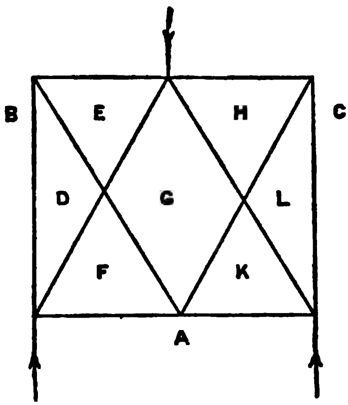


FIG. 11.—FRAME DIAGRAM.

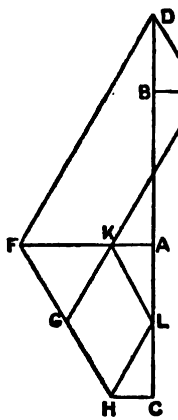


FIG. 12.—STRESS DIAGRAM WITH UNEQUAL STRESSES IN BRACES.

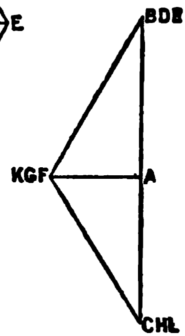


FIG. 13.—STRESS DIAGRAM WITH EQUAL STRESSES IN BRACES.

TWO BAY LATTICE GIRDER.

whilst the vertical component of the stress in the member G K would be equal to the other half of the shear, that is also one quarter of B C. Similarly, for the left-hand bay, according to the above assumption, the vertical components of the stresses in E G and F G would each be one quarter of B C.

Let us suppose the vertical component of the stress in G H and K L, to be one-quarter of B C, and then draw the Stress Diagram, Fig. 12 There A L is equal in length to the vertical component of K L and also equal to one-quarter of B C. The Stress Diagram may now be completed in the usual way. It shows that if the stress in G H has a vertical com-

ponent of one-quarter of B C, then the vertical component of the stress in E G is three-quarters of B C. This is a consistent result, because three-quarters and one-quarter of B C acting vertically together at the joint E B C H G will balance B C. But, if the members have been properly put together, there is no reason at all why the stress in E G should be greater than the stress in G H. The most sensible assumption to make is, that

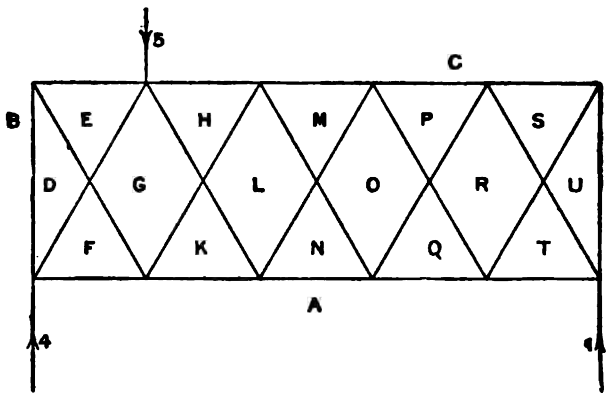


FIG. 14a.—FRAME DIAGRAM FOR FIVE-BAY LATTICE GIRDER.

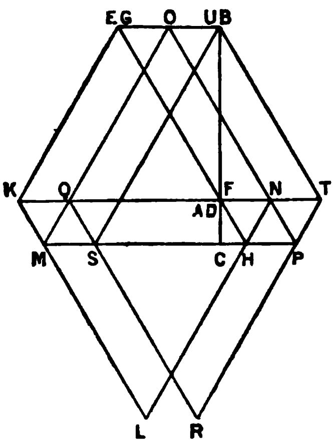


FIG. 14b.—STRESS DIAGRAM. WHEN BRACES E G AND D F ARE REMOVED.

the members E G and G H will each have a stress of which the vertical component is one-half of B C. The Stress Diagram for this assumption is worked out in Fig. 13.

Lattice Girder of Five Bays.—TO DETERMINE WHAT PROPORTION OF A SINGLE LOAD IS TRANSMITTED ALONG EACH BRACE.—In the Frame Diagram, Fig. 14a, we have assumed a load of 5 units at the joint B C H G E. From inspection, it will be evident that the reaction A B will be 4 units and A C will be 1 unit. The reactions may be determined by a substituted triangular frame with

its vertex at the loaded joint and rafter ends at the abutments. The Stress Diagram, Fig. 14*b*, has been drawn on the assumption that the braces E G and D F have been withdrawn. If the bar D F is removed, then the spaces D and F will have only one letter, let this be called D. Then B D and A D fix the point D; D F and A F the point F; D E and B E the point E; E G and F G the point G and so on, until the Stress Diagram is completed.

From the Stress Diagram, Fig. 14*b*, we see that the vertical components of the stresses in the braces * G H, K L, L N, M O, O P, Q R, R T, S U and U C are each equal to the load B C, and that the vertical components of the stresses in the remaining braces T U, R S, on to B D are each equal to four units; also that the kind of stress alternates between a push and a pull throughout the braces. The vertical component of the push in the lattice bar G H, is balanced by the vertical component of the pull in L N. Also, the vertical component of the pull in M O, is balanced by the vertical component of the push in O P, and similarly for Q R and R T, S U and U C. Now, the push in U C is balanced by the reaction C A, combined with the vertical component of the pull in T U, and since the reaction C A is one unit the vertical component of the pull in T U must be four units. This four units of vertical component of the pull in T U, is transmitted through the remaining lattice bars as a push and a pull alternately, until it reaches B D as a push, and is there balanced by the reaction A B.

The stress in each lattice bar produces a strain that will cause the load to dip. Now, since the stresses are severe and every lattice bar is stressed, the removal of the brace E G produces a very yielding frame.

In Fig. 15 we have the Stress Diagram which is obtained from the Frame Diagram, Fig. 14*a*, by removing the brace G H. The removal of this brace means, that there will be no stress in the bars L N, M O, O P, Q R, R T and S U. Since there is no stress in S U there can be none in C S or C U. Therefore, S and U will coincide with C and the Stress Diagram can now be completed.

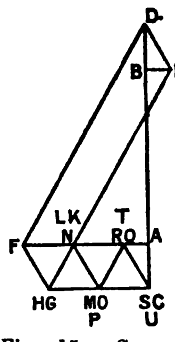


Fig. 15. — Stress Diagram with Brace G H Removed.

* Since the two members G H and K L, Fig. 14*a*, are parts of the same lattice bar or brace and the stresses in them are the same in every element, we may refer to them as the lattice bar or brace G H or K L and similarly for the others.

Fig. 15 shows that the lattice bar E G is the only one having the vertical component of its stress equal to B C. Also that each of the bars B D, D E, G K, L M, O Q and R S has the vertical component of its stress equal to one unit of the load. Further, that the stresses are alternately push and pull and that the remaining lattice bars are not stressed.

The vertical component of the push in the lattice bar E G is balanced by the reaction A B, in combination with the stress in B D. Since the vertical component of the push in the brace E G must balance the load, its value is 5 units and the reaction A B has a value of 4 units; therefore, the stress in the bar B D must be a pull of 1 unit. This vertical component of the pull in B D is balanced by the vertical component of the push stress in D E and so on, until the vertical component of the push in T U is balanced by the reaction A C of one unit.

Since the stress is severe in only one lattice bar and a number of the bars are unstressed, the removal of the brace G H produces a very much less yielding frame than the removal of the brace E G.

When the frame is complete as shown in the Frame Diagram, Fig. 14*a*, the vertical components of the pushes in the lattice bars E G and G H must together carry the load. The amount which each carries will be inversely proportional to the yieldingness of the system of bars to which each is connected. The lattice bar E G will therefore carry more than the lattice bar G H.

In this case, we have made the following assumptions:—

(1) That the Frame is not initially stressed.

(2) That the vertical component of the push stress in the left-hand lattice bar meeting in a joint at which a load is applied is equal to that portion of the left-hand reaction which that load produces. Also, that a similar relation exists between the vertical component of the push stress in the right-hand lattice bar and the right-hand reaction.

Lattice Girder Loaded at Top Joints.—For the lattice girder, Fig. 16*a*, we must first determine the reactions. This may be done by one of the methods for Figs. 1 and 2, of which the latter one, is the better and simpler. By joining the lower left-hand corner with each of the upper joints we obtain a Simple Triangular Frame, and by drawing the Stress Diagram for it, we can determine the reactions. Since the frame is redundant we must first calculate the stress in one member before we can begin the Stress Diagram. The bar B K is the most suitable one.

From the second assumption we observe that:—

(1) The load B C, produces no stress in K L and a push stress of 3 units in B K.

(2) The load C D, produces a push stress having a vertical component equal to 4 units ($\frac{4}{5}$ C D) in K M. This component is entirely balanced by the reaction which the load C D produces in A B — viz., 4 units. This load produces therefore no stress in B K.

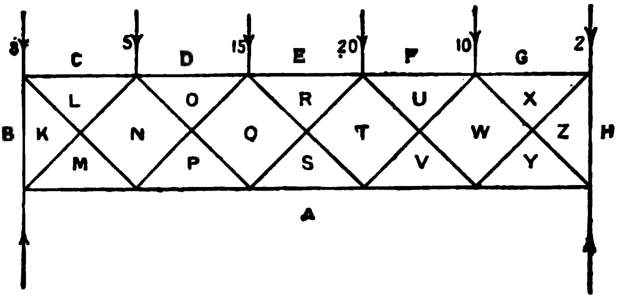


Fig. 16*a*.—Frame Diagram for a Loaded Lattice Girder.

(3) The load D E, produces a push stress having a vertical component equal to 9 units ($\frac{3}{5}$ D E) in O Q. This stress induces a pull stress in M N having a vertical component equal to 9 units and this pull stress induces in B K a push stress of 9 units.

(4) The load E F, produces a push stress having a vertical component equal to 8 units ($\frac{2}{5}$ E F) in R T. This stress induces a pull in P Q, and a push in N L, the vertical component of which is balanced by the reaction produced in A B. This load produces therefore no stress in B K.

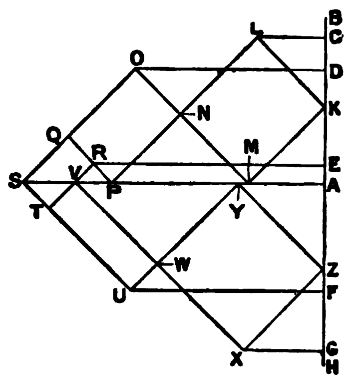


Fig. 16*b*.—Stress Diagram.

(5) The load F G, produces a push stress having a vertical component equal to 2 units ($\frac{1}{5}$ F G) in U W. This stress is transmitted as push and pull until it reaches B K as push and is balanced by the reaction A B.

(6) The load G H, produces no stress in B K.

Summing up the push stresses in B K we have :—

3 units due to B C,
0 units due to C D,
9 units due to D E,
0 units due to E F,
2 units due to F G,
0 units due to G H.

This gives a total push stress of 14 units in the bar B K.

From the point B, in the line of loads, Fig. 16*b*, draw a line B K parallel to the bar B K, and make the length of B K equal to 14 units to the same scale as the line of loads. This determines the point K in the Stress Diagram. Then K L and C L fix the point L and so on point by point until the Stress Diagram is finished. The finishing line forms a check line.

In calculating the stress in the lattice bars, we have sometimes a push stress and sometimes a pull stress. We must therefore pay due regard to the sign of the stress when adding up the various stresses. The force C D produces a push stress in the bar N O, having a vertical component of 1 unit, while E F produces a push stress in R T having a vertical component of 8 units. This push induces a pull in N O also having a vertical component of 8 units. Therefore, the resultant stress in N O is a pull having a vertical component of 7 units.

Referring to Fig. 16*a*, we may observe that sometimes, vertical ties are put in the spaces N, Q, T and W. The action of these ties is to prevent distortion of the rectangles in which they lie. Therefore unless these rectangles have become distorted, they will not be stressed. This distortion will depend upon the relative yieldingness of the two systems of bars forming the rectangle. In this example suppose a tie in the space N and a load applied at the lower joint. Then, before this load can stress the tie, the lower joint must come down more than the upper joint. The safest assumption to make is, that the ties carry no portion of the loads. Some lattice bridges have struts in the spaces N, Q, T and W of Fig. 16*a* and the lattice bars are all ties—that is, they are only able to stand a pull stress. On drawing the Stress Diagram for such a case, we must omit a bar if a push stress comes in it, and use the other tie.

One tie in each bay must carry the shear in that bay. This will enable us to calculate the pull in that tie. This is the best method of drawing the Stress Diagram for such cases.

Bending Moment.—DEFINITION.—**The Bending Moment at any point in a beam, is the algebraic sum of the moments with respect to that point, of all the external forces acting on the portion of the beam on either side of that point.**

In order to draw the Bending Moment Diagram of Fig. 17, we must proceed as if we were going to find the reactions, by means of the Funicular Polygon and Polar Diagram as explained for Fig. 1*a*.

The Funicular Polygon drawn in this way is a Bending Moment Diagram.—That is, if a vertical line be drawn from a point in the beam, to cut the bounding lines of the Funicular Polygon,

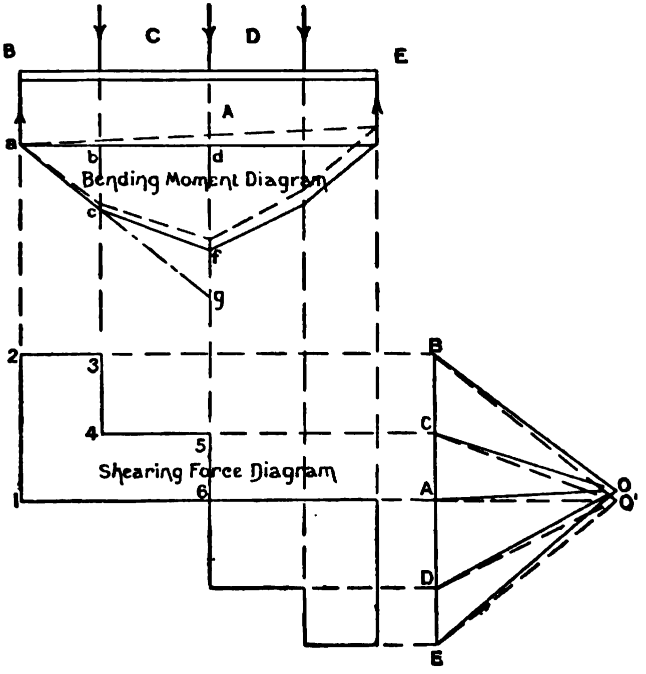


Fig. 17.—Bending Moment and Shearing Force on a Beam.

the intercept on this line which lies between those bounding lines, represents to a certain scale the bending moment on the beam at that point.

Having found the point A in the line of loads, by drawing A O′ horizontal and of any suitable length, we draw a Polar Diagram with this point O′ as the Pole and the corresponding Funicular Polygon, when we obtain a Bending Moment Diagram on a horizontal base.

Proof.—The Bending Moment at the point where the load

B C acts, is equal to A B × *a b* Units of Moment. The Triangles *a b c* and O′ A B are similar, having the sides *a b*, *b c* and *c a* of the one respectively parallel to the sides O′ A, A B and B O′ of the other.

Therefore, $bc : ab :: BA : AO'$

Hence, $$bc = \frac{BA \times ab}{AO'}$$

Similarly, $$df = \frac{BA \times ad - BC \times bd}{AO'}.$$

That is, the number of units of length in *b c*, when measured with the scale for the load line, would give the Bending Moment, if O′ A measured 1 unit on the scale of length for the Beam.

SCALE FOR BENDING MOMENT DIAGRAM.—**Subdivide the unit of the scale used for the line of loads, into as many parts as the line O′ A contains the unit of the scale used for the length of the beam. Then, one of these subdivisions will be the unit for the Bending Moment scale.** It is found convenient to make O′ A ten units of the length scale.

Shearing Force.—DEFINITION.—**The Shearing Force on any transverse section of a beam is equal to the algebraic sum of all the external forces acting on the portion of the beam on either side of that section.**

In order to draw the **Shearing Force Diagram** of Fig. 17, no explanation is necessary, beyond following out the lines of the figure. The Shearing Force on any transverse section of the beam lying between the loads B C and C D, is, from the definition, equal to the force A B minus the force B C. Therefore, the length between the line 4—5 and the line 1—6 will measure the Shearing Force to the scale of the line of loads.

Cantilever Uniformly Loaded.—The cantilever shown in Fig. 18 may be considered as 12 feet long. The loads indicated are therefore equivalent to a uniform load per foot run. They act at the centre of each of the portions. By drawing from A, B, C, &c., on the Load Line, horizontal lines in the spaces A, B, C, &c., as shown, we determine the Shearing Force Diagram. If we divide the beam into smaller divisions and draw the Shearing Force Diagram, the stepped line will become more nearly a straight line. Consequently, when divided into infinitely small parts, the Shearing Force Diagram becomes the Triangle R P Q. The length of the line Q R is the total load on the Beam.

The Bending Moment Diagram of Fig. 18 is determined by drawing in the spaces A, B, C, &c., lines parallel to the lines O A, O B, O C, &c. The limiting form of the curve Q S will be a parabola with its vertex at S, and the value of the length Q T will be the Maximum Bending Moment.

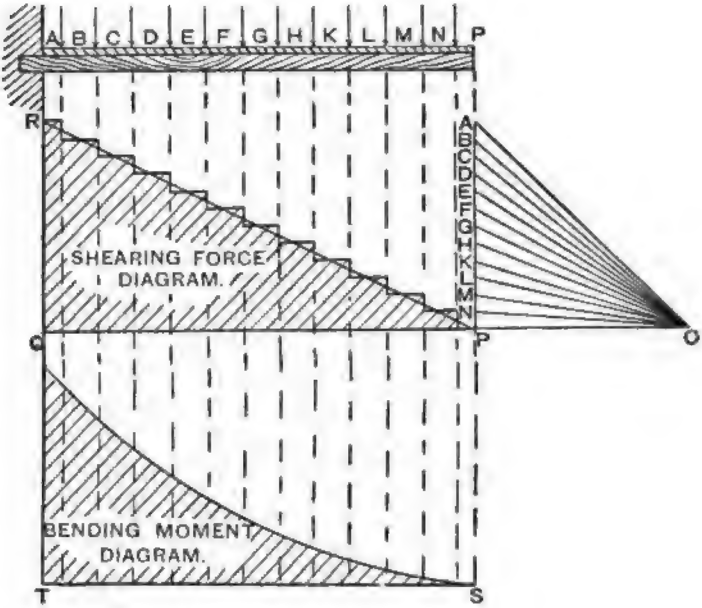


FIG. 18.—UNIFORMLY LOADED CANTILEVER.

Beam Uniformly Loaded and with Concentrated Loads.—Draw, as already explained, the Shearing Force Diagram for the concentrated loads. This is H A *1 2 3*, &c., on Fig. 19. Set off H P and K Q, each equal to half the total uniform load on the beam, and join P with Q. Then H P Q K is the Shearing Force Diagram for the Uniform Load. Adding the ordinates of the two diagrams together we derive the Combined Shearing Force Diagram H *a b c d e 5 6 f*, &c., of Fig. 19.

Draw the Bending Moment Diagram (L *n* M, Fig. 19) for the concentrated loads as described for Fig. 17. Then draw on the opposite side of L M a parabola, having its axis bisecting L M at right angles, and the ordinate at the centre of L M equal to the Maximum Bending Moment due to the uniform load. This

ordinate must be measured to the same scale as that of the ordinates of the concentrated Bending Moment Curve. The combined ordinate measures the Combined Bending Moment.

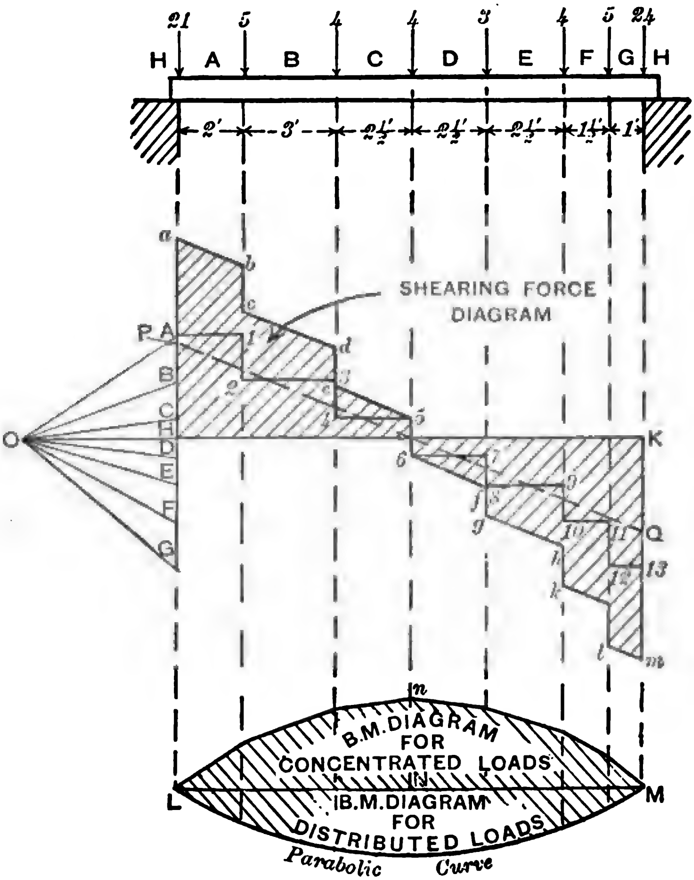
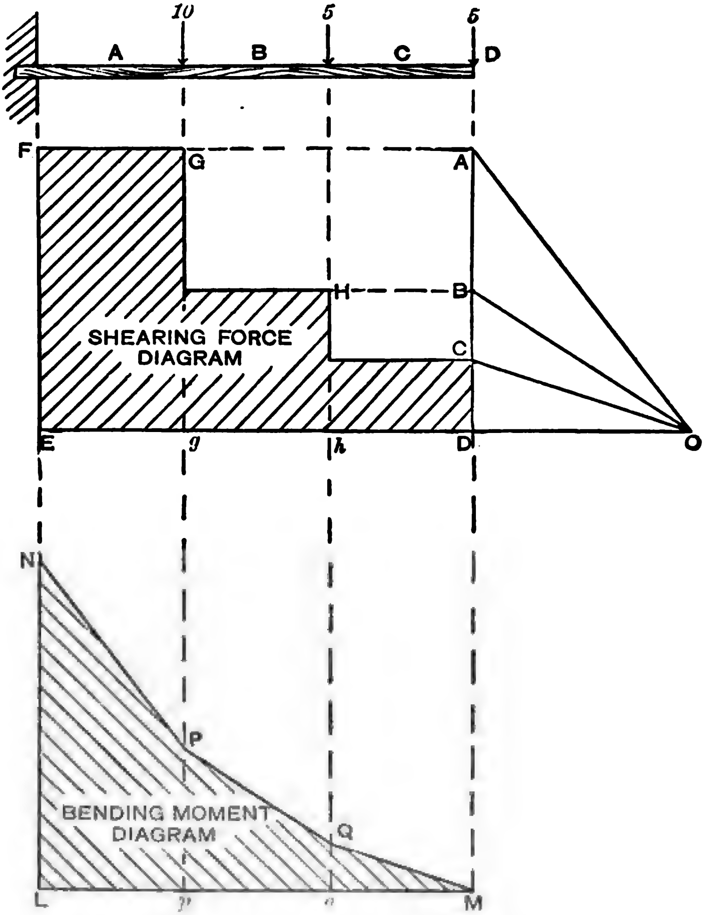


Fig. 19.—Beam with Uniform and Concentrated Loads.

Example II.—A cantilever 15 feet long has a load of 5 tons at its outer end, 5 tons at 5 feet from it, and 10 tons at a point 10 feet from the end. Find graphically the diagrams of shearing force and bending moment.

Answer.—The upper part of the figure shows the cantilever

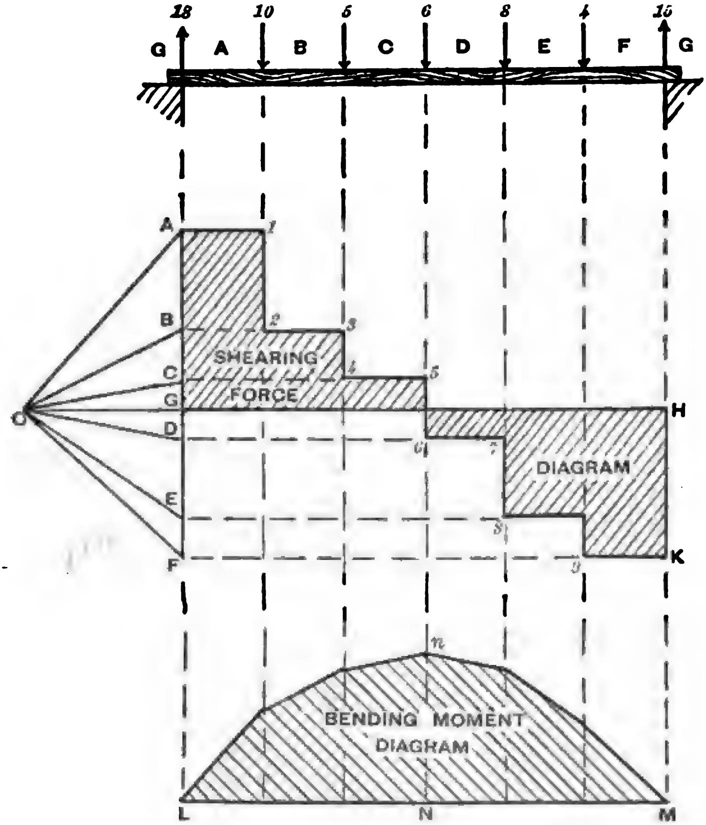
and the positions of the loads. Project down from these positions and the inner end of the beam, and then set out A B = 10 units, B C = 5, and C D = 5, to represent the forces A B, B C,



CANTILEVER IN EXAMPLE II.

and C D respectively. Draw horizontal lines through A, B, C, and D to intersect the lines of the forces. This gives us the Shearing Force Diagram as shown shaded.

To obtain the Bending Moment Diagram take any point O in E D, and join it to A, B, and C. Then take a base line M L parallel to O D, and draw M Q in the space C parallel to O C, Q P in the space B parallel to O B, and P N in the space A parallel to O A. Then L M Q P N is the Bending Moment



BEAM IN EXAMPLE III.

Diagram, the scale being that adopted for the shear multiplied by the length of O D measured on the scale employed in setting out the length of the beam.

EXAMPLE III.—A beam of 12 feet span carries five loads equally spaced along its length, the first and last being each 2

feet from the nearest end. The values of the loads are 10, 5, 6, 8, and 4 tons respectively. Obtain graphically diagrams showing the shear and bending moment at every point of the beam.

ANSWER.—In this case we shall determine the reactions by calculation, thus:—

Reaction at left hand	due to	A B	is $\frac{5}{6} \times 10 = 8\frac{1}{3}$	tons.
,,	,,	B C	,, $\frac{4}{6} \times 5 = 3\frac{1}{3}$	,,
,,	,,	C D	,, $\frac{3}{6} \times 6 = 3$	,,
,,	,,	D E	,, $\frac{2}{6} \times 8 = 2\frac{2}{3}$	,,
,,	,,	E F	,, $\frac{1}{6} \times 4 = 0\frac{2}{3}$	,,

$\therefore$ Total left-hand reaction $G A = 18$,,

The whole load is 33 tons, and therefore the right-hand reaction must be $33 - 18$, or 15 tons.

We can now proceed as before, making $G A = 18$, $A B = 10$, $B C = 5$, &c., and drawing horizontal lines through A, B, C, &c., to obtain the Shearing Force Diagram.

Take a point O in the horizontal through G, and join it to A, B, C, &c.

Then the part of the Bending Moment Diagram in the space A is parallel to O A, in the space B to O B, in C to O C, and so on, as in Example II.

We might, of course, have determined the reactions from the Funicular Polygon L *n* M N in the first instance; but had we done so we would probably not have got the line O G horizontal, and would have had to redraw it as explained in the text.

Centre of Gravity of an Area.—Divide the area into elements, such as parallelograms, triangles, &c., the centres of gravity of which can be easily determined.

If the area is bounded by a curved line, divide it into very narrow strips, so that they may be considered approximately as parallelograms.

We have divided the area shown in Fig. 20 into three rectangles, and have found the centre of gravity of each. We first, assume a line lying in any direction, such as the line X X, along which the pull of gravity acts. The centre of gravity of each area is a point in the line of action of the pull of gravity on that area. The line of action of gravity will be parallel to this assumed line X X. The way may be towards either the left or the right as may be found most suitable, and the magnitude will be proportional to the area. The forces B C, C D, and D E represent completely the action of gravity on the top, the centre, and the bottom rectangles respectively.

Proceed to find the resultant of the three forces B C, C D,

and D E, as explained for Fig. 1*a*. This is shown in Fig. 19 by the Polar Diagram B C D E O, and the corresponding funicular polygon 1 2 3 4. B C, C D, and D E, in the Polar Diagram are proportional to the areas of the three rectangles.

The line of action of the resultant of the three forces B C, C D, and D E passes through the centre of gravity of the whole area. This line is represented by the line 4—M. Now, assume a line at right angles to X X as a line along which the pull of gravity acts Proceed in exactly the same way with regard to this line as has been done for the line X X, and we obtain another line passing through the centre of gravity of the

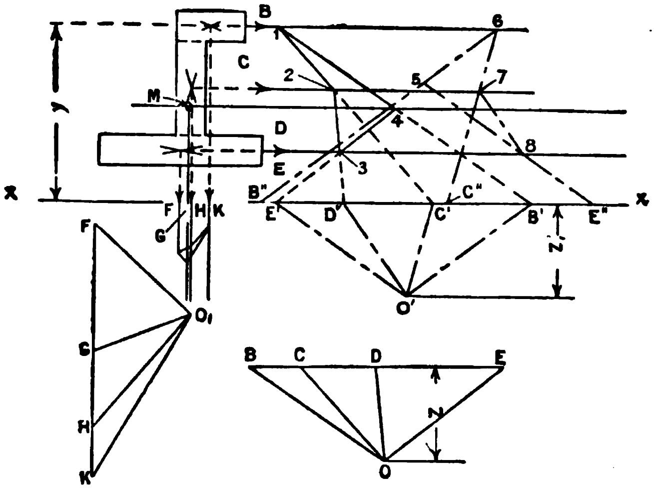


Fig. 20.—Centre of Gravity and Moment of Inertia.

whole area. The intersection M of these two resultants gives the centre of gravity of the whole area. The forces in the second case are called F G, G H, and H K, and the Polar Diagram F G H K O_1, with its corresponding Funicular Polygon, is shown in the figure.

Moment of Inertia of an Area.—If we wish to find approximately the Moment of Inertia round the line X X of the area in Fig. 20, we must first of all divide the area into elements just as in find ng the centre of gravity. Then proceed to draw the polar diagram B C D E O and the corresponding funicular polygon 1 2 3 4

Now consider the top polygon and how we may determine its Moment of Inertia.

Produce the two lines derived from the polar diagram, which meet in the line of action of the pull of gravity on that area (viz., 1—2 and 1—4) until they intersect the line X X in the points B′, C′. Do the same for the lines 1—2 and 2—3, and 2—3 and 4—3, which meet on the lines of action of the pull of gr.vity on the middle and bottom areas. These lines intersect X X in the points C′. D′, a d D′, E′, respectively.

Consider B′ C′, C′ D′, nd D′ E′ as the magnitudes of the forces acting along the lines B C, C D, and D E respectively. Proceed as if to find their resultant by drawing the polar diagram B′ C′ D′ E′ O′ and the corresponding funicular polygon 5 6 7 8.

Produce, as before, the lines which meet in B C (viz., 6—5 and 6—7) to intersect the lines X X in the points B″ C″. Do the same for the lines 5—8 and 7—8, or, as we have done in the figure, produce the one which will cut X X in a point furthest from B″. B″ C″ measures to a certain scale the moment of inertia of the top area round the line X X, and B″ E″ the moment of inertia of the whole area round the same line. Greater accuracy would be obtained by dividing the area into smaller elements.

Proof.—Let y in Fig. 20 represent the distance the centre of gravity of the top area is from the line X X.

Now, since the two triangles B C O and B′ C′ 1 are similar:—

$$BC : Z :: B'C' : y.$$

Then, $$B'C' = \frac{BC \times y}{Z}.$$

Again, the two triangles B′ C′ O′ and B″ C″ 6 are similar:—

Hence, $$B'C' : Z_1 :: B''C'' : y.$$

And, $$B''C'' = \frac{B'C' \times y}{Z_1}.$$

Substituting the above value of B′ C′ we get:—

$$B''C'' = \frac{BC \times y^2}{Z \times Z_1}.$$

But, $BC \times y^2$ is the moment of inertia for the top area with respect to the line X X, provided the depth of the area is small in comparison with y. B″ C″ measured with the scale called the "area scale," as used for drawing B C (in order to represent the area of the top rectangle), gives the value of this moment of

inertia, if Z and Z_1 are 1 unit of the scale which is used for setting off the lengths in drawing the section.

SCALE FOR MEASURING THE MOMENT OF INERTIA.—**Subdivide the unit of the scale used for representing the areas, into as many divisions as is represented by the number found by multiplying Z and Z_1, which are both measured by the length scale. One of these subdivisions will be the unit for the Moment of Inertia Scale. Or, measure B″ E″ with the area scale and multiply the reading first by Z and then by Z_1.**

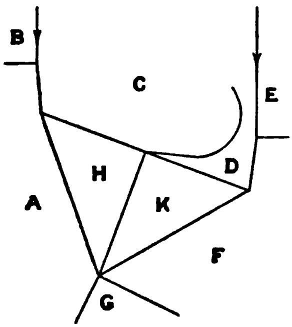


FIG. 21*a*.—FRAME DIAGRAM. ENGINE MECHANISM.

Engine Mechanism. — In the Frame Diagram, Fig. 21*a*, the bars B C and C E represent the centre lines of the piston-rods of a compound engine the heads of which are guided in parallel straight lines. The bars A C and D F are short

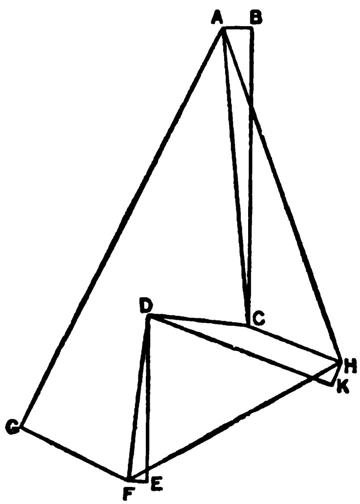
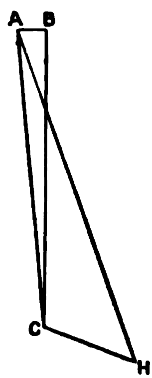


FIG. 21*b*.—STRESS DIAGRAM.



STRESS DIAGRAM FOR JOINT A C H.

connecting-rods, driving the crank F G, by means of the triangular frame shown. The joint C D K H is constrained to

move in an arc of a circle round a point in the bar C D produced towards the right. The bar C D is called a radius rod. The lines of action of all the external forces acting in the structure are shown. A B and E F are the guide pressures, C D the push or pull in the radius rod, F G the push or pull in the crank, and G A the crank effort or tangential resistance.

First Method.—We commence the Stress Diagram, Fig. 21*b*, by drawing D E to represent in magnitude the total pressure on the right-hand piston-rod. Then, E F and D F fix the point F, while F K and D K determine K.* But, we can get no further until we draw the Stress Diagram for the joint A C H. This is done by drawing B C to represent to the same scale as before the total pressure on the left hand piston-rod. Then, the points A and H are determined.

We must now fit the Stress Diagram for the joint A C H, to the Stress Diagram already drawn; so that the point C shall lie on the line drawn through D parallel to the bar C D and the point H on the line drawn through K parallel to the bar K H, C H being kept parallel to the bar C H. Then the Stress Diagram, Fig. 21*b*, can be completed in the usual way.

Second Method.—Find the forces acting in A C and D F, and then find their resultant. Produce the line of action of this resultant to cut the line of action of the force C D; when, by joining this point with the crank pin, we get the line of action of the resultant force acting on the said crank pin. Finally, draw the Stress Diagram from the supplementary data.

The following is a list of books and papers on Graphic Statics and the Design of Structures:—

The Design of Structures, Bridges, Roofs, &c., by S. Anglin, C.E. (Chas. Griffin & Co., London, 1895.)

A Practical Treatise on Bridge Construction, by Prof. T. Claxton Fidler. (Chas. Griffin & Co., London.)

Graphical Determination of Forces in Engineering Structures, by James B. Chalmers, C.E. (Macmillan & Co., London.)

Graphic and Analytic Statics, by Robert Hudson Graham, C.E. (Crosby Lockwood & Co., London.)

Graphics, by Prof. R. H. Smith, M.Inst.M.E. (Longmans, Green & Co., London.)

Mechanics, vol. ii., by A. Jay Du Bois, C.E., Ph.D. (Chapman & Hall, London.)

Applied Mechanics, by Gaetano Lanza. (Chapman & Hall, London.)

Theory of Structures and Strength of Materials, by Henry T. Bovey, M.A., D.C.L. (Chapman & Hall, London.)

Graphic Methods of Computing Stresses in Jointed Structures. Paper by C. O. Burge, Proc. Inst. C.E. Vol. lxxiv., p. 192.

Graphic Methods of Engine Design, by A. H. Barker. (The Technical Publishing Co., Ltd., Manchester.)

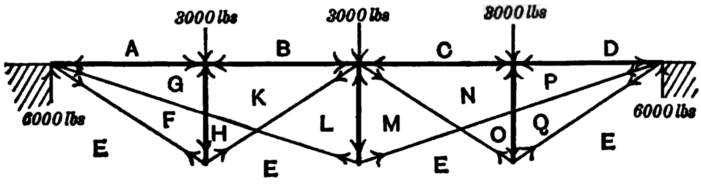
* The line F K has been omitted in the diagram.

Mechanical Graphics, by G. Halliday. (London, 1889.)
Elements of Graphic Statics, by K. von Ott, translated by G. S. Clark. (E. & F. N. Spon, London, 1888.)
Principles of Graphic Statics, by G. S. Clark. (E. & F. N. Spon, London, 1888.)
Elements of Graphic Statics, by L. M. Hoskins. (Macmillan & Co., London, 1892.)
Economics of Construction, by Robt. H. Bow. (E. & F. N. Spon, London.)
Applied Mechanics, 2nd edition, by Prof. James H. Cotterill, (Macmillan & Co., London.)

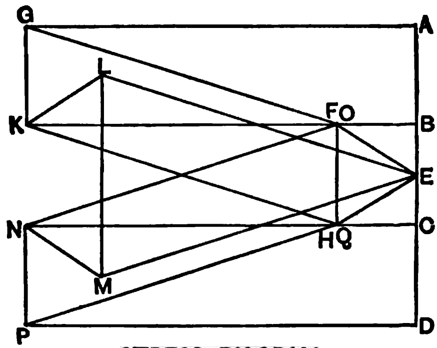
Re **Funicular Polygon** at pp. 232 and 256.—The word funicular, as used in mathematics, is an epithet for a curve, which is the same as the catenary; and also for a polygon hanging freely by its extremities. The Funicular Polygon or Link Polygon is really a Stress Diagram drawn from a Frame, such as that shown by Fig. 13 on page 164. When forces in equilibrium act at the corners of a series of links or bars jointed together at their extremities, the force acting along each link can be readily found by a special application of the triangle of forces, as in the above-mentioned case.

Lecture XXVIII.—Questions.

1. Draw the Stress Diagram for the Fink Truss shown below, and verify the stress diagram accompanying it.



FRAME DIAGRAM



STRESS DIAGRAM

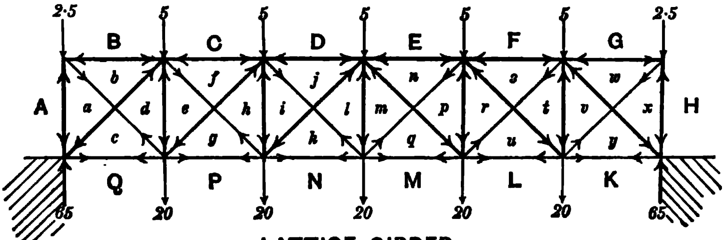
Fink Truss for Question 1.

2. A triangular frame is at rest under the action of three external forces. Prove that a certain diagram will represent the stresses in the bars of the frame. Extend this proposition to the case of a lattice girder of the Warren construction with four bays in the lower boom and three bays in the upper boom, loaded in the centre of the lower boom and supported at the ends, giving the Stress Diagram and showing how to distinguish the portions which are in compression or extension.

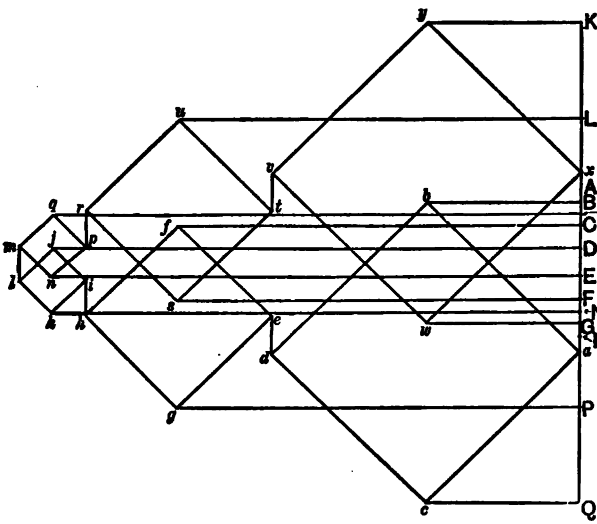
3. A triangular frame is acted on by three forces applied at its respective angular points and in equilibrium; investigate a method of constructing the diagram of all forces brought into play. Taking the case of a frame on the principle of the Warren Girder having four bays in the lower boom and three in the upper boom, and loaded at the centre of the lower member with a weight W, explain the method of constructing the diagram of forces,

drawing the same, and distinguishing those bars which act as struts from those which act as ties.

4. The lower boom of a Warren Girder, supported at both ends, is divided into three bays. The upper boom has two bays, and the bracing bars are each inclined at 60° to the horizon. Find by graphic construction the stresses in the several pieces when the frame is loaded with 1,000 lbs. at the middle of the top boom.



LATTICE GIRDER.



STRESS DIAGRAM.

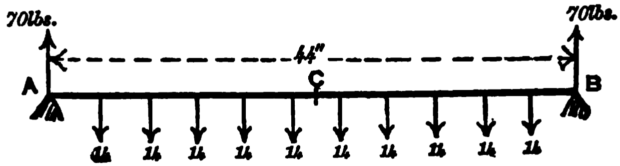
Lattice Girder for Question 7.

5. A Warren Girder has five bays consisting of equilateral triangles. If it be supported at each end and loaded at the two bottom central joints with loads of 18,000 lbs., find graphically the stress on each member, and show whether it is tensile or compressive. Explain fully the reasons and theory of the method you employ in obtaining your result.

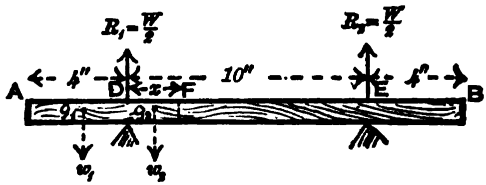
6. A Warren Girder of six bays with equilateral bracing, each bay being 10 feet long, is loaded with a distributed weight of 1 ton per foot run placed along the top of the girder; obtain the diagram of stress, and calculate the stresses in the various members, assuming that cotan. 60° = ·577, cosec. 60° = 1·155. *Ans.*

7. A lattice girder is loaded in the manner shown by the foregoing figure. Draw the Stress Diagram by the method explained in the text, and see if you get the same results as shown.

8. A bar of pine 44 inches long rests on props at its extremities, and just supports 10 weights, of 14 lbs. each, hung at equal intervals of 4 inches along the rod. Find graphically the B M at the centre of the bar and the amount of a single weight, which, if hung at the centre of the bar, would stress it to the same extent (see figure). *Ans.* 43·27 lbs.



BEAM FOR QUESTION 8.



BEAM FOR QUESTION 9.

9. A horizontal uniform bar, 18 inches long, is laid over two supports, each 4 inches from its ends, as shown in the figure. Find graphically two points at which the bending moment is zero, the bar being loaded by its own weight (see figure). *Ans.* 2 inches from inside of supports.

10. Given an iron arched rib, hinged at both ends, and a system of vertical loads, show how we find the stress at any point of any section. Prove the rule for stress at any point of a section when we know the resultant of all the forces acting on the structure on one side of the section.

11. A beam, ABCDE, has a vertical supporting force at A; at E there is a pin joint support. AB is 5 feet, BC is 2 feet, CD is 6 feet, DE is 4 feet. There are vertical loads of 2 tons at B and 3 tons at D, and at C there is a load of 5 tons inclined at 30° to the vertical, its horizontal component being towards A. All forces in one plane. Find the supporting forces, graphically or otherwise. (S. & A. Adv. Exam., 1897.)

12. Suppose the vertical loads and supporting forces of a horizontal beam to be known, show how we find (1) the total shearing force at any section, (2) the position of the neutral line, (3) the tensile or compressive stress at any place. Prove your statements. (S. and A. Adv., 1899.)

13. In calculating by graphical methods the tensile or compressive forces in the several members of braced girders, why is it usual to

represent the weight of, or upon, the several bars, by equivalent forces acting at the joints, and to assume that the joints are frictionless pin joints or hinges? A Warren girder has 6 equal bays on the bottom flange, the girder is 90 feet span and 13 feet deep. It is loaded with a weight of 20 tons resting on the apex of the top flange, which is 37 feet 6 inches from the left abutment. Determine either graphically or analytically the forces in the diagonals and flanges of the girder produced by this load.

(S. and A. Adv., 1899.)

14. A railway bridge over a road is 40 feet span. An engine with its tender stands upon the bridge. The weights on the leading, the driving, and the trailing axles of the engine are 9, 15, and 7 tons respectively, while the load on the three axles of the tender is 7 tons on each. The engine stands so that the leading axle is $2\frac{1}{2}$ feet from the end of the bridge, and the distance between the centres of the engine axles is 8 feet, and between the tender axles is 4 feet 6 inches, while between the trailing axle of the engine and the leading axle of the tender is 8 feet. Draw the bending moment and shearing force diagrams for the above position of the engine, and write down the maximum value of the bending moment and also the value and position of the maximum shearing force.

(S. and A. Adv., 1899.)

Lecture XXVIII.—A.M.Inst.C.E. Exam. Questions.

1. A framed girder, 8 feet deep, supported at the ends, with double triangulation, has the bracing bars inclined at 45°. The girder has seven bays, each 12 feet long, and carries at each joint of the bottom boom a load of 3 tons and at each top joint a load of 6 tons. Find the total stress in each of the inclined bracing bars and in the top boom in the second bay from the left end. (I.C.E., *Oct.*, 1897.)

2. A plate-girder is required to carry a fixed load of 1 ton per foot run and a rolling load of $1\frac{1}{2}$ ton per foot run. Clear span 75 feet. Draw the diagrams of maximum bending moment and maximum shearing force. Curves may be sketched in if three or four points are determined. (I.C.E., *Oct.*, 1897.)

3. Design a centre section for the girder in the previous question, taking the depth of girder at 8 feet, and the working stresses at 4 tons per square inch of gross section in compression, and 5 tons per square inch of net section in tension. Assume thickness of plates and size of rivets as you think best. (I.C.E., *Oct.*, 1897.)

4. The platform of a bridge 160 feet span is carried by cross-girders resting on the lower joints of a pair of Warren girders (the members inclined at 60°). Assuming eight bays, find the stress on each member of the girders due to a uniform load of 4 tons per foot run covering the platform. (I.C.E., *Oct.*, 1899.)

5. If a uniformly distributed travelling load of 4 tons per foot run traverse the bridge of the preceding question, describe how the stress due to it on the centre diagonals of the Warren girders varies during the passage, and find the magnitude of that stress when the load extends from one end to the centre. Compare it with that due to the fixed load. (I.C.E., *Oct.*, 1899.)

6. A cantilever girder bridge consists of a central girder span $2a$, the extremities of which are jointed to the ends of cantilevers of length c projecting from the piers. The bridge is uniformly loaded throughout the span; find the straining actions at any point, and explain how they are reduced by the cantilever construction as compared with a simple girder bridge of the same total span. (I.C.E., *Oct.*, 1899.)

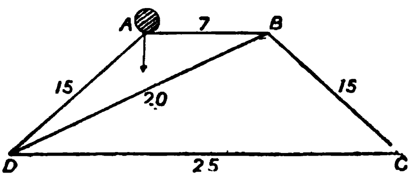
7. A beam is strengthened by the addition of tension rods below attached to the lower end of a vertical strut supporting the centre, the whole forming a simple triangular truss of given span and depth. Find the stress on each member of the truss when loaded with a weight in the centre. (I.C.E., *Oct.*, 1899.)

8. Find the centre of gravity of the area in one corner of an equilateral triangle cut off by its inscribed circle. (I.C.E., *Feb.*, 1900.)

9. A beam is strengthened by the addition of tension rods below attached to the lower ends of two vertical struts dividing the span into three equal parts, the whole forming a simple trapezoidal truss. Find the stress on each member when the beam carries a uniformly distributed load, and describe the effect of a load which is not uniformly distributed. (I.C.E., *Feb.*, 1900.)

10. Explain how you would find experimentally, and also by calculations the centre of gravity of a uniform board of any given shape. A ship, with its equipment, weighs 6,000 tons. How far will its centre of gravity move if a gun weighing 30 tons is moved a distance 20 feet across the deck. (I.C.E., *Feb.*, 1901.)

11. A light framework, consisting of four jointed rods in the form of a trapezoid A B C D, and a fifth rod connecting D and B, and having A B parallel to C D, is supported at D and C with D C horizontal. It is now loaded with 100 lbs. at A; find the pressures on the supports at D and C and the stresses in the various rods. The dimensions are A B = 7, B C = A D = 15, D B = 20, D C = 25. (I.C.E., *Feb.*, 1901.)



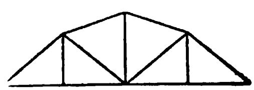
12. A beam, A B C D E, has a vertical supporting force at A. At E there is a pin-joint support. A B is 5′, B C is 2′, C D is 6′, D E is 4′. There are vertical loads of two tons at B, and 3 tons at D, and at C there is a load of 5 tons inclined at 30° to the vertical, its horizontal component being towards A. The beam is symmetrical in section and weighs ½ ton. All the forces are in one plane. Find graphically the supporting force at A and the reaction of the pin-joint support at E. (I.C.E., *Oct.*, 1901.)

13. The cross section of a cast-iron girder has the following dimensions: total depth 12 inches, top flange 3 inches by 1 inch, bottom flange 9 inches by 2 inches, thickness of web 1¼ inch. Assuming the web to be of uniform thickness throughout, and all the corners and edges to be square, find *graphically* the position of the "centre of area," or "centre of gravity" of the section. (Draw the section half-full size.) (I.C.E., *Feb.*, 1902.)

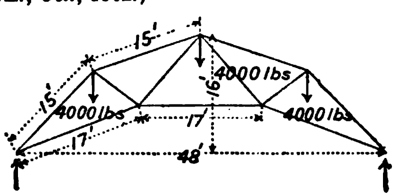
14. A continuous girder consists of two spans. One span of 100 feet is loaded with 1¾ tons per foot run, the second span of 80 feet is loaded with 2½ tons per foot run. Find the values of the supporting forces, and the maximum bending moment for the whole girder. Both ends of the girder are free. (I.C.E., *Feb.*, 1902.)

15. Find, by graphic construction, the centre of gravity of a section of an I beam, top flange 4 inches by 1 inch, web, between flanges, 14 inches by 1½ inch, bottom flange 9 inches by 2 inches. (I.C.E., *Oct.*, 1902.)

16. A parabolic girder has 4 bays, verticals from the intersection of the straight members of the upper chord support the cross-girders: the upper ends of the two verticals at quarter and three-quarters the length are connected by braces to the feet of the centre vertical, show that the length of these braces is a measure of the maximum stress they will have to sustain, on the same scale that the length of the lower chord represents the horizontal stress when the live load covers the span. (I.C.E., *Oct.*, 1902.)



17. A roof, as in sketch, is loaded as shown; find the stresses in its members. (I.C.E., *Oct.*, 1902.)



18. A girder 100 feet long is supported 20 feet from each end, and carries a uniform load of 2 tons per foot run; draw the diagrams of bending moment and shearing force. (I.C.E., *Oct.*, 1902.)

19. A Warren girder, length 100 feet, is divided into five bays on the lower flange, the length of the inclined braces being 20 feet; if loads of 30 tons are carried by the girder at the joints 20 feet and 40 feet from one end, find the stresses in the members. (I.C.E., *Feb.*, 1903.)

20. If a uniform live load moves over the bridge in the previous question, in what positions of it would maximum stresses in the various members be produced? (I.C.E., *Feb.*, 1903.)

21. A girder 100 feet long is supported at each end and in the middle, and carries a uniform load of 2 tons per foot run. Draw the bending moment and shearing force diagrams, and find the pressure on each support. (I.C.E., *Feb.*, 1903.)

PART V.—STRENGTH OF MATERIALS.—STRESS, STRAIN, ELASTICITY, FACTORS OF SAFETY, RESILIENCE, CYLINDERS, CHAINS, SHAFTS, BEAMS, AND GIRDERS.

LECTURE XXIX.

CONTENTS.—Stress—Definition of Intensity of Stress—Relation between Normal and Tangential Stresses—Strain—Example I.—Coefficient or Young's Modulus of Elasticity—Limit of Elasticity—Work done in Stretching a Bar—Resilience—Example II.—Sudden Pull or Live Load—Shrunk Rings—Example III.—Strength of Thin Cylinders—Helical Seams—Strength of Thick Cylinders—Example IV.—Strength of Suspended Chains and Wires—Example V.—Questions.

Stress.—When a piece of material is subjected to the action of external forces they tend to cause the material to change its shape or form. The particular way in which the change takes place depends upon the manner in which the load is applied. This tendency gives rise to certain forces within the material which offer resistance to the change. These internal forces are generally called *stresses;* but the term *Stress* which we have now to consider has a somewhat more definite meaning. By the principle of the equality of action and reaction, we know that so long as no rupture of the material takes place, the algebraic sum of the components of the internal forces in the direction of the load at any section of the material must be equal to the load. This principle enables us to express the *internal* in terms of the *external* forces. It is a fundamental fact that, for a given load, the amount of resistance to be contributed by each individual fibre or part composing a section will be less or greater, according as the number of such fibres or parts is greater or less; or as we usually regard it, according as there is more or less area of section. This introduces us to the conception of *distributed* force, and paves the way towards gaining definite and clear ideas regarding the strength of materials.

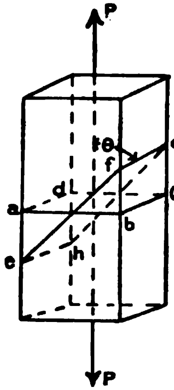
DEFINITION.—**Intensity of stress is the resistance or reaction due to a load per unit area of section.** For brevity it is usually called the Stress. Stresses may be of three different kinds, depending on the direction of the applied force with reference to the section on which the stress is estimated.

(1) If the applied force is normal or at right angles to the section, and acting *away* from it, the stress is called *tensile.*

(2) If acting *towards* the section, the stress is termed *compressive.*
(3) If the direction of the applied force be *parallel* to the section, then the stress is named a *shearing* stress.

It is evident that if the applied force be acting in a direction inclined to the given section, it will cause both a shearing and a direct stress, the latter being tensile or compressive, according as the force is directed away from or towards the section.

When the applied force acts in such a way that we know that its effect is uniformily distributed over the section we are considering, then we estimate the stresses as follows:—



ILLUSTRATING NORMAL AND TANGENTIAL STRESSES.

Let P_n = The applied load (or its component) acting normally to the section in lbs or tons.
„ A = The area of the section (usually in square inches).
„ f = The direct stress, which may be either tensile or compressive.
„ P_t = The applied load (or its component) acting tangentially to the section in lbs. or tons.
„ f_s = The shearing stress.

$$\left.\begin{aligned}\text{Then,}\qquad f &= \frac{P_n}{A}.\\ \text{And,}\qquad f_s &= \frac{P_t}{A}.\end{aligned}\right\} \quad \ldots\ldots\ldots \quad \text{(I)}$$

Relation between Normal and Tangential Stresses.—Let *a b c d* be the section of a bar normal to the direction of the applied force P, and *e f g h* another section making an angle θ, with the direction of P; and let the area of *a b c d* be A square inches.

Thus, the stress on *a b c d* is:—

$$f = \frac{P}{A};$$

But, on the area *e f g h*, we have a normal force:—

$$P_n = P \sin \theta,$$

And a tangential force:—

$$P_t = P \cos \theta.$$

Now, the area *e f g h* $= \dfrac{\text{area } abcd}{\sin \theta} = \dfrac{A}{\sin \theta}$

If f_n and f_t be the normal and tangential stresses on the section $efgh$,

We have:— $$f_n = \frac{\frac{P_n}{A}}{\sin\theta} = \frac{P}{A} \cdot \sin^2\theta = f \cdot \sin^2\theta.$$

Similarly, we get:—$f_t = f \cdot \sin\theta \cdot \cos\theta$.

Strain.—When a piece of material, such as a bar of iron, is in tension or compression under the action of an applied force P, the bar will, in consequence, be lengthened or shortened by an amount depending on the extent to which it is *stressed.* The *ratio* which this *change* of length bears to the original length of the bar is called the *strain* due to P. Or in symbols,—

If, L = Original length of bar in inches.

And, l = Change of length of bar also in inches.

We have:— $$\text{Strain} = \frac{l}{L} \quad \ldots \ldots \quad \text{(II)}$$

Since L and l are both actual lengths, measured by some common unit, the student should carefully note that strain, as thus defined, is merely an *abstract ratio*, and *not* a quantity, for it is independent of the units employed.

Example I.—A tie-rod, 100 ft. long, is stretched $\frac{3}{4}$ of an inch by the action of a certain force; what is the strain?

Here, $L = 100 \times 12 = 1{,}200$ inches,

And, $l = 0{\cdot}75$ inch.

$$\therefore \quad \textbf{Strain} = \frac{0{\cdot}75}{1{,}200} = \mathbf{0{\cdot}000625.}$$

Coefficient, or Modulus of Elasticity.—Experiment has demonstrated that for most materials used in engineering there is a very simple law connecting stress and strain, which is fairly well defined within certain limits. The stress is proportional to the strain, so long as the stress does not exceed a certain value, which, of course, is different for different materials and for different qualities of the same material. For example, if the stress be doubled, the strain will be doubled, or if the stress be reduced to one-half, the strain will also be halved, and so on. The limit beyond which this law does not hold is termed the **Limit of Elasticity.** When this limit is exceeded, the strain increases at a much greater rate than the stress producing it. Within the limit of elasticity, the material returns to its original state when the load is removed; but when stressed

beyond this, the material does not do so, but retains a *permanent set*. In the following investigations the stress, in all cases, is assumed to be within the elastic limit:—

Consequently, $$\frac{\text{Stress}}{\text{Strain}} = \text{E (a constant)}. \quad . \quad . \quad . \quad . \quad \text{(III)}$$

This E is termed Young's **Modulus of Elasticity**, or more appropriately by some writers the **Coefficient of Elasticity.**

Another way of exhibiting the relation subsisting among the various quantities we have been discussing, is to combine equations (I), (II), and (III) in such a way, as to express the stress and strain in terms of loads and dimensions.

Thus, $$\text{E} = \frac{\text{P}}{\text{A}} \div \frac{l}{\text{L}};$$

Or, $$\text{P L} = \text{A}\, l\, \text{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IV)}$$

Ultimate Strength.—The ultimate strength of a material is the resultant stress which produces rupture. It is usually reckoned as so many lbs. or tons per square inch of the section of the material. It is always understood that the section taken is the original section of the bar, and not that at the instant and point of fracture.

Working Load.—The working stress on a member of any structure is the maximum stress to which it is subjected in actual practice.

Factors of Safety.—The ratio of the *ultimate strength* or limiting stress to the safe working load, is termed the *factor of safety* of the material. To determine the proper value of the "factor of safety," a number of considerations must be taken into account. A great deal depends on the quality of the materials and upon the manner in which different structures are stressed, such as the action and frequency of the working load.

1st. The load may be a constant *dead load*—*i.e.*, one which is steady and produces no appreciable vibration.

2nd. The load may be a *live load*, such as a regiment of walking soldiers, or a rolling load, in the case of a moving railway train, passing over a bridge.

3rd. Where the quality of materials is variable or liable to change, the factor of safety must be larger, than for more uniform materials and for those which are less affected by exposure to atmospheric and other deteriorating influences.

4th. In structures where the whole load cannot be ascertained with accuracy, the factor of safety must be increased, to allow for the unknown stressing actions.

5th. In some structures, there is a liability to a sudden increase in the working load. Thus, in the case of a crane, where the weight may be allowed to descend rapidly and then be suddenly stopped, the maximum stress may be very much greater, than that due to the statical weight.

It is, therefore, necessary, that these special stresses be duly allowed for in the "factor of safety."

TABLE OF ULTIMATE STRENGTH AND WORKING STRESS OF MATERIALS WHEN IN TENSION, COMPRESSION, AND SHEARING.

Materials.	Ultimate Strength. Tons per sq. inch.			Working Stress. Tons per sq. inch.		
	Tension.	Compression.	Shearing.	Tension.	Compression.	Shearing.
Cast iron, ...	7·5	45	14	1·5	9	3
Wrought-iron bars, .	25	20	20	5	3·5	4
Steel bars, . . .	45	70	30	9	9	5
Copper bolts, . .	15	25	...	3	5	...
Brass sheet, . .	14	...	...	3	...	...

TABLE OF FACTORS OF SAFETY.*

Material.	FACTORS OF SAFETY FOR			
	A Dead Load.	A Live or Varying Load producing Stress of one kind only.	A Live or Varying Load producing Equal Alternate Stresses of different kinds.	In Structures subject to Varying Loads and Shocks.
Cast iron, and brittle metal and alloys, ...	4	6	10	15
Wrought iron and mild steel,.	3	5	8	12
Cast steel, ...	3	5	8	15
Copper and other soft metals and alloys, ...	5	6	9	15
Timber, ...	7	10	15	20
Masonry and brickwork, .	20	30	...	...

* These numbers were taken from Prof. Unwin's *Elements of Machine Design, &c.* They are all larger and safer than those given in Prof. Rankine's *Rules and Tables.*—A.J.

To find the Factor of Safety for a Mixed Load.—"Given the proportions of *live* and *dead* load on a structure, to find the factor of safety for a mixed load; multiply the factor of safety for a dead load, by a number proportional to the dead part of the load, and the factor of safety for a live load by the number proportional to the live part of the load; add together the products, and divide by the sum of the multipliers."—(*Rankine.*)

EXAMPLE.—In an iron bridge, suppose

$$\text{Dead load} : \text{live load} :: 5 : 4.$$

Then, from the above table, we get $(3 \times 5) + (5 \times 4) = 35$; and $35 \div (5 + 4) = 4$, as the factor of safety for mixed loads.

Work done in Stretching a Bar.—Resilience.—If a load of *gradually* increasing amount be applied to a bar so as to stretch it, the amount of actual stretch, or elongation of the bar will, with the limitations already specified, be directly proportional to the load producing it. A diagram might, therefore, be drawn to represent graphically the work done in stretching the bar, as explained in Lecture II. of Volume I. The area of the diagram would represent the work done. The load will increase uniformly from 0 to P. The mean value of the force doing the work is, therefore, $\frac{1}{2}$ P, and the stretch or displacement is l. Hence, we have for the work done:—

$$W = \tfrac{1}{2} P\, l.$$

But from equations (I) and (III)—

$$P = f A, \qquad \text{and } l = \frac{f L}{E}.$$

$$\left.\begin{aligned} &\text{Hence,} \quad W = \frac{f^2}{E} \times \frac{A L}{2} \\ &\text{Or,} \quad W = \frac{f^2}{E} \times \tfrac{1}{2} \text{ volume of the bar.} \end{aligned}\right\} \quad . \quad . \quad . \quad \text{(V)}$$

The work done is therefore proportional to the volume of the bar, or to its weight.

When the bar is loaded to its elastic limit, or *proof stress*, as it is sometimes called, then the *work done* in stretching it is termed the **Resilience** of the bar, and the ratio $\frac{f^2}{E}$ is its **Modulus or Coefficient of Resilience.**

Example II.—What is the resilience of a material? If a wrought-iron tie bar, 5 feet long and 3 inches in diameter, has a limit of elasticity of 15 tons per square inch, and a modulus of elasticity of 30,000,000 lbs. per square inch, what is its resilience? (Take $\pi = \frac{22}{7}$.) (Adv. S. & A. Exam. 1893).

ANSWER.—$f = 15 \times 2240$ lbs., $E = 30,000,000$ lbs. per square inch, $A = \frac{1}{4} \times \frac{22}{7} \times 3^2$ square inches, and $L = 5$ feet.

$$\therefore \textbf{Resilience} = \frac{(15 \times 2240)^2}{30,000,000} \times \frac{\frac{11}{14} \times 3^2 \times 5}{2} = \textbf{665·28 ft.-lbs.}$$

Sudden Pull, or Live Load.—We have just seen that a *constant* force of $\frac{1}{2}$P lbs. acting through a distance of l feet will do the same amount of work in stretching a bar as would a load gradually increasing from zero to P lbs.; therefore, the strain produced by a *sudden pull* of $\frac{1}{2}$P lbs. is the same as that due to P lbs. applied gradually. It follows, therefore, that if P be applied *suddenly*, but without initial velocity, the strain will be doubled, and the work done will be:—

$$W = P \times 2\,l = 2\,P\,l \text{ ft.-lbs.}$$

Or, in words, the work done on the bar by a suddenly applied or *live load* P, is *four* times that done by a gradually applied or *dead load* of the same amount.

Shrunk Rings.—In the construction of built-up guns, the process consists in shrinking on a series of concentric rings, each ring griping the next inner one with a certain pre-determined tension.

The reason for this arrangement will be better understood when we come to deal with the strength of thick cylinders. The principles set forth in the preceding sections enable us to calculate the dimensions of rings to give a certain grip.

Let D = The external diameter of an inner ring.
,, d = The internal diameter of the next outer ring.
,, f = The required tension.

When the outer ring is shrunk on, its diameter is then D. The inner fibres of this ring are then stretched by an amount $\pi\,(D - d)$; and by definition, we have:—

$$\text{Strain} = \frac{\pi\,(D - d)}{\pi\,d} = \frac{D - d}{d}.$$

If E denote the modulus of elasticity of the material of the ring, then :—

$$E = \frac{\text{stress}}{\text{strain}} = \frac{f}{\frac{D - d}{d}}$$

Hence,
$$d = D\left(\frac{E}{E + f}\right).$$

EXAMPLE III.—The external diameter of an inner ring is 20 inches. Work out the diameter which the outer ring must have in order to grip the inner one with an initial tension of 8 tons per sq. inch. Take the modulus of elasticity as 30,000,000.

ANSWER.—Here $D = 20$ inches, and $f = 8 \times 2240 = 17{,}920$ lbs. per sq. inch.

$$\therefore \qquad d = 20 \times \frac{30{,}000{,}000}{30{,}017{,}920} = 19{\cdot}98 \text{ inches.}$$

Strength of Thin Cylinders.—By *thin* cylinders are meant cylindrical vessels whose thickness is small compared with their diameter. The resistance which such vessels offer to forces tending to burst them, both longitudinally and circumferentially, is easily deduced as follows:—Consider a cylindrical ring, whose breadth is b inches, thickness t inches, and internal diameter is d inches. Let p denote the intensity of the internal pressure, in lbs. per sq. inch, tending to burst the ring, and f the induced stress within the material of the ring, also in lbs. per sq. inch.

Then the magnitude of the total internal force tending to tear asunder the ring at the ends of a diameter is $p\,d\,b$ lbs. And the resistance which the ring offers to this bursting force is $2\,t\,b\,f$ lbs.

These being equal, we have :—

$$2\,t\,b\,f = p\,d\,b \quad \therefore \quad f = \frac{p\,d}{2\,t}. \qquad . \quad . \quad . \quad \text{(VI)}$$

This result shows that the stress, in a circumferential direction, is independent of the length of the cylinder.

Whatever be the form of the ends of the cylinder—whether they be flat or hemispherical—the total force tending to cause rupture circumferentially is $p\,\frac{\pi}{4}\,d^2$ lbs.; resisting this force, we have a ring of material whose total sectional area is $\pi\,d\,t$ sq. inches.

Let f_1 be the longitudinal stress due to the longitudinal bursting force; then the total resistance is $\pi\, d\, t\, f_1$ lbs.

And $$\pi\, d\, t\, f_1 = p\,\frac{\pi}{4}\,d^2$$

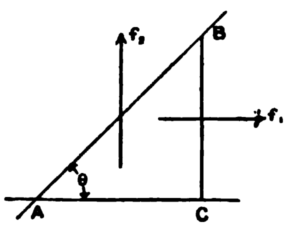
Hence, $$f_1 = \frac{p\,d}{4\,t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(VII)}$$

From this we see that:—

$$f_1 = \tfrac{1}{2} f.$$

So that in a cylindrical boiler, which comes within the category of thin cylinders, the stress in a longitudinal direction is only one-half of the stress circumferentially.

Helical Seams.—If we made a boiler of rings, joined together circumferentially, then, so long as the strength of those joints was greater than one-half that of the solid plate, the boiler would still be as strong as one without joints, because the solid plate longitudinally would still be weaker than the circumferential joints. When, instead of solid rings, these are made up of pieces joined together longitudinally, it is obvious that the strength of the boiler is determined entirely by that of its longitudinal joints, unless the circumferential joints are less than half as strong.



ILLUSTRATING STRESS ON HELICAL SEAMS.

As a compromise, it has been proposed to have, instead of circumferential and longitudinal joints, one continuous seam running spirally, called a helical joint.

Let the accompanying figure represent a portion of such a boiler flattened out. A B is the helical seam, which, when flattened out, becomes a straight line, making the angle θ with the longitudinal direction. The longitudinal and circumferential stresses are represented by f_1 and f_2 respectively. The intensities of those stresses on A B being denoted by f_1' and f_2', we have :—

$$f_1' \times AB = f_1 \times BC; \qquad \text{and } f_2' \times AB = f_2 \times AC.$$

$$\therefore \qquad f_1' = f_1 \sin\theta; \qquad \text{and } f_2' = f_2 \cos\theta.$$

Resolving f_1' and f_2' normally to A B, we have, for the total normal stress :—

$$f_n = f_1' \sin \theta + f_2' \cos \theta$$

$$\text{,,} = f_1 \sin^2 \theta + f_2 \cos^2 \theta.$$

But, $\quad f_1 = \frac{1}{2} f_2.$

$\therefore \quad f_n = \frac{1}{2} f_2 \sin^2 \theta + f_2 \cos^2 \theta.$

Or, $\quad \dfrac{f_n}{f_2} = 1 - \frac{1}{2} \sin^2 \theta.$

Let, $\quad n = \dfrac{\text{B C}}{\text{A C}};$

Then, $\quad \sin^2 \theta = \dfrac{\text{B C}^2}{\text{A B}^2} = \dfrac{\text{B C}^2}{\text{B C}^2 + \text{A C}^2} = \dfrac{n^2}{n^2+1}$

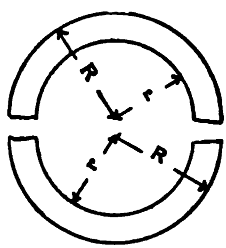
Hence, $\quad \dfrac{f_n}{f_2} = 1 - \frac{1}{2} \cdot \dfrac{n^2}{n^2+1} = \dfrac{n^2+2}{2n^2+2} \quad . \quad . \quad . \quad . \quad \text{(VIII)}$

For example, if $n = 1$, *i.e.*, $\theta = 45°$,

Then, $\quad \dfrac{f_n}{f_2} = \dfrac{3}{4}.$

That is to say, that the normal stress on a spirally-running joint, making an angle of 45° with the axis of the boiler, would be three-fourths of that on a longitudinal joint. With joints of equal efficiency, therefore, the helical seam would be 33·3 per cent. stronger than the longitudinal one.

Strength of Thick Cylinders.—When the thickness of a cylindrical vessel, subjected to internal pressure, is *not* small in comparison with its internal diameter, the problem requires to be treated differently.



ILLUSTRATING STRAIN IN THICK CYLINDERS.

A complete determination of the strength of thick cylinders of all proportions is not an easy matter; and as for an *accurate* solution of the problem, the thing is simply impossible.

For moderate proportions of cylinders, such as are used in hydraulic appliances, the following demonstration yields results fairly substantiated by practice.

If such a cylinder were to give way under internal pressure, the plane of rupture would evidently contain the axis of the cylinder; whilst the rupture itself would appear as shown in the accompanying figure. From this figure it is clear that the circumferential *stretch* is the same from the inner to the outer surface. Now, remembering the definition of strain previously given, it is obvious that in this case, the strain in any cylindrical

Example II.—What is the resilience of a material? If a wrought-iron tie bar, 5 feet long and 3 inches in diameter, has a limit of elasticity of 15 tons per square inch, and a modulus of elasticity of 30,000,000 lbs. per square inch, what is its resilience? (Take $\pi = \frac{22}{7}$.) (Adv. S. & A. Exam. 1893).

ANSWER.—$f = 15 \times 2240$ lbs., $E = 30{,}000{,}000$ lbs. per square inch, $A = \frac{1}{4} \times \frac{22}{7} \times 3^2$ square inches, and $L = 5$ feet.

$$\therefore \textbf{Resilience} = \frac{(15 \times 2240)^2}{30{,}000{,}000} \times \frac{\frac{11}{14} \times 3^2 \times 5}{2} = \textbf{665·28 ft.-lbs.}$$

Sudden Pull, or Live Load.—We have just seen that a *constant* force of $\frac{1}{2}$P lbs. acting through a distance of l feet will do the same amount of work in stretching a bar as would a load gradually increasing from zero to P lbs.; therefore, the strain produced by a *sudden pull* of $\frac{1}{2}$P lbs. is the same as that due to P lbs. applied gradually. It follows, therefore, that if P be applied *suddenly*, but without initial velocity, the strain will be doubled, and the work done will be :—

$$W = P \times 2\,l = 2\,P\,l \text{ ft.-lbs.}$$

Or, in words, the work done on the bar by a suddenly applied or *live load* P, is *four* times that done by a gradually applied or *dead load* of the same amount.

Shrunk Rings.—In the construction of built-up guns, the process consists in shrinking on a series of concentric rings, each ring griping the next inner one with a certain pre-determined tension.

The reason for this arrangement will be better understood when we come to deal with the strength of thick cylinders. The principles set forth in the preceding sections enable us to calculate the dimensions of rings to give a certain grip.

Let D = The external diameter of an inner ring.
,, d = The internal diameter of the next outer ring.
,, f = The required tension.

When the outer ring is shrunk on, its diameter is then D. The inner fibres of this ring are then stretched by an amount $\pi\,(D-d)$; and by definition, we have :—

$$\text{Strain} = \frac{\pi\,(D-d)}{\pi\,d} = \frac{D-d}{d}.$$

If E denote the modulus of elasticity of the material of the ring, then :—

$$E = \frac{\text{stress}}{\text{strain}} = \frac{f}{\frac{D - d}{d}}$$

Hence, $$d = D\left(\frac{E}{E + f}\right).$$

Example III.—The external diameter of an inner ring is 20 inches. Work out the diameter which the outer ring must have in order to grip the inner one with an initial tension of 8 tons per sq. inch. Take the modulus of elasticity as 30,000,000.

Answer.—Here D = 20 inches, and $f = 8 \times 2240 = 17{,}920$ lbs. per sq. inch.

$$\therefore \qquad d = 20 \times \frac{30{,}000{,}000}{30{,}017{,}920} = 19\cdot 98 \text{ inches.}$$

Strength of Thin Cylinders.—By *thin* cylinders are meant cylindrical vessels whose thickness is small compared with their diameter. The resistance which such vessels offer to forces tending to burst them, both longitudinally and circumferentially, is easily deduced as follows :—Consider a cylindrical ring, whose breadth is b inches, thickness t inches, and internal diameter is d inches. Let p denote the intensity of the internal pressure, in lbs. per sq. inch, tending to burst the ring, and f the induced stress within the material of the ring, also in lbs. per sq. inch.

Then the magnitude of the total internal force tending to tear asunder the ring at the ends of a diameter is $p\,d\,b$ lbs. And the resistance which the ring offers to this bursting force is $2\,t\,b\,f$ lbs.

These being equal, we have :—

$$2\,t\,b\,f = p\,d\,b \quad \therefore \quad f = \frac{p\,d}{2\,t}. \qquad . \quad . \quad . \quad \text{(VI)}$$

This result shows that the stress, in a circumferential direction, is independent of the length of the cylinder.

Whatever be the form of the ends of the cylinder—whether they be flat or hemispherical—the total force tending to cause rupture circumferentially is $p\,\frac{\pi}{4}\,d^2$ lbs. ; resisting this force, we have a ring of material whose total sectional area is $\pi\,d\,t$ sq. inches.

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$$\therefore \text{ Resilience} = \frac{(15 \times 2240)^2}{30{,}000{,}000} \times \frac{\frac{11}{14} \times 3^2 \times 5}{2} = 665{\cdot}28 \text{ ft.-lbs.}$$

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Weight of Chains.—The weight of chains in lbs. per foot may be expressed from the equation, $w = 9\,d^2$, where d = diameter of iron in inches.

The following tables are given by Professor Unwin, M. Inst. C. E., in his *Elements of Machine Design;* also, in *Design of Structures,* by Mr. S. Anglin, C.E., Wh.Sc. The breaking strengths were calculated from the Woolwich experiments.

Table of Strength and Weight of Close-Link Crane Chains, and Size of Equivalent Hemp Cable.

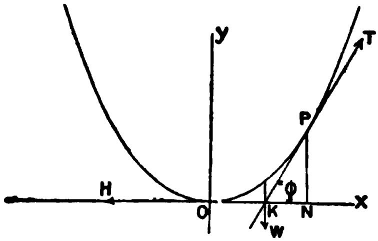
Diameter of iron d in inches.	Weight of chain per fathom.	Breaking strength in tons.	Testing load in tons.	Girth of equivalent rope in inches.	Weight of rope in lbs. per fathom.
$\frac{1}{4}$	3·5	1·9	0·75	2	$1\frac{3}{8}$
$\frac{5}{16}$	6·0	3·0	1·10	$2\frac{1}{2}$	$1\frac{1}{4}$
$\frac{3}{8}$	8·5	4·3	1·6	$3\frac{1}{4}$	$2\frac{1}{8}$
$\frac{7}{16}$	11·0	5·9	2·3	4	$3\frac{3}{8}$
$\frac{1}{2}$	14·0	7·7	3·0	$4\frac{3}{4}$	5
$\frac{9}{16}$	18·0	9·7	3·8	$5\frac{1}{2}$	7
$\frac{5}{8}$	24·0	12·0	4·6	$6\frac{1}{4}$	$8\frac{1}{4}$
$\frac{11}{16}$	28·0	14·6	5·6	7	$10\frac{1}{2}$
$\frac{3}{4}$	31·5	17·3	6·8	$7\frac{1}{2}$	12
$\frac{13}{16}$	37·0	20·4	7·9	$8\frac{1}{4}$	15
$\frac{7}{8}$	44·0	23·1	9·1	9	$17\frac{1}{2}$
$\frac{15}{16}$	50·0	26·1	10·5	$9\frac{1}{2}$	$19\frac{1}{2}$
1	56·0	29·3	12·0	10	22
$1\frac{1}{8}$	71·0	36·3	15·3	$11\frac{1}{4}$	$27\frac{3}{8}$
$1\frac{1}{4}$	87·5	44·1	18·8	$12\frac{1}{2}$	$34\frac{1}{8}$
$1\frac{3}{8}$	105·8	52·8	22·6	$13\frac{3}{4}$	$41\frac{1}{2}$
$1\frac{1}{2}$	126·0	62·3	27·0	15	$49\frac{1}{2}$

Table of Strength and Weight of Studded-Link Cable.

Diameter of Iron d in Inches.	Weight in Lbs. per Fathom.	Breaking Strength in Tons.	Test Load in Tons.	Girth of Equivalent Rope in Inches.	Weight of Rope in Lbs. per Fathom.
$\frac{5}{8}$	24	9·5	7	$6\frac{1}{2}$	9
$\frac{11}{16}$	28	11·4	$8\frac{1}{2}$	$7\frac{1}{2}$	12
$\frac{3}{4}$	32	13·5	$10\frac{1}{8}$	8	14
$\frac{7}{8}$	44	20·4	$13\frac{3}{4}$	$9\frac{1}{2}$	$19\frac{1}{2}$
1	58	24·3	18	$10\frac{1}{2}$	$22\frac{1}{2}$
$1\frac{1}{8}$	72	29·5	$22\frac{3}{4}$	12	$30\frac{3}{4}$
$1\frac{1}{4}$	90	38·5	$28\frac{1}{8}$	$13\frac{1}{2}$	$39\frac{1}{4}$
$1\frac{3}{8}$	110	48·5	34	15	$48\frac{1}{4}$
$1\frac{1}{2}$	125	59·5	$40\frac{1}{2}$	16	55
$1\frac{5}{8}$	145	66·5	$47\frac{1}{2}$	17	62
$1\frac{3}{4}$	170	74·1	$55\frac{1}{8}$	18	$68\frac{1}{4}$
$1\frac{7}{8}$	195	92·9	$63\frac{1}{4}$	20	86
2	230	99·5	72	22	104
$2\frac{1}{8}$	256	112	$81\frac{1}{4}$	24	124
$2\frac{1}{4}$	285	126	$91\frac{1}{8}$	26	145

Strength of Suspended Chains and Wires.—When a uniformly heavy chain or wire is suspended between two points, to find the equation to the curve in which it hangs, and the tension at any point. Let T be the tension at any point P; and H, that at the lowest point O. If W be the weight of the part O P of the chain, it is evident that O P will be in equilibrium under the action of three forces—the tensions at O and P and its own weight W, acting through its centre of gravity. These three forces, therefore, must pass through some point K, in O x, such that K P will be a tangent to the curve at the point P. Let the curve be referred to the co-ordinate axes, O x, O y, and let

$ON = x$, and $NP = y$, $OP = s$. Also let w be the weight of



ILLUSTRATING STRENGTH OF SUSPENDED CHAINS.

a unit length of chain, and for H write $m\,w$. Resolving vertically and horizontally, we get:—

$$T \sin \varphi = W = s\,w. \quad T \cos \varphi = H = m\,w.$$

Hence, $\tan \varphi = \frac{s}{m}$; or, $\frac{dy}{dx} = \frac{s}{m}$. . . . . (1)

But, $\frac{dy}{ds} = \sin \varphi = \frac{1}{\operatorname{cosec} \varphi} = \frac{1}{\sqrt{\cot^2 \varphi + 1}} = \frac{s}{\sqrt{m^2 + s^2}}$

$\therefore \quad dy = \frac{s\,.\,ds}{\sqrt{m^2 + s^2}}.$

Integrating this expression, we have:—

$$y = \sqrt{m^2 + s^2} + C.$$

To find the value of the constant C, we know that $s = 0$, when $y = 0$.

$\therefore \quad 0 = m + C,$

Or, $\quad C = -m.$

So that, $\quad y + m = \sqrt{m^2 + s^2}.$

and, therefore, $\quad s = \sqrt{(y + m)^2 - m^2}.$

Substituting this value of s in (1), and inverting:—

$$\frac{dx}{dy} = \frac{m}{\sqrt{(y + m)^2 - m^2}}.$$

Multiplying each side by dy, and integrating, we get:—

$$x = m\,.\,\log_e \{(y + m) + \sqrt{(y + m)^2 - m^2}\} + C.$$

When $\quad x = 0,\ y = 0,$ then:—$C = -m\,.\,\log_e m.$

Hence, $\quad x = m\,.\,\log_e \left\{ \frac{y + m) + \sqrt{(y + m)^2 - m^2}}{m} \right\}$ . . (2)

Equation (2) is sometimes useful in the solution of problems. In order to get the equation to the curve in which the chain hangs, or, in other words, the relation between x and y, we write (2) as follows :—

$$\frac{x}{m} = \log_e \left\{ \left(\frac{y+m}{m}\right) + \sqrt{\left(\frac{y+m}{m}\right)^2 - 1} \right\}$$

That is, $$e^{\frac{x}{m}} = \left(\frac{y+m}{m}\right) + \sqrt{\left(\frac{y+m}{m}\right)^2 - 1}; \quad . \quad . \quad . \quad . \quad (3)$$

Where e is the base of the Napierian or hyperbolic logarithms.

Now, since :—

$$\left\{ \left(\frac{y+m}{m}\right) + \sqrt{\left(\frac{y+m}{m}\right)^2 - 1} \right\} \cdot \left\{ \left(\frac{y+m}{m}\right) - \sqrt{\left(\frac{y+m}{m}\right)^2 - 1} \right\} = 1$$

$$\therefore \left\{ \left(\frac{y+m}{m}\right) - \sqrt{\left(\frac{y+m}{m}\right)^2 - 1} \right\} = \frac{1}{\left\{ \left(\frac{y+m}{m}\right) + \sqrt{\left(\frac{y+m}{m}\right)^2 - 1} \right\}}$$

$$= \frac{1}{e^{\frac{x}{m}}}.$$

Hence, $$e^{-\frac{x}{m}} = \left(\frac{y+m}{m}\right) - \sqrt{\left(\frac{y+m}{m}\right)^2 - 1}. \quad . \quad . \quad . \quad . \quad (4)$$

Now, adding (3) and (4) and reducing, we have finally :—

$$\left.\begin{aligned} y &= \frac{m}{2}\left\{e^{\frac{x}{m}} + e^{-\frac{x}{m}} - 2\right\} \\ \text{Or,} \quad y &= \frac{m}{2}\left\{e^{\frac{x}{2m}} - e^{-\frac{x}{2m}}\right\} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (XI)$$

The curve whose equation is (XI) is called a *catenary*.

To find T, the tension at any point, we have :—

$$T = \frac{s\,w}{\sin \varphi} = s\,w \div \frac{s}{\sqrt{s^2 + m^2}}$$

$$= w \, . \sqrt{s^2 + m^2}.$$

But $$\sqrt{s^2 + m^2} = y + m.$$

$$\therefore \qquad T = w\,(y + m). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (XII)$$

When the curve is very flat, as in the case of telegraph wires, then $s = x$ approximately, and (1) becomes $\frac{dy}{dx} = \frac{x}{m}$.

Multiplying by dx and integrating, we get:—

$$y = \frac{x^2}{2m} \quad \ldots \ldots \quad \text{(XIII)}$$

This requires no correction, because x and y vanish together.

EXAMPLE V.—A telegraph wire, which weighs $\frac{1}{10}$ of a lb. per yard, is stretched between poles on level ground, so that the greatest dip of the wire is 3 feet. Find approximately the distance between the poles when the tension at the lowest point of the wire is 140 lbs. (Hons. S. and A. Exam., 1891.)

ANSWER.—Here, H = 140, y = 3 ft., and $w = \frac{1}{30}$; and since:—

$$\text{H} = mw$$

$$\therefore \quad 140 = m \times \frac{1}{30}.$$

$$\text{Or,} \quad m = 4200.$$

Putting these values of y and m in equation (XIII), it becomes:—

$$3 = \frac{x^2}{2 \times 4200}.$$

$$\therefore \quad x = \sqrt{3 \times 2 \times 4200} = 158{\cdot}7 \text{ ft.}$$

Distance between poles:—

$$= 2x = 2 \times 158{\cdot}7 = \mathbf{317{\cdot}4 \text{ ft.}}$$

If the more exact equation (2) be used, then:—

$$2x = 2 \times 4200 \times \log_e \left\{ \frac{4203 + \sqrt{4203^2 - 4200^2}}{4200} \right\}$$

$$,, \quad = 8400 \times \log_e \frac{218}{210}$$

$$,, \quad = 8400 \times 0{\cdot}0374 = \mathbf{314{\cdot}16 \text{ ft.}}$$

LECTURE XXIX.—QUESTIONS.

1. What do you understand by the terms *strain*, *stress*, and *modulus of elasticity?* A tie rod 100 feet long, and of 2 square inches sectional area, is stretched three-quarters of an inch under a tension of 32,000 lbs. What is the intensity of the stress, the strain, and the modulus of elasticity under these circumstances? *Ans.* 16,000 lbs. per square inch; 0·000625; 25,600,000.

2. A ship is moored by two cables of 90 feet and 100 feet in length respectively. The first cable stretches $2\frac{5}{8}$ inches, and the second stretches 3 inches, under the pull of the ship; find the strain of each cable. (S. and A. Exam., 1889.) *Ans.* 0·00243; 0·0025.

3. Define the term *Resilience*. Show that the work done on a material by a *live load* is four times that done by an equal *dead load*. A wrought-iron tie rod 20 feet long and ·5 square inch cross sectional area bears a dead load of 5,000 lbs. Find the work done on stretching the rod by this load. What live load would produce an instantaneous elongation of another $\frac{1}{16}$ inch? Take E = 30,000,000. *Ans.* $33\cdot\dot{3}$ ft.-lbs.; 3,125 lbs.

4. A rod of iron 25 feet long and 2 square inches cross sectional area checks a weight of 80 lbs., which falls from a height of 20 feet before beginning to strain it. Find the greatest stress and strain produced. Take E = 25,000,000. *Ans.* 39,960 lbs. per square inch; ·0016.

5. If the modulus of elasticity of a piece of steel in lbs. per square inch is 32,000,000, how much would a bar $\frac{3}{4}$ of an inch in diameter and 25 inches long extend under a load of 10 tons? If its limit of elasticity is 21 tons per square inch, what is its resilience?

6. What is the resilience of a bar? A bar of steel is $\frac{3}{4}$ inch in diameter, and 30 inches in length, and is under a tensile pull of 10 tons, what is the work stored up in the bar, the modulus of elasticity being 32,000,000 lbs. per square inch?

7. Built-up guns are made of concentric rings, the outer hoops, or rings, being shrunk or forced upon inner tubes with a regulated tension. Supposing the external diameter of the inner tube to be 12 inches, and that the substance of its covering hoop is to have given to it an initial grip of 4 tons per square inch of its sectional area; the exterior diameter of this second hoop is 18 inches, and is to be covered with a third hoop, having an initial grip of 8 tons per square inch of its sectional area; will you work out in arithmetic the difference of dimensions that will afford the above conditions?

8. Prove that when a thin spherical shell is exposed to the bursting pressure of gas or liquid the stress in the material is half as great as that within the curved surface of a thin cylindrical shell exposed to the like pressure, each shell being of the same thickness and diameter.

9. A long thin pipe of given internal radius is subjected to fluid pressure; find the tension of the material of the pipe. If the internal radius of the pipe is 6 inches, and the thickness of the pipe 0·5 inch, what fluid pressure per square inch would increase the radius of the pipe by 0·001 inch? The modulus of elasticity being 20,000,000, and the elasticity of the material being supposed to continue perfect. *Ans.* 277·7 lbs. per square inch.

10. A *steel* hydraulic cylinder, 10 feet long and 6 inches in diameter, acts as a brake on a lift. It has a movable piston fitted with a spring valve, the cylinder being full of liquid when the lift is at its highest position, and the piston and rod at the end of the stroke inside the cylinder. It was found that when the lift began to descend the internal pressure was 1,000 lbs. per square inch, which gradually rose to 2,000 lbs. when the piston had travelled 9 feet. Treating the cylinder as a thin one, what would be the law of variation of thickness at different points? Prove the formula.

11. A uniformly heavy chain is suspended from two given points: find the equation to the curve in which it hangs, and the tension at any point of the curve.

12. Prove that the tendency of a thin cylindric pipe to burst laterally (neglecting the strength of flanges, &c.) is twice as great as to burst endwise.

A wrought-iron pipe is 2 feet diameter, $\frac{1}{2}$ inch thick, its working stress is 5 tons to the square inch, but strength of plate is diminished 30 per cent. because of riveted joint. What is the working pressure? What head of water does this correspond to? (S. & A. Adv. Exam., 1897.)

13. Prove the law for the tensile stress produced in a thick cylinder by internal fluid pressure. Describe how we attempt by chilling to give maximum strength. (Hons. S. & A. Exam., 1897.)

14. A steel tube 5 inches internal and 7 inches external diameter has steel strip wound on it to the external diameter of 12 inches under a constant winding tensile stress of 15 tons per square inch. What is the stress at any place in the solid metal or the winding? (S. & A. Hons. Exam., Part II., 1898.)

15. If a thin vessel is subjected to fluid pressure, p, inside (in excess of the outside pressure), prove that the total bursting force at any plane section is pA if A is the area of the whole section. How do we calculate the tensile stress in the section a of the metal? Find the rule for the stress in a thin spherical vessel. Does the rule apply to a thick vessel? Give reasons for your answer. (S. and A. Adv., 1899.)

16. State clearly how we arrive at a rule for the proper thickness of a pipe, proving any formula used by you. In fixing the proper value of the stress that the material (say cast-iron) will stand, why do we not use the results of experiment on cast-iron test pieces in tension?

(B. of E. Adv., 1901.)

17. Knowing the axial and lateral strains in a tie bar of homogeneous material when subjected to a tensile force, show how we calculate the modulus of rigidity of the material. (B. of E. H., Part I., 1901.)

18. A tube 3 inches internal and 8 inches external diameter is subjected to a collapsing pressure of 5 tons per square inch; show by curves the radial and circular stresses everywhere. Prove your formulæ. The limits of elasticity are not supposed to be exceeded. (B. of E. H., Part I., 1901.)

19. A horizontal circular tube of steel is 7 feet diameter, $\frac{1}{2}$ inch thick, 100 feet long supported at the ends, its total load distributed uniformly all over being 30 tons, what are the greatest stresses in the metal? The tube is filled with compressed air, what must its pressure be if there is just no compressive stress in the metal? State what is now the nature of the stress in the metal at the place where it is greatest.

(B. of E. H., Part I., 1902.)

Lecture XXIX.—C. & G. Exam. Questions.

1. A steam engine has a piston 18 inches in diameter, and the greatest difference of steam pressure between the two sides of the piston when the engine is at work is 120 lbs. per square inch: what must the piston-rod diameter be in the body of the rod if the greatest intensity of stress per square inch is not to exceed 2,500 lbs.? (C. & G., 1900, O., Sec. B.)

2. The links of a chain are made out of $1\frac{1}{8}$-inch round bar iron, having a tenacity in pure tension of 22·5 tons per square inch: what load could be safely lifted with such a chain if the stress is not to exceed $\frac{1}{4}$th of the breaking stress of the chain? You may assume that only three-quarters of the full tenacity would be developed in the material before rupture, when worked up into the form of a link. (C. & G., 1900, O., Sec. B.)

3. How much would a steel tie-bar 3 inches in diameter and 25 feet 6 inches long extend under a total load of 33 tons? The modulus of elasticity of the steel is 12,500 tons per square inch. (C. & G., 1900, O., Sec. B.)

4. A cylinder 10 inches in diameter has a cover fixed on by $1\frac{1}{8}$-inch studs. The internal fluid pressure is 200 lbs. per square inch above the atmospheric pressure. How many such studs would you employ if the tensile stress per square inch at the bottom of the threads is not to exceed 2,500 lbs.? (The diameter at the bottom of the thread of a bolt = 0·9 diameter of bolt − ·05 inch.) (C. & G., 1901, O., Sect. B.)

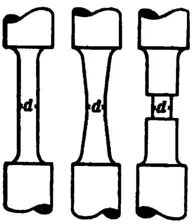
Lecture XXIX.—A.M.Inst.C.E. Exam. Questions.

1. Show that a chain supported at its ends and carrying a load which is uniform per foot of the span hangs in a parabolic curve. Hence find approximately the pull in a telegraph wire when the span, the dip, and the weight of the wire are given. (I.C.E., *Oct.*, 1897.)

2. Distinguish between stress and strain. A round bar 20 feet long and 2 inches in diameter is stretched $\frac{1}{20}$ inch by a load of 7 tons, applied along the axis. Find the intensity of stress on a cross-section and the coefficient of elasticity of the material. (I.C.E., *Oct.*, 1897.)

3. The greatest stress permissible being 5,000 lbs. per square inch, find the shortest rod of 1 square inch section which can be safely used to check a weight of 50 lbs. falling 3 feet. Take E = 12,000 tons per square inch. (I.C.E., *Oct.*, 1897.)

4. Compare, in general terms, the three rods shown in the sketch as regards their suitability to withstand tensional stresses produced by impacts of given energy, dealing both with the magnitude and distribution of the stresses. (I.C.E., *Feb.*, 1898.)



5. Prove that the shape of a hanging chain loaded uniformly horizontally is parabolic. The points of support are 50 feet and 70 feet above the lowest point in the chain, and the horizontal distance between the points of support is 300 feet. The load is 3 tons per foot horizontal, state the pull in the chain at the points of support and the middle. (I.C.E., *Oct.*, 1898.)

6. Describe the particular distributions of load under which an equilibrated suspension chain would hang in the following curves:—(*a*) a parabolic arc, (*b*) the common catenary, (*c*) the curve of sines. (I.C.E., *Oct.*, 1898.)

7. Explain the meaning of the following terms, giving some verbal illustrations: (*a*) modulus of elasticity; (*b*) elastic limit; (*c*) permanent set; (*d*) plastic elongation; (*e*) ultimate elongation; (*f*) contraction of area. (I.C.E., *Feb.*, 1899.)

8. By how much would a wrought-iron tie-bar be elongated under a direct pull of 30 tons, if its length were 30 feet, and its cross-section 5 inches by 1 inch? (Assume E = 12,000 tons.) (I.C.E., *Feb.*, 1899.)

9. A standard test bar of cast iron, 1 inch broad and 2 inches deep, laid upon supports 3 feet apart, is found to deflect just $\frac{1}{10}$ inch under a central load of 25 cwts. Calculate the modulus, E, and also the apparent tensile stress in the extreme fibre. (I.C.E., *Feb.*, 1899.)

10. Explain the terms (*a*) stress, (*b*) strain, (*c*) elasticity, and state the law connecting (*a*) and (*b*) when the elasticity of a material is perfect. A steel tie-rod, 1 inch diameter 20 feet long, stretches $\frac{1}{8}$ inch under a pull of 5 tons: find the modulus of elasticity. (I.C E., *Oct.*, 1899.)

11. Prove the ordinary formula which gives the stress on the material of a cylindrical boiler-shell of given diameter and thickness under a given internal fluid pressure, explaining carefully the conditions under which it is approximately correct and stating the reasons why it cannot generally be used for an external pressure. (I.C.E., *Oct.*, 1899.) (*See my "Text-Book on Steam and Steam Engines" for further treatment.*)

12. Find the curve in which a string, whose weight is neglected, uniformly loaded horizontally, would hang. (I.C.E., *Oct.*, 1899.)

13. If the load on the platform of a suspension bridge be uniformly distributed show that the chains hang in a parabola, the weight of the chains and suspending rods being omitted from the calculation. Find the pull at any point of the chain, and assuming the chains attached to the top of the piers, find the moment tending to overturn them. Describe the arrangements adopted in practice to prevent this.
(I.C.E., *Feb.*, 1900.)

14. What is the modulus of elasticity (Young's modulus) and how is it measured? Mention some of the practical questions which depend upon its value in different materials. (I.C.E., *Oct.*, 1900.)

15. Assuming the modulus to be 12,000 tons in iron wire, what would be the elongation of a wire originally 1000 feet in length, due to its own weight when suspended vertically? (I.C.E., *Oct.*, 1900.)

16. An iron telegraph wire is stretched between a pair of supports 160 feet apart, and hangs with a sag of 2 feet in the middle. Assuming its weight to be uniformly distributed over the span, find the tensile stress per square inch at the centre. The weight of a cubic inch of iron may be taken at 0·27 lb. (I.C.E., *Feb.*, 1901.)

17. Explain the meaning and uses of a factor of safety, stating what factor you would select for the following cases:—(*a*) A girder of mild steel carrying the dead weight of a wall. (*b*) A steel cross-girder in a railway bridge. (I.C.E., *Feb.*, 1901.)

18. In a tensile test of a piece of flat wrought-iron bar, the following results were obtained:—

(i.) Original dimensions of cross-section 2·010 inches by 0·505 inch.
(ii.) Final dimension of cross-section at point of fracture 1·520 inch by 0·410 inch.
(iii.) Gross load at limit of elasticity 34,500 lbs.
(iv.) Gross load at fracture 53,100 lbs.
(v.) Total extension on length of 10 inches = 1·46 inch.
(vi.) Extension on 10″ length under a gross load of 20,000 lbs. = $\frac{7}{1000}$″.

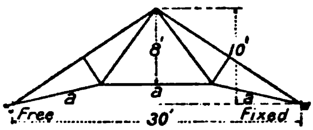
Find from the above data:—(*a*) The modulus of elasticity of the material; (*b*) the limit of elasticity and tenacity per square inch; (*c*) the reduction of area per cent.; (*d*) the approximate work done per cubic inch of the material in fracturing the bar. Would you consider this good material?
(I.C.E., *Oct.*, 1901.)

19. A foot-bridge 10 feet in width is carried over a river 100 feet in width by two cables of uniform section, with a dip of 10 feet in the centre. Find the greatest pull on the cables, their cross-sectional area, length and weight for the following data:—Maximum load on platform 120 lbs. per square foot. Working-stress in metal of cables 4 tons per square inch. Weight of cable material per cubic foot 484 lbs. (I.C.E., *Oct.*, 1901.)

20. A short cylindrical block is subject to a simple push, P, in the direction of its axis. Find an expression for the normal and tangential forces on a plane whose normal is inclined to the axis of the block at an angle, θ; and show that the tangential force has its maximum value when θ is 45°.
(I.C.E., *Feb.*, 1902.)

21. A suspension bridge has a span of 200 feet, the dip of the chains is 64 feet, and the weight of the roadway carried 2,800 lbs. per foot run. Calculate the tension at the middle and at the two ends of each of the two chains. (I.C.E., *Feb.*, 1902.)

22. In the roof truss given find the stresses in the different members due to a normal wind pressure of 27 lbs. per square foot, acting on the left-hand side of the roof if the principals are 8 feet apart. (I.C.E., *Feb*, 1902.)



23. Assuming that the stresses due to the dead loads on the above roof have been similarly determined, show how you would determine the sizes of the tie-bars (a) say. Assume some value for the stresses due to dead load, and also assume that the severest stresses due to the wind are those obtained in the previous question. (I.C.E., *Feb*, 1902.)

24. A connecting-rod of an engine is 6 feet long and 4 inches in diameter at the centre. What factor of safety does it possess when subjected to a pure compressive force of 14½ tons? The rod is steel. (I.C.E., *Feb.*, 1902.)

25. A mild steel tie-rod is made of angle-bar 4 inches by 4 inches by ½ inch and is 24 feet long. If it is subjected to a direct pull of 21 tons, what is the intensity of tensile stress per square inch, and how much will it elongate under the load. (I.C.E., *Feb.*, 1902.)

26. A round wrought-iron rod is 25 feet long and 1 inch in diameter, find the tensile load which would, if suddenly applied, instantaneously elongate the bar by 0·08 inch. (I.C.E., *Feb.*, 1902.)

27. Wrought iron expands $\frac{1}{10000}$th of its length for every 15° rise of temperature (Fahrenheit). Find the highest temperature at which a rivet should be closed, if the stress due to cooling is not to exceed 15 tons per square inch. Modulus of elasticity 28,000,000 lbs. per square inch. (I.C.E, *Oct.*, 1902.)

28. A chain, with its two ends level, carries concentrated loads at 5 points along its length; show how to find graphically the position and amount of the resultant of the loads and the amount of the vertical reaction of the supports (I.C.E., *Feb.*, 1903.)

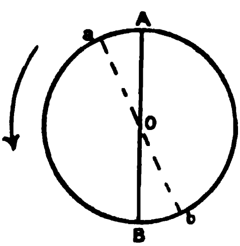
LECTURE XXX.

CONTENTS.—Torsional Strength of Shafts—Examples I., II., and III.—Strength of Shafts subjected to Combined Twisting and Bending—Theorem—Examples IV. and V.—Stiffness of Shafts—Angle of Twist—Example VI.—Table of Powers Transmitted by Shafts—Questions.

Torsional Strength of Shafts.—In order to transmit energy through a shaft, the driving force must be applied at some distance from its centre. The driving force and its effective leverage, therefore, constitute what is termed a **Turning** or **Twisting Moment** (T.M.) which puts the shaft in a state of twist or torsion. The tendency of a purely torsional moment applied to a shaft is to cause the shaft to shear in planes normal to its axis, and this has to be met by the shearing resistance of the material, which resistance must, of course, be of the nature of a moment. The resistance the shaft offers to twisting we term its **Torsional Resistance** (T.R.); and as this balances the turning moment, we have :—

$$\text{T.M.} = \text{T.R.}$$

We have now to find the value of T.R., as depending on the material and dimensions of the shaft, and shall confine ourselves to shafts of circular section—solid and hollow. Suppose the accompanying figure to represent an end view of a shaft; and suppose A B and $a\,b$ to have been parallel diameters of two sections very near to each other when the shaft was at rest; then, when the shaft is at work transmitting energy, the diameters, A B and $a\,b$, will no longer be parallel, but will make an angle with each other, as shown. A longitudinal section, through the axis of a shaft, which is a plane when the shaft is at rest, thus becomes a screw surface when the shaft is working. We shall have occasion later to measure this angle of twist; but in the meantime we are mainly concerned with the distribution of shearing stress within the shaft.



ILLUSTRATING STRAIN IN A SHAFT.

Looking at the figure, we easily see that the *strain* in any ring of fibres must be proportional to the arc of this ring

which is included between the diameters A B and $a\,b$, when these are twisted out of parallelism by the turning moment. Within the elastic limit of the material, therefore, it follows that the shearing *stress* in any ring of fibres is proportional to the radius of that ring.

Therefore, let f = the greatest shearing stress, in lbs. per sq. inch, permissible in the material of the shaft.

D = the outside diameter,

d = the inside diameter of the shaft, both in inches.

And x = the radius of *any* ring of fibres within the material of the shaft.

Then the shaft must be so proportioned that f shall be the value of the stress in its outermost fibres which are $\frac{1}{2}$ D inches from the centre. Consequently, from what has already been said, we have :—

$$\text{Stress at } x = \frac{x}{\frac{1}{2}D} f = \frac{2x}{D} f.$$

Consider, now, the ring of fibres at x inches from the shaft centre, whose radial thickness is $d\,x$ inches. The sectional area of this elementary ring will $= 2\pi x\,d\,x$ sq. inches; and its resistance to shearing will be

$$2\pi x\,d\,x \times \frac{2x}{D} f \text{ lbs.} = \frac{4\pi f}{D} x^2\,d\,x \text{ lbs.}$$

Now, the leverage at which this resisting ring of fibres acts, is x inches; therefore, its *moment* of resistance is $\frac{4\pi f}{D} x^2\,d\,x \times x$, or $\frac{4\pi f}{D} x^3\,d\,x$ inch-lbs.

Hence, summing up the moments of resistance of all such elementary rings which go to make up the shaft, we get:—

$$\text{T.R.} = \frac{4\pi f}{D} \int_{\frac{1}{2}d}^{\frac{1}{2}D} x^3\,d\,x,$$

$$\text{,,} = \frac{4\pi f}{D} \left\{ \frac{(\frac{1}{2}D)^4}{4} - \frac{(\frac{1}{2}d)^4}{4} \right\}$$

$$\text{,,} = \frac{\pi}{16} f \left(\frac{D^4 - d^4}{D} \right).$$

Hence, for hollow shafts, we have:—

$$\text{T.R.} = \frac{\pi}{16}\left(\frac{D^4 - d^4}{D}\right)f \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(I)}$$

For solid shafts, we make $d = 0$, and get:—

$$\text{T.R.} = \frac{\pi}{16} D^3 f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(II)}$$

It is instructive to compare the torsional resistances of solid and hollow shafts of the same weight and material. For this purpose let D_1 be the outer diameter of hollow shaft.

Then, if we neglect couplings, and consider the shafts to be of equal length, the weights will simply be proportional to their sectional areas; *i.e.*:—

$$\frac{\text{Weight of hollow shaft}}{\text{Weight of solid shaft}} = \frac{D_1^2 - d^2}{D^2}.$$

For equal weights, this ratio is unity; therefore we have the relation:—

$$D^2 = D_1^2 - d^2$$

Or,
$$D = \sqrt{D_1^2 - d^2}.$$

Now, we have from equations (I) and (II):—

$$\frac{\text{T.R. of hollow shaft}}{\text{T.R. of solid shaft}} = \frac{D_1^4 - d^4}{D_1 \times D^3} = \frac{D_1^2 + d^2}{D_1 \times D} \times \frac{D_1^2 - d^2}{D^2}$$

$$\text{,,} \qquad \text{,,} \qquad = \frac{D_1^2 + d^2}{D_1 \times D} = \frac{D_1^2 + d^2}{D_1 \times \sqrt{D_1^2 - d^2}}.$$

It will simplify matters if we put $d = x \times D_1$, where x is a proper fraction, we then have:—

$$\frac{\text{T.R. of hollow shaft}}{\text{T.R. of solid shaft}} = \frac{1 + x^2}{\sqrt{1 - x^2}}.$$

For example, let $x = \frac{1}{2}$, then:—

$$\frac{1 + x^2}{\sqrt{1 - x^2}} = \frac{1 + \frac{1}{4}}{\sqrt{1 - \frac{1}{4}}} = \frac{5}{2\sqrt{3}} = 1{\cdot}443.$$

This result shows that for the same length and weight, the hollow shaft having outer and inner diameters in the proportion of 2 to 1 will be 44·3 per cent. stronger than the solid one.

The turning moment driving a shaft may either be uniform or variable in amount. Shafts driven by means of gearing, and revolving at a uniform speed, are generally considered as cases

of uniform turning moment. As a typical example of variable turning moment, we have the case of the steam engine crank-shaft, where both the driving force of the steam on the piston and its effective leverage are continually varying throughout the stroke.

When the turning moment is uniform—that is, when the shaft revolves uniformly at n revolutions per minute, and transmits energy at the rate of so many H.P., this is all the data we require to know in order to estimate T. M. We have already seen (see Vol. I., Lect. III.) that the work done by a turning couple in one minute is equal to the magnitude of the turning couple multiplied by its angular displacement in the same time. Now our turning couple, or turning moment, as we call it, is T.M. inch-lbs., or $\frac{1}{12}$ T.M. foot-lbs., and the angular velocity of our shaft is $n \times 2\pi$ radians per minute.

Therefore, the

$$\text{Work done} = \frac{\text{T.M.}}{12} \times 2\pi n \text{ ft.-lbs. per minute}$$

And the
$$\text{H.P.} = \frac{\frac{\text{T.M.}}{12} \times 2\pi n}{33{,}000} = \frac{n \times \text{T.M.}}{63{,}025}.$$

$$\therefore \quad \text{T.M.} = 63{,}025 \cdot \frac{\text{H.P.}}{n}. \quad \ldots \ldots \quad \text{(III)}$$

EXAMPLE I.—Find the moment of resistance to torsion of a hollow shaft. Compare the strengths to resist torsion of a solid and hollow shaft of the same length and weight, the extreme diameter of the hollow shaft being double its internal diameter. A hollow shaft, the external and internal diameters of which are 20 inches and 8 inches respectively, runs at 70 revolutions per minute, with a surface stress of 6,000 lbs. per square inch; find the twisting moment and the horse-power transmitted. (S. & A. Hons. Exam., 1895.)

ANSWER.—The first two parts of this question have already been answered in the text.

With regard to the last part, we are asked to find the values of T.M. and H.P., being given:—

$D_1 = 20$ inches. $\qquad f = 6000$ lbs. per sq. in.
$d = 8$ inches. $\qquad n = 70$ per min.

Since $\quad \text{T.R.} = \text{T.M.}$

$$\text{,,} \quad = \frac{\pi}{16} \cdot \frac{D_1^4 - d^4}{D_1} \cdot f.$$

$$\therefore \qquad \text{T.M.} = \frac{3 \cdot 1416}{16} \times \frac{20^4 - 8^4}{20} \times 6000.$$

$$,, \quad = 9{,}183{,}525 \text{ inch-lbs.}$$

and
$$\text{H.P.} = \frac{\text{T.M.} \times n}{63{,}024}$$

$$,, \quad = \frac{9{,}183{,}525 \times 70}{63{,}024}$$

$$,, \quad = 10{,}200.$$

EXAMPLE II.—If a steel shaft revolving at 60 revolutions per minute be required to transmit 220 horse-power, what should be its diameter so that the maximum stress produced in it may not exceed one-fifth of th t at the elastic limit? The elastic limit in torsion is 18 tons per sq. inch. Prove any formula you may employ. (S. & A. Hons. Exam., 1894.)

ANSWER.—Combining formulæ (II) and (III) we have:—

$$\text{T.R.} = \text{T.M.},$$

i.e.,
$$\frac{\pi}{16} D^3 f = 63{,}024 \times \frac{\text{H.P.}}{n}.$$

$$\therefore \qquad D = 68 \cdot 5 \sqrt[3]{\frac{\text{H.P.}}{n f}}. \quad . \quad . \quad . \quad \text{(IV)}$$

Here, H.P. = 220. $n = 60$.

And, $f = \frac{1}{5} \times 18 \times 2240 = 8064$ lbs. per sq. in.

$$\therefore \qquad D = 68 \cdot 5 \times \sqrt[3]{\frac{220}{60 \times 8064}} = 5 \cdot 27 \text{ inches.}$$

In cases where the turning moment exerted on a shaft varies, it is, of course, necessary that the shaft should be of strength sufficient to withstand safely the maximum value of T.M. So that in dealing with an example like that of the steam engine crank-shaft we take as the turning force the product of the maximum effective steam pressure on the piston into the piston area; and for the leverage we take the crank radius, although this is not quite accurate; because, if the crank be driven by means of a connecting-rod, the virtual leverage of the steam force at a certain point in the stroke exceeds that of the crank radius by an amount depending on the relative lengths of the crank and connecting-rod.

But on the other hand, the effective steam pressure on the piston is, as a rule, much below its maximum value when the piston reaches the point of greatest leverage. On the whole, therefore, it is quite accurate enough for all practical purposes to estimate the maximum turning moment in the way we have indicated.

Thus, Let p = Greatest effective steam pressure acting on the piston, in lbs. per sq. inch.

„ A = Area of piston, in sq. inches.

„ r = Crank-radius, in inches.

Then, max. T.M. = $p \, A \, r$ inch-lbs.

By *effective* steam pressure, we mean the *difference* between the pressures behind, and in front of, the piston.

EXAMPLE III.—Find the diameter of the crank-shaft for a horizontal engine which is to be worked with an effective mean steam pressure of 45 lbs. per square inch throughout the stroke, the diameter of the cylinder being 36 inches, the stroke 5 feet, and the working load being taken at $\frac{1}{8}$ of the breaking load. The shaft is to be of wrought iron, such that a 1-inch shaft will break with the torsion produced by 800 lbs. acting at the end of a 12-inch lever. (S. & A. Hons. Exam.)

ANSWER.—Let f_B be the breaking stress of the experimental shatt, then the working stress in the crank shaft, according to the question, will be $\frac{1}{8} f_B$.

To find the value of f_B we are given that when T.M. $= 800 \times 12$ inch-lbs., and D = 1″, fracture takes place. From these data, therefore, we deduce :—

$$f_B = \frac{800 \times 12}{\frac{\pi}{16} \times 1^3} = \frac{800 \times 12}{\frac{\pi}{16}} \text{ lbs.}$$

The area of a 36-inch piston = 1017·87 square inches, and r is 30 inches.

$$\therefore \qquad \text{Max. T.M.} = 45 \times 1017{\cdot}87 \times 30 \text{ inch-lbs.}$$

$$\text{Also,} \qquad „ \qquad = \frac{\pi}{16} D^3 f.$$

$$\therefore \qquad D^3 = \frac{45 \times 1017{\cdot}87 \times 30}{\frac{\pi}{16} f};$$

but, $$f = \tfrac{1}{6} f_s = \frac{800 \times 2}{\frac{\pi}{16}}.$$

Hence, $$D = \sqrt[3]{\frac{45 \times 1017{\cdot}87 \times 30}{800 \times 2}},$$

$$,, = 9{\cdot}5 \text{ inches, nearly.}$$

Strength of Shafts subjected to combined Twisting and Bending.—In Example III. the diameter of the shaft has been calculated as for a purely twisting moment. But in no case of a shaft being driven by a crank is the effect of the load quite so simple as this. Besides the turning moment, which we have already seen how to deal with, there is always in action a *bending* moment of greater or less magnitude depending on the engine arrangement. The worst case is that in which the crank is overhung. When this is so, the bending moment is caused by the load on the piston acting along a line (the centre line of the cylinder) at a certain distance from the shaft bearing nearest to the crank.

Let l = the distance between the centre line of the cylinder and the middle of the nearest shaft bearing, in inches; and

p and A = (as before) the effective steam pressure and piston area respectively.

Then the magnitude of the bending moment which we have now to take into account is

$$\text{B.M.} = p\,A\,l \text{ inch-lbs.}$$

This bending moment is balanced by the moment of resistance of the shaft, which, as will be shown in the next lecture, is

$$\text{M.R.} = \frac{\pi}{32} D^3 f_t;$$

Where, D = diameter of the shaft journal, in inches,

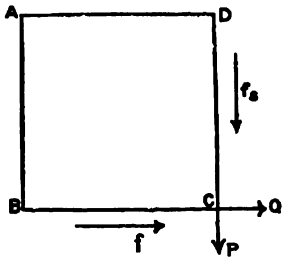
And, f_t = the tensile stress in the outer fibres of the journal, in lbs. per sq. inch.

Hence, we see that when a crank-shaft is being turned by the steam on the piston, it is subjected simultaneously to a shearing stress of intensity f_s, and a tensile stress of intensity f_t. The problem now before us is to combine these stresses so as to

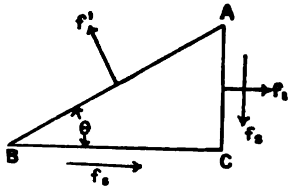
obtain what is termed the *Equivalent* tensile or shearing stress; but in order to render all the steps in the process clear and intelligible, we require to demonstrate the following theorem :—

Theorem.—A shearing stress on any plane produces a shearing stress of equal intensity on planes at right angles to it.

Let A B C D be a rectangular block of material whose thickness is 1 inch perpendicular to the plane of the paper. And let f_s be the intensity of the shearing stress over the face whose edge is



ILLUSTRATING SHEARING STRESS THEOREM.



ILLUSTRATING EQUIVALENT TENSILE STRESS.

C D. It is easy to see that the total shearing force on the face C D which tends to pull that face parallel to itself, must be accompanied by a similar effect on the face B C in order that the block may not be turned around A. To find the relation between those forces, take moments about A, and we get :—

$$P \times A D = Q \times A B.$$

Or, $$(f_s \,.\, C D) \times B C = (f \,.\, B C) \times C D.$$

$\therefore$ $$f_s = f.$$

Hence, we see that the shearing stress induced in a shaft by the turning moment is accompanied by a shearing stress of equal intensity on planes at right angles to it; that is, parallel to the axis of the shaft.

In the right-hand figure let A C represent the edge of a small portion of a plane normal to the axis of the shaft, and B C that of another plane at right angles to A C. On the former of these planes there is a shearing stress of intensity f_s, due to the turning moment, and a direct tensile stress of intensity f_t, due to the bending moment acting on the shaft. By the theorem just proved, we also have on B C a shearing stress f_s. Let f' denote the intensity of a tensile stress, which, acting on a

plane A B inclined to A C and B C, would balance the stresses on these latter planes. As before, let the width of the three planes perpendicular to the plane of the paper be unity.

Resolving vertically and horizontally, we have :—

$$(f' \,.\, AB)\cos\theta = (f_s \,.\, AC),$$

and $$(f' \,.\, AB)\sin\theta = (f_s \,.\, BC) + (f_t \,.\, AC).$$

From the first of these equations we get :—

$$\frac{f'}{f_s} = \frac{\frac{AC}{AB}}{\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta, \quad . \quad . \quad . \quad . \quad (1)$$

and from the second :—

$$f' = \frac{\frac{BC}{AB}}{\sin\theta} \cdot f_s + \frac{\frac{AC}{AB}}{\sin\theta} \cdot f$$

$$,, = \frac{\cos\theta}{\sin\theta} f_s + \frac{\sin\theta}{\sin\theta} f_t.$$

$$,, = f_s \cot\theta + f_t.$$

Or, $$\frac{f' - f_t}{f_s} = \cot\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Multiplying together (1) and (2), we get :—

$$\frac{f'}{f_s} \cdot \frac{f' - f}{f_s} = 1.$$

$$\therefore \qquad f'(f' - f) = f_s^2.$$

Which on being solved for f' gives :—

$$f' = \frac{f_t}{2} + \sqrt{\frac{f_t^2}{4} + f_s^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (V)$$

We take the positive sign in the solution of this quadratic equation, for obviously f' is greater than $\frac{1}{2} f_t$.

Being now in possession of the relation subsisting among the stresses, we next have to express these in terms of the T.M. and B.M. :—

Since $$\text{T.M.} = \frac{\pi}{16} D^3 f_s$$

And $$\text{B.M.} = \text{R.M.} = \frac{\pi}{32} D^3 f_t.$$

Hence, $$f_s = \frac{\text{T.M.}}{\frac{\pi}{16} D^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

And $$f = \frac{\text{B.M.}}{\frac{\pi}{32} D^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In like manner, we must have :—

$$f' = \frac{\text{B.M.}}{\frac{\pi}{32} D^3}$$

where B.M.′ stands for the *equivalent* bending moment.

Making these substitutions, and reducing, (V) becomes :—

$$\text{B.M.}' = \tfrac{1}{2} \{\text{B.M.} + \sqrt{\text{B.M.}^2 + \text{T.M.}^2}\} \quad . \quad . \quad . \quad (\text{VI})$$

Now, if T.M.′ denotes the *equivalent* twisting moment, it easily follows from equations (3) and (4) that for equal intensities of stress we have :—

$$\text{T.M.}' = 2\,\text{B.M.}'$$

Hence, $$\text{T.M.}' = \text{B.M.} + \sqrt{\text{B.M.}^2 + \text{T.M.}^2} \quad . \quad . \quad . \quad (\text{VII})$$

It will be found that equation (VII), giving the so-called equivalent twisting moment, is the one most generally applied. It should be noted, however, that the stress concerned here is a *tensile* one and not a shear as in a proper twisting moment.

Example IV.—Investigate an expression in terms of f_t, f', and f_s, which will give the resultant tensile stress, f', per square inch of section in a material which is subjected at the same time to a direct tensile stress of f_t lbs. per square inch, and to a shearing stress, f_s, lbs. per square inch. A bar of iron is at the same time under a direct tensile stress of 5,000 lbs. per square inch, and to a shearing stress of 3,500 lbs. per square inch. What would be the resultant equivalent tensile stress in the material? (S. & A. Hons. Exams., 1896.)

Answer.—The complete investigation referred to in the first part of this question is given in the text, and equation (V)

is the expression required. It only remains to find the numerical value of f', having given

$$f_t = 5{,}000 \text{ and } f_s = 3{,}500$$

$$\therefore \qquad f' = \frac{5{,}000}{2} + \sqrt{\frac{5{,}000^2}{4} + 3{,}500^2}$$

$$\text{,,} = 6{,}800 \text{ lbs. per sq. in. fully.}$$

EXAMPLE V.—A wrought-iron shaft is subjected simultaneously to a bending moment of 8,000 inch-lbs., and to a twisting moment of 15,000 inch-lbs. Find the twisting moment equivalent to these two, and the least safe diameter of the shaft. The safe stress against shearing is to be taken at 8,000 lbs. per square inch. Prove clearly the formula you employ. (S. & A. Hons. Exam., 1890.)

ANSWER.—Here we have :—

$$\text{B.M.} = 8{,}000 \text{ inch-lbs.}$$

And $$\text{T.M.} = 15{,}000 \quad \text{,,}$$

Hence, by formula (VII) we get :—

$$\text{T.M.}' = 8{,}000 + \sqrt{8{,}000^2 + 15{,}000^2}$$

$$\text{,,} = 25{,}000 \text{ inch-lbs.}$$

To find the diameter of the shaft to withstand this T.M.′ with a shearing stress of not over 8,000 lbs. per square inch, we employ formula (II) making :—

$$\text{T.M.} = \text{T.R.} = \frac{\pi}{16} D^3 f.$$

$$\therefore \qquad D = \sqrt[3]{\frac{\text{T.M.}'}{\frac{\pi}{16} \cdot f}} = \sqrt[3]{\frac{25{,}000}{\frac{3 \cdot 1416}{16} \times 8{,}000}} = 2 \cdot 51 \text{ inches.}$$

Stiffness of Shafts.—Angle of Twist.—We have already seen that the effect of a turning moment applied to a shaft is to twist one part relatively to another. Hitherto we have been dealing only with the resistance the shaft offers to being twisted—that is to say, we have been concerned only with the *strength* of the shaft without regard to the question of *stiffness*. In many cases

—especially in light machinery—the question of the stiffness of the shafting is of greater importance than that of strength.

The stiffness of a shaft is measured by the smallness of the angle of twist per unit length of the shaft.

Turning back to the figure illustrating strain in a shaft, let $d\,l$ be the axial distance, in inches, between the two sections whose diameters are A B, $a\,b$, and let $d\,\theta$ be the circular measure of the angle between those diameters when the shaft is twisted; then the torsional, or shearing *strain* at the surface of the shaft, is

$$= \left(\frac{D}{2}\right) \times \frac{d\,\theta}{d\,l}.$$

D, as before, being the extreme diameter of the shaft in inches,

Let f = Surface stress in the material of the shaft in lbs. per sq. inch.

„ C = Modulus or coefficient of shearing elasticity or of rigidity in lbs. per sq. inch.

Then, since

$$C = \frac{\text{stress}}{\text{strain}} = \frac{f}{\left(\frac{D}{2}\right) \cdot \frac{d\,\theta}{d\,l}}.$$

$$\therefore \qquad d\,\theta = \frac{2\,f}{C\,D} \cdot d\,l.$$

Hence, for a shaft L inches long we have, by a simple integration, the angle of twist.

$$\theta = \frac{2\,f}{C\,D}\int_0^L d\,l = \frac{2\,f\,L}{C\,D}.$$

To express this result in terms of the twisting moment and the diameter of the shaft, we have :—

$$f = \frac{\text{T.M.}}{\frac{\pi}{16}D^3} \text{ for solid shafts.}$$

And,

$$f = \frac{\text{T.M.}}{\frac{\pi}{16}\frac{D^4 - d^4}{D}} \text{ for hollow shafts.}$$

Making these substitutions and simplifying, we get :—

Angle of twist for solid shafts,

$$\left.\begin{aligned} \theta &= \frac{10{\cdot}2\,(\text{T.M.})\,\text{L}}{\text{C}\,\text{D}^4}\,.\ \text{radians.} \\ \text{Or,}\qquad \theta^\circ &= \frac{584\,(\text{T.M.})\,\text{L}}{\text{C}\,\text{D}^4}\,.\ \text{degrees.} \end{aligned}\right\} \quad . \quad . \quad . \quad (\text{VIII})$$

And, for hollow shafts,

$$\left.\begin{aligned} \theta &= \frac{10{\cdot}2\,(\text{T.M.})\,\text{L}}{\text{C}\,(\text{D}^4 - d^4)}\,.\ \text{radians.} \\ \text{Or,}\qquad \theta^\circ &= \frac{584\,(\text{T.M.})\,\text{L}}{\text{C}\,(\text{D}^4 - d^4)}\,.\ \text{degrees.} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (\text{IX})$$

By the equations just established, we see that, while the strength of shafts vary as the *third* power of their diameters, their stiffness varies as the *fourth* power.

Example VI.—Establish a formula for the moment of resistance to torsion of a solid shaft of circular section. The angle of torsion of a shaft is limited to 1° for each 10 feet of length; find the diameter of a solid round shaft to transmit 100 H.P. at 50 revolutions per minute, the modulus of resistance to torsion being 10,000,000 lbs. per sq. inch. (S. & A. Hons. Exam., 1892.)

Answer :—

Here, $\theta = 1^\circ$ when, $\text{L} = 10 \times 12 = 120$ inches

And, $\text{C} = 10{,}000{,}000.$

Also, $\text{T.M.} = 63{,}024 \times \dfrac{\text{H.P.}}{n} = 63{,}024 \times \dfrac{100}{50}$

$\quad$ „ $= 126{,}048$ inch-lbs.

Now, applying formula (VIII) the given conditions are that:—

$$1^\circ = \frac{584 \times 126{,}048 \times 120}{10{,}000{,}000 \times \text{D}^4}$$

Hence, solving for D, we get :—

$$\text{D} = \sqrt[4]{\frac{584 \times 126{,}048 \times 120}{10{,}000{,}000}} = 5{\cdot}45 \text{ inches.}$$

POWER THAT STEEL SHAFTING WILL TRANSMIT AT VARIOUS SPEEDS.

From *The Practical Engineer*, September 2, 1892. By A. G. Brown, M.E.

Revs. per Minute.	Diameters of Shafts in Inches.													
	1½	1¾	2	2¼	2½	3	3½	4	5	6	7	8	9	10
	Horse-Powers They Will Transmit.													
50	3.3	5.3	8.0	10.9	15.6	27	43	64	125	216	343	512	729	1000
60	4.0	6.4	9.6	13.1	18.8	32	51	77	150	259	412	614	875	1200
70	4.7	7.5	11.2	15.2	21.9	38	60	89	175	302	480	717	1021	1400
80	5.4	8.5	12.8	17.4	25.0	43	69	102	200	346	549	819	1166	1600
90	6.0	9.6	14.4	19.6	28.1	49	77	115	225	389	617	922	1312	1800
100	6.7	10.7	16.0	21.8	31.2	54	86	128	250	432	686	1024	1458	2000
110	7.4	11.8	17.6	23.9	34.4	59	94	141	275	475	755	1126	1604	2200
120	8.1	12.9	19.2	26.1	37.5	65	103	154	300	518	823	1229	1750	2400
130	8.7	13.9	20.8	28.3	40.6	70	111	166	325	562	892	1331	1895	2600
140	9.4	15.0	22.4	30.5	43.8	76	120	179	350	605	960	1434	2041	2800
150	10.1	16.1	24.0	32.6	46.9	81	129	192	375	648	1029	1536	2187	3000
160	10.8	17.1	25.6	34.8	50.0	86	137	205	400	691	1097	1638	2333	3200
170	11.5	18.2	27.2	37.0	53.1	92	146	218	425	734	1166	1741	2479	3400
180	12.2	19.3	28.8	39.2	56.3	97	154	230	450	778	1235	1843	2624	3600
190	12.8	20.4	30.4	41.3	59.4	103	163	243	475	821	1303	1945	2770	3800
200	13.5	21.4	32.0	43.5	62.5	108	172	256	500	864	1372	2048	2916	4000
225	15.2	24.1	36.6	49.0	70.3	122	193	288	563	972	1543	2304	3280	4500
250	16.9	26.8	40.0	54.4	78.1	135	214	320	625	1080	1715	2560	3645	5000
275	18.6	29.5	44.0	59.8	85.9	149	236	352	688	1188	1886	2816	4009	5500
300	20.3	32.2	48.0	65.3	93.7	162	257	384	750	1296	2058	3072	4374	6000
325	21.9	34.8	52.0	70.7	101.6	176	279	416	813	1404	2229	3328	4739	6500
350	23.6	37.5	56.0	76.2	109.4	189	300	448	875	1512	2401	3584	5103	7000
375	25.3	40.2	60.0	81.6	117.2	203	322	480	938	1620	2572	3840	5468	7500
400	27.0	42.9	64.0	87.0	125.0	216	343	512	1000	1728	2744	4096	5832	8000
425	28.7	45.6	68.0	92.5	132.8	230	364	544	1063	1836	2915	4352	6197	8500
450	30.4	48.2	72.0	97.9	140.6	243	386	576	1125	1944	3087	4608	6562	9000
475	32.1	50.9	71.0	103.4	148.4	257	407	603	1188	2052	3258	4864	6926	9500
500	33.7	53.6	80.0	108.8	156.2	270	429	640	1250	2160	3430	5120	7290	10000

For power of wrought-iron shafts take 70 per cent. of steel shafts of the same size.

Lecture XXX.—Questions.

1. A 10-inch shaft has a 4-inch hole run through it; what fraction of its weight is removed? To what extent is its strength in resisting torsion affected? *Ans.* 16 per cent ; 2·5 per cent. nearly.

2. A hollow shaft is 10 inches external diameter and 4 inches internal diameter; compare its strength to resist torsion with that of a solid shaft of the same weight.

3. Cylindrical bars of metal, each of 1 inch diameter, are exposed to torsion by weights applied at the end of a 12-inch lever. What would be the probable ultimate strength in the case of good specimens of wrought iron and cast iron State the law according to which the strength of shafting increases by increasing its diameter.

4. If a wrought-iron shaft of 1 inch diameter is broken by the torsion of a load of 800 lbs. acting at the end of a 12-inch lever, find the weight which, when applied to the end of the same lever, would break a shaft of the same material, but 3 inches in diameter. State, in general terms, the reasoning by which you arrive at the result. *Ans.* 21,600 lbs.

5. If a shaft of 3 inches diameter transmits safely 33 horse-power at 100 revolutions er minute, what size of shaft will transmit safely 20 horse-power at 150 revolutions per minute. *Ans.* 2·22 inches.

6. If 800 lbs. at the end of a 12-inch lever be a safe stress to apply to a wrought-iron bar 1 square inch in section, find the effort which a shaft 2 inches in diameter can transmit at the circumference of a pulley one foot in diameter, and making 300 revolutions per minute. Find also the horse-power transmitted. *Ans.* 8,893 lbs.; 254 H.P.

7. A shaft is of given material and given diameter, find an expression for the moment of resistance to torsion. Given the maximum stress to which the material may be subjected, find the diameter of a shaft which will transmit a given horse-power at a given number of revolutions per minute.

8. A twisting moment of 9,600 inch-pounds is sufficient to break a wrought-iron shaft of 1 inch diameter. Use 6 as a factor of safety, and hence determine what horse-power can be safely transmitted through a shaft of 3 inches diameter when running at 120 revolutions per minute. Prove the formula which you employ.

9. Investigate an expression for the moment of resistance to torsion of a given cylindrical shaft when subjected to a given twisting moment. What is the maximum horse-power which could be transmitted by a shaft 3 inches in diameter when making 150 revolutions per minute, it being given that the shearing stress in the material is not to exceed 7,500 lbs. per square inch? *Ans.* 94·5 H.P.

10. If θ be the angle of twist expressed in circular measure in a length of shafting l, M the twisting moment, C the modulus of transverse elasticity, and d the diameter of the shaft, prove that—

$$\theta = \frac{10{\cdot}2\,\mathrm{M}\,l}{\mathrm{C}\,d^4}.$$

11. A horizontal bar of round iron, 1 inch diameter, 6 feet long, hinged at the ends, is subjected to equal and opposite pushing forces of 1,000 lbs. at its ends, and a load of 10 lbs. is hung at the middle so that it is both a beam and a strut. Find the greatest stress anywhere. $\mathrm{E} = 29 \times 10^6$ lbs. per square inch. (Hons. S. & A. Exam., 1897.)

12. Find the inside and outside diameters of a hollow steel shaft, the internal diameter being $\frac{5}{8}$ of the external diameter. The shaft is to transmit 6,000 H.P. at 116 revolutions. Suppose the maximum twisting moment to be 1·3 times its mean value and the maximum stress allowed in the material to be 10,000 lbs. per square inch. Prove the truth of the formula which you use. (S. and A. H., Part I., 1899.)

13. A shaft transmits 35 horse-power at 130 revolutions per minute; what is the twisting moment in pound feet? What is the nature of the strain and stress in the shaft? *Ans.* 1,413 lbs. (B. of E. Adv., 1900.)

14. A wire of Siemens' steel, 0·1 inch diameter, is to be twisted till it breaks. Sketch the arrangement; show how the angle of twist and the twisting moment are measured; how the results may be plotted on squared paper, and the sort of results that may be expected In what way may a wire of twice the diameter be expected to behave? After twisting such a wire much beyond permanent set, suppose the twisting torque to be removed, in what state of internal strain might one expect to find the material? (B. of E. Adv., 1901.)

15. A hollow tube of aluminium bronze, 1 inch diameter inside, $1\frac{1}{4}$ inches diameter outside, is to be twisted till it breaks. How would you arrange the experiment without any special testing machine? Show on squared paper what sort of results you would expect. If the modulus of rigidity of the material is $5{\cdot}3 \times 10^6$ lbs. per square inch, what twisting moment will produce a twist of 0·001 radian per inch? What is now the greatest stress? (B. of E. H., Part I., 1901.)

16. Suppose that a shaft of 1 inch diameter may be safely subjected to a torque of 2,000 pound inches, what torque will a $2\frac{1}{4}$-inch shaft safely resist? Calculate the horse-power which may be transmitted by the latter shaft if its speed is 150 revolutions per minute. How does the shear stress in a circular shaft, subjected to twisting, depend upon distance from the centre? (B. of E. Adv., 1902.)

17. A steel shaft is to be used to transmit power a distance of 75 feet; the twist on the whole length is not to exceed $22\frac{1}{2}°$, nor the stress to exceed 8,500 lbs. per square inch: what must be the diameter of the shaft, and what H.P. can be passed through it at 135 revolutions a minute? (Modulus of transverse elasticity $= 12 \times 10^6$ lbs. per square inch.

(C. & G., 1900, H., Sec. A.)

18. Compare the strength of two cylindrical shafts, subjected to pure torsion, if their diameters are $1\frac{3}{8}$ and $2\frac{1}{4}$ inches respectively. Assuming that the cost of such shafting is directly proportional to its weight, what would be the relative costs? (C. & G., 1901, O., Sec. B.)

19. Obtain a formula for the diameter of a shaft, given the twisting moment to which it is exposed, and the maximum stress allowed in the material:—A shaft 15 inches in diameter is exposed to a twisting moment of 227,800 foot-pounds, and to a maximum bending moment of 43 foot-tons: find (*a*) the maximum stress set up in the material, (*b*) the twisting moment required to produce the same stress if there were no bending moment acting with it. (C. & G., 1901, H., Sec. A.)

20. The two halves of a flange coupling are fastened together by four round wrought-iron bolts. The diameter of the bolt circle is 10 inches, and the shaft transmits 120 H.P. when making 120 revolutions per minute. Assuming a suitable shear stress for the bolts, find their diameter.

(C. & G. 1902, O., Sec. B.)

21. Find the diameter of the propeller shaft for a ship, each engine of which develops 10,000 H.P. when going at 120 revolutions per minute.

You may assume that the shaft is subjected to pure torsion, and that the safe shear stress of the material is 5 tons per square inch.

(C. & G., 1902, O., Sec. B.)

22. The screw shaft of a high-speed vessel is 6 inches external and 3 inches internal diameter, and rotates 400 times a minute. If the intensity of stress is limited to 5 tons per square inch, find the maximum horse-power that can be safely transmitted, the shaft being supposed subjected to twisting only. If the length of the shaft between the thrust block and the screw be 60 feet, find the angle of torsion of the shaft, the modulus of rigidity being 4,500 tons per square i ch. (C. & G , 1902, H., Sec. A.)

23. In an overhanging crank, the crank-arm radius is 16 inches and the distance between the centre of the crank-pin and the centre of the near crank-shaft bearing is 12 inches. When the connecting-rod is at right angles to the crank, the thrust along the rod is 5,000 lbs. Estimate the maximum tensile and shearing stresses in the crank-shaft, the diameter of the crank-shaft being 5 inches. (C. & G., 1902, H., Sec. A.)

Lecture XXX.—A.M.Inst.C.E. Questions.

1. The bolts of a cylinder cover are screwed down until they are subject to an initial stress of, say, 1 ton per square inch. Discuss the question of the effect of steam-pressure in the cylinder on the tension in the bolts. (I.C.E., *Feb.*, 1898.)

2. Find the relative weights of a solid shaft and of a hollow shaft which will transmit a given torque (twisting moment) with the same maximum stress, the external and internal diameters of the hollow shaft being as 3 to 2. Find the ratio of the torsional stiffnesses of the two shafts. (I.C.E., *Feb.*, 1898.)

3. For a shaft subject to torsion and bending deduce a formula for the simple torque that is equivalent—as regards the maximum stress induced—to the given torque and bending moment combined. (I.C.E., *Feb.*, 1898.)

4. Define the terms:—Efficiency of machine, H.P.-hour, torque, moment of momentum, moment of inertia, radius of gyration. If a torque of 0·245 ton-foot is acting on a shaft which makes 100 revolutions per minute, find the H.P. transmitted. (I.C.E., *Feb.*, 1899.)

5. Show that the resistance to twisting of a shaft varies as the cube of the diameter. Compare the strengths of two shafts of the same diameter, one solid, the other hollow of half the weight. (I.C.E., *Oct.*, 1899.)

6. Find a formula for the diameter of a shaft to transmit N horse-power at n revolutions per minute, and state the circumstances on which the value of the coefficient depends. Obtain a numerical result for 2,700 H.P. at 100 revolutions, using such a value of the coefficient as you consider suitable for some specified case. (I.C.E., *Oct.*, 1899.)

7. When a force is applied to the pin of an overhung crank, show that it is equivalent to a couple tending to turn the crank shaft, a couple tending to bend the shaft at a section between the crank and its journal, and a shearing force at the same section. Find their amounts in terms of P the applied force, r the length of crank, and a the distance of section considered from the point of application of P. (I.C.E., *Feb.*, 1900.)

8. Distinguish between "sheering stress" and "normal stress," and find their values at any point of an oblique section (1) of a compressed block, (2) of a tube subjected to torsion. What is the essential difference of these two cases? (I.C.E., *Feb.*, 1900.)

9. Find the relation between the stress on a twisted shaft and the moment producing it. Find the least diameter of a solid steel shaft to transmit 269 H.P. at 150 revolutions per minute, the stress being limited to 9,000 lbs. per square inch. (I.C.E., *Feb.*, 1901.)

10 A steel shaft has to transmit 25 H.P. at 75 revolutions per minute. Of what diameter would you make this shaft? Choose your own safe working-stress. (I.C.E., *Oct.*, 1901.)

11. If the end of a rod of 1 inch diameter is twisted by the turning effort of a force of 80 lbs. acting at the end of a 12-inch lever, find the force which, when applied to the end of the same lever, would twist equally the end of a rod of the same material but of 1½ inch diameter and of half the length. (I.C.E., *Feb.*, 1902.)

12. Deduce an expression for the amount of twist in a shaft under a given twisting moment, and apply your result to find the angle through which a 2½ inch steel shaft will be twisted if it is 80 feet long and subjected at one end to a twisting moment of 19,000 inch-lbs. (I.C.E., *Feb.*, 1902.)

13. A shaft transmits 120 H.P. at 75 revolutions a minute. The diameter of the circle of the bolt centres of the couplings is 10½ inches. Each coupling has 6 bolts, and the shearing stress per square inch allowed in these bolts is 2½ tons. Find the diameter of these bolts.

(I.C.E., *Feb.*, 1902.)

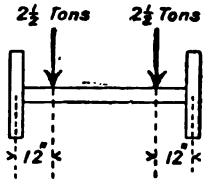
14. Obtain an expression for the work done by a couple. The pitch of a screw propeller is 14 feet and the twisting moment is 120 ton-inches. Find the thrust. (I.C.E., *Oct.*, 1902.)

15. How is the strain caused by shearing stress measured, and what is the modulus of rigidity? If a rod ½ inch in diameter and 20 inches long is fixed at one end and the other end is twisted through an angle of 15° relatively to it, what is the strain in the outer fibres of the rod?

(I.C.E., *Oct.*, 1902.)

16. A bar of iron ½ inch diameter is twisted to destruction; calculate what twisting moment is required for this purpose, assuming that the shearing stress becomes uniform all over the section and equals in the limit 19 tons per square inch. (I.C.E., *Oct.*, 1902.)

17. An axle fixed to two wheels is driven at the centre by a chain and supports two bearings, distant 12 inches from the centre of the wheels, the load on each bearing being 2½ tons. If the power exerted is 20 H.P. and



the rate of turning is 30 revolutions per minute, find the maximum intensity of stress in the axle if its diameter is 5 inches.

(I.C.E., *Oct.*, 1902.)

18. How is the strain measured in the case of a bar in tension or compression; also in the outside fibres of a shaft subject to torsional stress? If the modulus of elasticity is given in the first case, and the modulus of rigidity in the second, how would you find the stresses from the measured strain? (I.C.E., *Feb.*, 1903.)

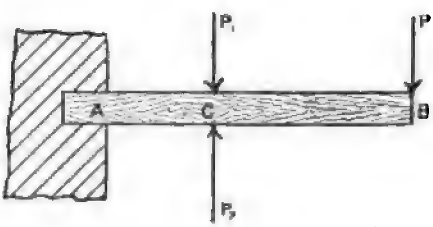
LECTURE XXXI.

CONTENTS.—Strength of Beams and Girders—Definitions of Shearing Force and Bending Moment—Beam Fixed at one end and Loaded at the other —Beam Fixed at one end and Loaded Uniformly—Beam Supported at both ends and Loaded in the middle—Example I.—Beam Supported at ends and Loaded anywhere — Beam Supported at both ends and Loaded Uniformly—Examples II. and III.—Floating Beams—Travelling Loads—Two Loads Moving at a Fixed Distance apart—Example IV.—Distributed Travelling Load – Questions.

Strength of Beams and Girders.—The subject under this heading is one that naturally divides itself into two portions. (1) The determination of the resultant effects of the applied loads at any section of a beam or girder; and (2) the nature and amount of the resistance offered by the beam or girder to rupture at that section.

When the section under consideration is in the same plane as the load, the only effect the load has at that section is a tendency to *shear* the beam; but in the more general case, where the load acts at a distance from the given section, we have, in addition, a tendency to curve or bend the beam at the section. Hence the name *Bending Moment* is given to this latter effect.

In the accompanying figure, let A B represent a cantilever



ILLUSTRATING SHEARING AND BENDING ACTION.

or beam fixed at one end, with a load P applied at the free end; and let C be any section in the beam. At C let there be applied two equal and opposite forces P_1, P_2, of the same magnitude as P. The introduction of these forces does not affect the equilibrium of the system, as P_1 and P_2 balance each other. Hence, the effect of P at the section C is equivalent to that of a

couple $P P_2$, with a single force P_1. A general proof of this important theorem is given in Vol. ~~I., Lecture~~ III., Prop. II. The couple constitutes the **Bending Moment** (B.M.), and the single force P_1, the **Shearing Force** (S.F.) at the section C.

Definition.—**The Shearing Force at any section of a beam is the algebraic sum of all the forces acting on either side of that section.**

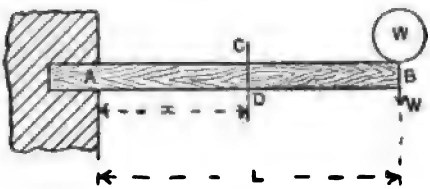
Definition.—**The Bending Moment at any section is the algebraic sum of the moments of all the forces acting on either side of that section.**

Or, in symbols, if P denote any one of the forces acting on one side of a section, and at a distance x, from it; consider all the forces on the same side of the section as P, paying due regard to their sign—that is, if we reckon forces acting upwards as positive, we must regard those acting downwards as negative.

$$\left.\begin{array}{lr} \text{Then,} & \text{S.F.} = \Sigma P. \\ \text{And,} & \text{B.M.} = \Sigma P x. \end{array}\right\} \quad \ldots \ldots \ldots \quad \text{(I)}$$

Beam Fixed at one end and Loaded at the other.—Let C D be a cross-section anywhere within the length of the beam at a distance of x inches from the fixed end A. To find the S.F. and B.M. at C D, we observe that the only force acting to the right of the section is W lbs. Hence:—

$$\text{S.F.} = W \text{ lbs.} \quad \ldots \ldots \ldots \quad \text{(II)}$$



Beam Fixed at one end, Loaded at other.

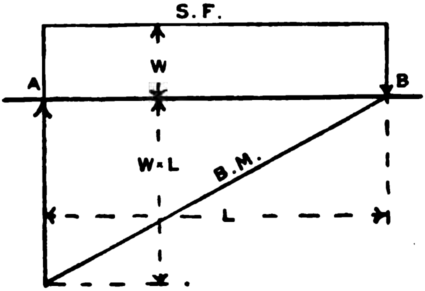
It is independent of x, and therefore the same for all such sections as C D.

The B.M. at C D is W multiplied by its distance from the section in inches. Hence:—

$$\text{B.M.} = W \times BD = W(L - x) \text{ inch-lbs.} \quad \ldots \quad \text{(III)}$$

This equation is true whatever may be the position of W on the beam, so long as L denotes its distance in inches from the fixed end, and C D is between W and the support.

In this case, the diagram of the S.F. is a straight line parallel to the base and at a distance of W lbs. from it. Since (III) is the equation of a straight line, the B.M. is therefore a quantity increasing uniformly from zero, where $x = L$, to W L inch-lbs., where $x = 0$, as shown by the accompanying figure.



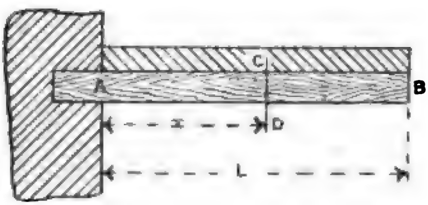
S.F. curve is a straight line.

B.M. curve is a straight line.

DIAGRAM OF S.F. AND B.M. FOR BEAM FIXED AT ONE END AND LOADED AT THE OTHER.

Beam Fixed at one end, and Loaded Uniformly.—Let the load on the beam be w lbs. per inch-run, it is required to find the shearing force and bending moment at any section C D, at x inches from the fixed end. As before, consider the part of the beam to the right of C D. The only force is the weight of that portion of the load carried by B D, so that :—

$$\text{S.F.} = w \times BD = w(L - x) \text{ lbs.} \quad . \quad . \quad . \quad \text{(IV)}$$



BEAM FIXED AT ONE END AND LOADED UNIFORMLY.

The moment of the portion of the load on B D with respect to C D is the same as if it were all concentrated at the middle point of B D. Hence :—

$$\text{B.M.} = w \times BD \times \tfrac{1}{2} BD = \tfrac{1}{2} w \times BD^2 = \tfrac{1}{2} w (L - x)^2 \text{ inch-lbs.} \quad \text{(V)}$$

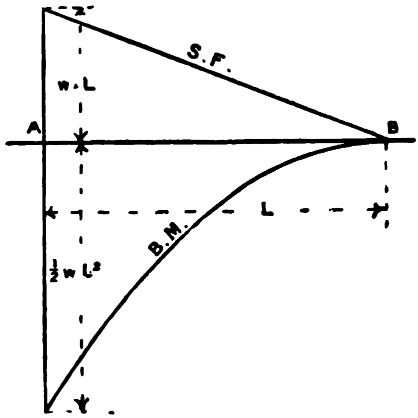
Equations (IV) and (V) show us that both the S.F. and B.M. vanish when $x = L$; and when $x = 0$, we get :—

$$\text{S.F.} = w \text{ L lbs.} \qquad (\text{IV}_a)$$

And,

$$\text{B.M.} = \tfrac{1}{2} w \text{ L}^2 \text{ inch-lbs.} \qquad (\text{V}_a)$$

The diagrams of S.F. and B.M. for this case take the forms shown in the accompanying figure.



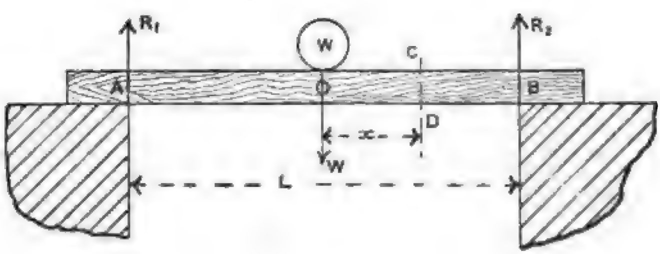
S.F. curve is a straight line.

B.M. curve is a parabola with vertex at B.

DIAGRAM OF S.F. AND B.M. FOR BEAM FIXED AT ONE END AND LOADED UNIFORMLY.

Beam Supported at both ends, and Loaded at the Middle.—In this case we measure x from the middle point of the beam. Since W is equidistant from A and B, the reactions at those points, R_1 and R_2, are equal to each other, and since their sum is W, we have:—

$$R_1 = R_2 = \tfrac{1}{2} \text{W lbs.}$$



BEAM SUPPORTED AT BOTH ENDS AND LOADED AT MIDDLE.

The only force to the right of C D is R_2, and its leverage is B D.

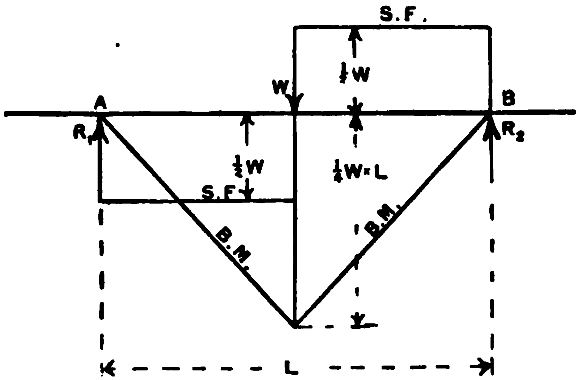
Hence, $\text{S.F.} = R_2 = \frac{1}{2} W$ lbs. . . . . (VI)

And, $\text{B.M.} = R_2 \times BD = \frac{1}{2} W (\frac{1}{2} L - x)$ inch-lbs. . (VII)

Here, the **B.M.** vanishes when $x = \frac{1}{2}$ L, and increases uniformly from this until $x = 0$, when it attains its maximum value, $\frac{1}{4}$ W L inch-lbs.

Or, **Maximum B.M.** $= \frac{1}{4}$ **W L inch-lbs.** . . . (VII_a)

The following figure shows the diagrams of **S.F.** and **B.M.** for this case :—



S.F. curve is a pair of straight lines.

B.M. curve is a pair of straight lines.

DIAGRAM OF S.F. AND B.M. FOR BEAM SUPPORTED AT BOTH ENDS AND LOADED IN MIDDLE.

EXAMPLE I.—In a beam of length L, supported at both ends and loaded at the middle with a load W, show that the bending moment is greatest at the centre of the beam and equal to $\frac{1}{4}$ W L. Then determine graphically the bending moment and shearing force at a point 6 ft. from one support in a beam of 25 ft. span loaded with 5 tons at its centre. (Adv. S. & A. Exam., 1890.)

ANSWER.—We have already seen from equation (VII) that for a beam loaded as in this example, the **B.M.** at any distance x, from its middle point, is :—

$$\text{B.M.} = \tfrac{1}{2} W (\tfrac{1}{2} L - x).$$

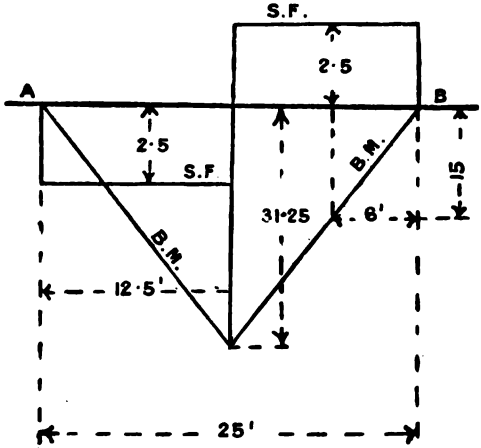
This is obviously greatest when $x = 0$—that is, at the centre. Then :—

$$\text{Maximum B.M.} = \tfrac{1}{4} W L ; \text{ and S.F.} = \tfrac{1}{2} W.$$

For the values of W and L given in the example, we get :—

$$\text{Maximum B.M.} = \tfrac{1}{4} \times 5 \times 25 = 31{\cdot}25 \text{ ft.-tons.}$$

And, $$\text{S.F.} = \tfrac{1}{2} \times 5 = 2{\cdot}5 \text{ tons.}$$



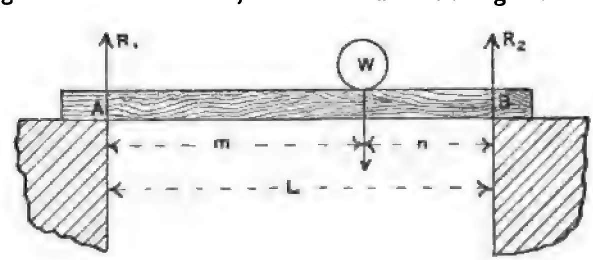
S.F. AND B.M. CURVES FOR EXAMPLE I.

The accompanying figure shows the diagrams of B.M. and S.F. as constructed from these data.

At 6 feet from one end the B.M. measures 15 ft.-tons. This is easily verified by means of the formula for B.M., because $x = 12{\cdot}5 - 6 = 6{\cdot}5$.

$$\therefore \qquad \text{B.M.} = \tfrac{1}{2} \times 5 \times (12{\cdot}5 - 6{\cdot}5) = 15 \text{ ft.-tons.}$$

Beam Supported at both ends, and Loaded Anywhere.—With a single concentrated load, the maximum bending moment will



BEAM SUPPORTED AT BOTH ENDS, AND LOADED ANYWHERE.

always occur immediately under the load, whether it be at the middle of the beam or not.

For the B.M. at any section at a distance x, from one end is $R \times x$, and this is greatest when x is largest; that is, when the section is under the load.

To find the reactions at the supports, we take moments about A and B, and get $R_2 \times L = W \times m$.

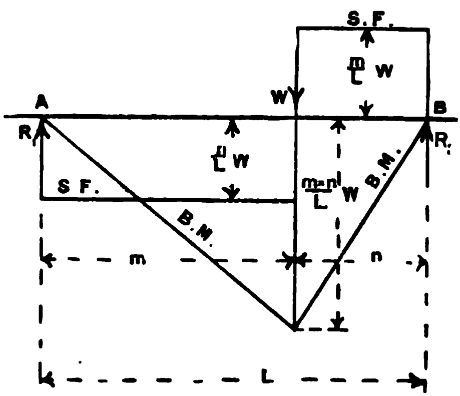
$\therefore R_2 = \frac{m}{L} W$ lbs. and $R_1 = \frac{n}{L} W$ lbs. These are the values of the S.F. to the right and left of W respectively.

$$\left.\begin{aligned} \text{S.F. (to right)} &= \frac{m}{L} W \text{ lbs.} \\ \text{S.F. (to left)} &= \frac{n}{L} W \text{ lbs.} \end{aligned}\right\} \quad . \quad . \quad . \quad \text{(VIII)}$$

Multiplying the first of these equations by n, or the latter by m, we get :—

$$\text{Maximum B.M.} = \left(\frac{m\,n}{L} W\right) \text{ inch-lbs.} \quad . \quad . \quad . \quad \text{(IX)}$$

We can now construct the diagrams of S.F. and B.M. :—



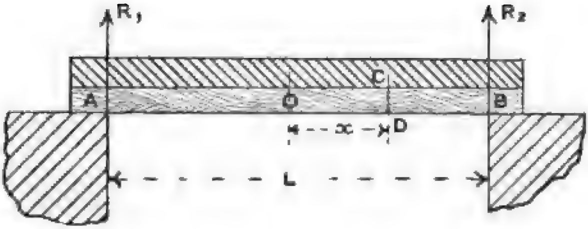
S.F. curve is two straight lines.

B.M. curve is two straight lines.

DIAGRAM OF S.F. AND B.M. FOR SINGLE LOAD IN ANY POSITION.

Beam Supported at both ends and Loaded Uniformly.—As before, let the weight per inch-run be denoted by w, then the total load carried by the beam will be w L lbs., and the reactions

R_1 and R_2, will each be $\frac{1}{2} w L$ lbs. Taking the forces to the right of the section C D.



BEAM SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

We get, S.F. $= R_2 - w \times BD = \frac{1}{2} w L - w(\frac{1}{2}L - x) = w\, x$ lbs. (X)

And, B.M. $= R_2 \times BD - w \cdot BD \times \frac{1}{2} BD$

„ $= \frac{1}{2} w L \times BD - \frac{1}{2} w \cdot BD^2$

„ $= \frac{1}{2} w \cdot BD(L - BD)$

„ $= \frac{1}{2} w (\frac{1}{2} L - x)(\frac{1}{2} L + x)$.

$\therefore$ B.M. $= \frac{1}{2} w (\frac{1}{4} L^2 - x^2)$ inch-lbs. . . . . . (XI)

The limiting values of S.F. and B.M. are :—

When, $x = \frac{1}{2} L$; then, S.F. $= \frac{1}{2} w L$ lbs.; and, B.M. $= 0$. (X_a)

When, $x = 0$; then, S. F. $= 0$; and :—

Maximum, B. M. $= \frac{1}{8} w L^2$ inch-lbs. . . (XI_a)

Plotting our diagrams of S.F. and B.M., we get the figure shown on next page.

When a beam carries more than one load, or is loaded in more ways than one, the simplest and safest way is to consider each load separately, without regard to the others, and then combine the separate effects so as to obtain the resultant action, as in Example II.

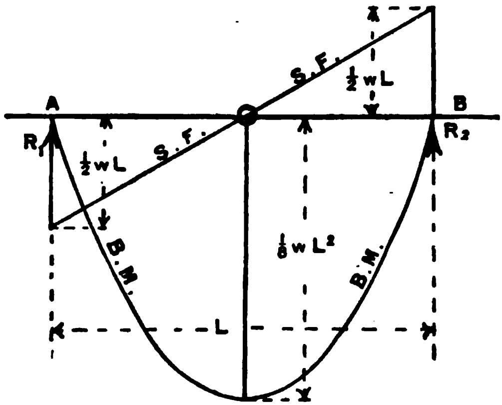
EXAMPLE II.—Draw the bending moment and shearing force diagrams for a beam 12 feet long, supported at both ends, and loaded with weights of 4 and 6 tons at distances of 3 and 8 feet respectively, from one end of the beam. Explain fully the mode of arriving at these diagrams. (Adv. S. & A. Exam., 1887.)

ANSWER.—Measuring distances from the left end of the beam, and considering each load separately, we have, for the 4 tons, to the left of the load :—

$$S.F._1 = \frac{n}{L} W = \frac{9}{12} \times 4 = 3 \text{ tons.}$$

And, to the right of it :—

$$S.F._1 = \frac{m}{L} W = \frac{3}{12} \times 4 = 1 \text{ ton.}$$



S.F. curve is a straight line.

B.M. curve is a parabola with vertex below the middle of the beam.

DIAGRAM OF S.F. AND B.M. FOR A BEAM SUPPORTED AT BOTH ENDS AND LOADED UNIFORMLY.

The maximum $B.M._1$ due to this load is :—

$$B.M._1 = \frac{m \times n}{L} W = \frac{3 \times 9}{12} \times 4 = 9 \text{ ft.-tons.}$$

It occurs immediately under the load.

Next taking the 6-tons load, we have to the left of it :—

$$S.F._2 = \frac{n}{L} W = \frac{4}{12} \times 6 = 2 \text{ tons;}$$

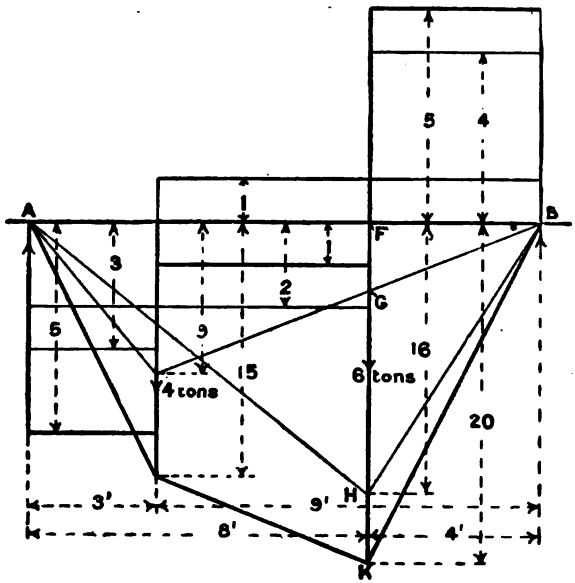
And to the right of it :—

$$S.F._2 = \frac{m}{L} W = \frac{8}{12} \times 6 = 4 \text{ tons.}$$

The maximum $B.M._2$ due to the 6 tons is :—

$$B.M._2 = \frac{m \times n}{L} W = \frac{8 \times 4}{12} \times 6 = 16 \text{ ft.-tons.}$$

Plotting these results, we get the accompanying figure :—



S.F. AND B.M. CURVES FOR EXAMPLE II.

The thin lines show the actions of the separate loads, and the full lines their combined results, obtained by taking the algebraic sum of the former.

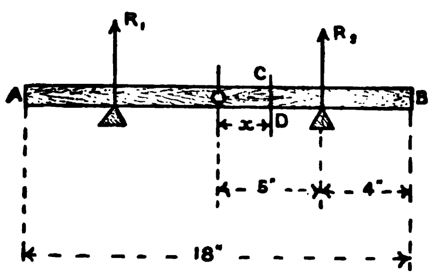
The student should here carefully observe the necessity of attending to the *sign* of the shearing force. Thus, between the weights we have a shearing force of 2 tons, which, on account of its sign, is drawn below the base line; also a shearing force of 1 ton drawn above the base line. The resultant shearing force between the loads is therefore the difference of these, and is drawn on the same side of the base line as the greater of its components.

The bending moments everywhere along the beam are of the same sign; therefore, to obtain the combined bending moment diagram, we have simply to add the ordinates of each separate diagram. Thus, to get the total bending moment at the section under the 6 tons load, we add F G (viz., that due to the 4 tons at that point) to F H (that due to the 6 tons). The result F K is therefore the total B.M. at that point.

It is quite sufficient to do this for the sections under each load, and then to join each of the points so obtained with each other and with the ends of the beam by straight lines. If drawn to scale, the B.M. at any other point can then be obtained by measuring the corresponding ordinate.

EXAMPLE III.—A horizontal uniform bar, 18 inches long, is laid over two supports, each 4 inches from its ends. Find two points at which the bending moments are zero. (Hons. S. & A. Exam.)

ANSWER.—Let w be the weight in lbs. per inch-run of the bar. Then the total weight of the bar will be $18\,w$ lbs., and the reactions will each be $9\,w$ lbs.



ILLUSTRATING EXAMPLE III.

Taking moments to the right of the section C D at a distance x inches from the centre of the bar, we get:—

$$\begin{aligned} \text{B.M.} &= R_2(5 - x) - w\,.\,BD \times \tfrac{1}{2} BD \\ ,, &= 9\,w(5 - x) - \tfrac{1}{2} w\,.\,BD^2 \\ ,, &= 9\,w(5 - x) - \tfrac{1}{2} w(9 - x)^2 \\ ,, &= \tfrac{1}{2} w(9 - x^2) \text{ inch-lbs.} \end{aligned}$$

The B.M. will be zero when $9 - x^2 = 0$; *i.e.*, when $x = \pm 3$ inches.

Hence, the required points are 3 inches on each side of the centre, or 2 inches inside of the supports.

Floating Beams.—When a *solid* body, such as a piece of wood of *uniform density*, floats in *still* water its weight and its buoyancy, or the resultant upward pressure of the water on the body, will at all points balance each other. There are consequently no shearing or bending stresses on the body, and each part is in equilibrium independently of the other parts.

But whenever those conditions are departed from, such as (1) when the floating body carries weights; (2) when it is not of uniform density, due to want of homogeneity in its material, if solid, or to its being hollow, or of a boat form; or (3) when it crosses waves, then bending and shearing stresses are set up.

Consider the case of a uniform beam of wood of rectangular section floating in still water. The beam will displace an amount of water exactly equal to its own weight. This is true, not only for the beam as a whole, but also for every individual segment of the beam. Any segment of the beam will displace just as much, and no more, water than it would do if floating by itself. The beam, therefore, is as free from stress as it would be if it were lying on a perfectly flat surface.

Suppose now that a weight W, be placed on the middle of a floating beam. This will cause the beam to sink to a greater depth and displace an *extra* volume of water. The *weight* of this *extra* displacement is exactly equal to W. What, now, is the condition of the beam as regards straining forces? Evidently, we need only consider the weight W, and the extra displacement, due to its being carried by the beam; because the upward reaction of the displacement due to the beam's own weight, is still at all points balanced by the downward weight of the beam. In other words, the condition of the beam, so far as its own weight and displacement are concerned, is in no way affected by the addition of the load.

To give definiteness to our ideas, let W be expressed in lbs., and let L denote the length of the beam in inches.

Then the forces we have to consider are:—

(1) W lbs. concentrated at the middle of the beam and acting downwards.

(2) The displacement of W lbs. of water uniformly distributed along the whole length of the beam and acting upwards, with an intensity of $\frac{W}{L}$ lbs. per inch of length.

The case is, therefore, analogous to that of a beam uniformly loaded and supported at its centre; or what is virtually the same thing, two beams of length equal to $\frac{1}{2}$ L, fixed at one end and loaded uniformly. For, in order to obtain the shearing force and the bending moment at any section of the beam, x inches to either side of W, we have simply to substitute $\frac{W}{L}$ for w, and $\frac{1}{2}$ L for L in equations (IV) and (V), and we get:—

$$\text{S.F.} = \frac{W}{L}\left(\tfrac{1}{2}L - x\right) \text{ lbs.} \quad . \quad . \quad . \quad . \qquad \text{(XII)}$$

And, $$\text{B.M.} = \frac{W}{2L}\left(\tfrac{1}{2}L - x\right)^2 \text{ inch-lbs.} \quad . \quad . \quad . \quad (\text{XIII})$$

Under W the shearing force and bending moment are each a maximum. Their values may be found by making $x = 0$.

Then the $$\text{Maximum S.F.} = \tfrac{1}{2}W \text{ lbs.} \quad . \quad . \quad . \quad (\text{XII}_a)$$

And the $$\text{Maximum B.M.} = \tfrac{1}{8}WL \text{ inch-lbs.} \quad . \quad . \quad (\text{XIII}_a)$$

The diagrams of S.F. and B.M. for this case are constructed in identically the same way as for a beam fixed at one end and carrying a uniform load, but taking $\frac{1}{2}$ L as a base line instead of L.

Suppose that, instead of one weight in the middle, the beam is loaded with two weights, one at each end, and each equal to W lbs., it is easy to see that the condition now is that of a beam uniformly loaded with $\frac{2W}{L}$ lbs. per inch-run and supported at each end. We have, therefore, only to apply formulæ (X) and (XI), substituting $\frac{2W}{L}$ for w, when we get:—

$$\text{S.F.} = \frac{2W}{L} \cdot x \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{XIV})$$

And, $$\text{B.M.} = \frac{W}{L}\left(\tfrac{1}{4}L^2 - x^2\right) \text{ inch-lbs.} \quad . \quad . \quad . \quad . \quad (\text{XV})$$

Here the shearing force is a maximum when $x = \frac{1}{2}$ L, and the bending moment a maximum when $x = 0$.

Or, $$\text{Maximum S.F.} = W \text{ lbs.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{XIV}_a)$$

And, $$\text{Maximum B.M.} = \tfrac{1}{4}WL \text{ inch-lbs.} \quad . \quad . \quad (\text{XV}_a)$$

The diagrams of S.F. and B.M. are, therefore, in every way similar to those for a uniformly loaded beam supported at the ends.

Travelling Loads.—The simplest case of a movable load is that, wherein we are given a weight, say a heavy cylindrical body, rolling along a beam, to find the equations of maximum S.F. and B.M. for any position of the load, and exhibit these results in a diagram.

Referring to formulæ (VIII) and (IX), and the diagrams already deduced for a fixed load in *any* position on a beam, we

have for the maximum S.F. to the immediate right of the load :—

$$\text{S.F.} = \frac{m}{\text{L}} \text{W lbs.}$$

And, to the immediate left of the load :—

$$\text{S.F.} = -\frac{n}{\text{L}} \text{W lbs.}$$

For the maximum B.M., which occurs immediately under the load :—

$$\text{B.M.} = \frac{m\,n}{\text{L}} \text{W inch-lbs.}$$

Putting $m = x$ so that $n = \text{L} - x$, we obtain, when the load is x inches from the left end of the beam :—

$$\left.\begin{aligned} &\textbf{The Maximum S.F.}\ (\textit{just to right of the section}) = \frac{\text{W}}{\text{L}}x \\ &\quad ,, \qquad ,, \qquad (\textit{just to left of the section}) = \frac{\text{W}}{\text{L}}(x - \text{L}) \end{aligned}\right\} \text{(XVI)}$$

$$\textbf{And,} \qquad \textbf{Maximum B.M.} = \frac{\text{W}}{\text{L}}(\text{L} - x)x \; . \; . \; . \; . \; . \qquad \text{(XVII)}$$

To construct the diagram of S.F., we observe that its equation is that of a straight line, and that to the right of the section considered its value is zero when the load just starts from the left end of the beam, and increases uniformly as the load approaches the other end. That is :—

When, $\quad x = 0$; then, S.F. $= 0$.

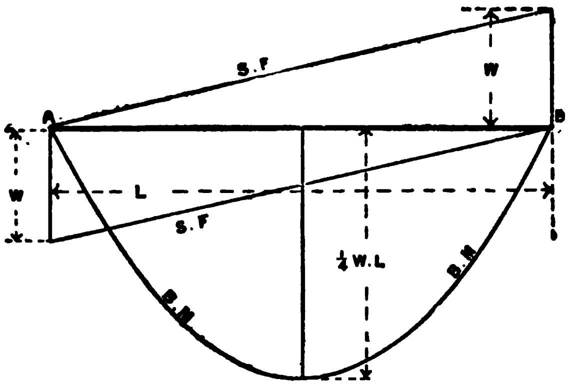
Also when, $\quad x = \text{L}$; then, S.F. $=$ W lbs.

There is also another line for the shear at all positions just to the left of the load. This line passes through B, and its ordinate is $-$W at the end A.

The equation of the B.M. curve is that of a parabola, whose axis is vertical, and passes through the middle point of the beam, where, of course, the maximum value of B.M. occurs. To construct this diagram, we have :—

When, $\quad x = 0$, or $x = \text{L}$; then B.M. $= 0$.

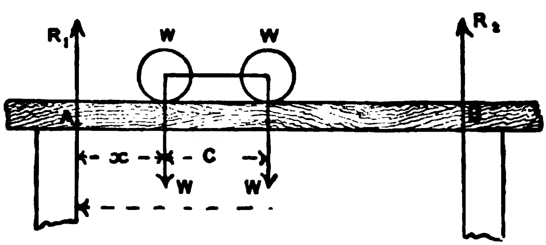
Also when, $\quad x = \frac{1}{2}\text{L}$; then B.M. $= \frac{1}{4}$ W L inch-lbs.



DIAGRAMS OF MAXIMUM S.F. AND B.M. FOR ROLLING LOAD.

Two Loads moving at a fixed distance apart.—From the above simple case we may easily pass to a very important practical example of moving loads—viz., overhead travelling cranes.

Here the crane rests on a carriage with four wheels running on two rails carried by girders, the weight of the whole machine together with the load being equally distributed over the wheels. Hence, considering one girder only, our problem is reduced to that of two equal loads moving along the girder at a fixed distance apart.



ILLUSTRATING TRAVELLING CRANE PROBLEM.

In the figure let W be the weight resting on each wheel, and c be the distance between their centres. If the motion be supposed to be in the direction shown by the arrow, it is evident that until the carriage gets to the middle of the girder the maximum

shearing force and bending moment will occur under the leading wheel—that is, if we estimate the shearing force to the immediate right of the wheel. But as the same thing takes place in the reverse order when the carriage moves from the other end of the girder in the opposite direction, we shall take the section of the girder immediately under the following wheel and estimate the shearing force and bending moment for that position. This method of procedure will be found to lead to simpler equations than if we had taken the leading wheel as our point of reference.

Now, considering the forces acting to the left of the wheel, we easily see :—

That, $\quad$ S.F. $= R_1$.

And, $\quad$ B.M. $= R_1 \times x$.

To find R_1 we take moments about B, which gives us :—

$$R_1 \times L = W\{L - (x + c)\} + W(L - x)$$

$$\text{,,} \quad \text{,,} = W\{2(L - x) - c\}.$$

$$\therefore \quad R_1 = \frac{W}{L}\{2(L - x) - c\}.$$

Hence, $\quad$ $$\text{S.F.} = \frac{W}{L}\{2(L - x) - c\}. \quad . \quad . \quad . \quad \text{(XVIII)}$$

And, $\quad$ $$\text{B.M.} = \frac{W}{L}\{2(L - x) - c\}x. \quad . \quad . \quad . \quad \text{(XIX)}$$

The equation for the S.F. is that of a straight line, and for the B.M. a parabola. To find the position and dimensions of those diagrams, we see that :—

When $\quad x = 0$, S.F. $= \frac{W}{L}(2L - c)$, and B.M. $= 0$.

Again, both S.F. and B.M. will vanish when $2(L - x) - c = 0$; that is, when $x = L - \frac{c}{2}$.

To find the maximum ordinate of the B.M. curve, we have the condition that, when the B.M. is a maximum :—

$$\frac{d}{dx}(\text{B.M.}) = 0.$$

That is, $\quad$ $$\frac{d}{dx}\{2(L - x) - c\}x = 0.$$

Or,
$$\frac{d}{dx}\{(2\,L-c)x-2x^2\}=0.$$
$$(2\,L-c)-4\,x=0.$$

Hence,
$$x=\frac{L}{2}-\frac{c}{4}.$$

The shearing force diagram will, therefore, consist of two straight lines parallel to each other, and the bending moment diagram will consist of two equal parabolas intersecting at the middle of the girder. The axes of these equal parabolas will be equidistant from the middle of the girder and $\frac{1}{2}$ c units apart.

The following numerical example will elucidate this important case much better than a bare examination of formulæ:—

EXAMPLE IV.—In a travelling crane of 40 feet span the load is supported on a carriage which runs upon two similar girders, the axles of the carriage being 8 feet apart, and a load of $2\frac{1}{2}$ tons coming upon each wheel. Obtain a diagram showing the maximum bending moment at every section of the girder, and give the numerical values at distances of 10, 15, and 20 feet from one end. (Hons. S. & A. Exam., 1880.)

ANSWER.—Applying our general formulæ, we have, for the bending moment at any distance x ft., from one end:—

$$\text{B.M.}=\frac{W}{L}\left[(2\,L-c)-2\,x\right]x.$$

Here, $W=2\cdot5$ tons, $L=40$ ft., and $c=8$ ft.

$$\therefore \qquad \text{B.M.}=\frac{2\cdot5}{40}\left[80-8-2\,x\right]x.$$

Or,
$$\text{B.M.}=\tfrac{1}{8}\,(36-x)\,x \text{ ft.-tons.}$$

For the numerical values asked for, we have:—

When $x=10$ ft.; B.M. $=\frac{1}{8}\,(36-10)\,10=$ **32·5 ft.-tons.**

When $x=15$ ft.; B.M. $=\frac{1}{8}\,(36-15)\,15=$ **39·375 ft.-tons.**

When $x=20$ ft.; B.M. $=\frac{1}{8}\,(36-20)\,20=$ **40 ft.-tons.**

We have seen, that the **B.M.** attains its maximum value when:—

$$x=\frac{L}{2}-\frac{c}{4}=\frac{40}{2}-\frac{8}{4}=18\text{ ft.}$$

Hence, the maximum B.M. $=\frac{1}{8}\,(36-18)\,18=$ **40·5 ft.-tons.**

The S.F. is not asked for in the question, but we here add it so as to make the example more complete.

The maximum S.F. occurs when $x = 0$, and when $x = L$, its value for this case then being :—

$$\text{Maximum S.F.} = \frac{2 \cdot 5}{40}(80 - 8) = 4 \cdot 5 \text{ tons.}$$

And, like the B.M., it is zero when $x = 40 - \frac{8}{2} = 36$ feet.

The following figure shows the S.F. and B.M. diagrams as required for this example.

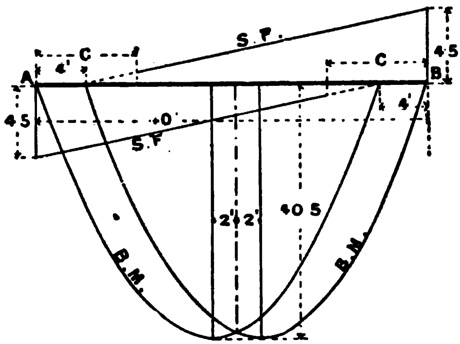
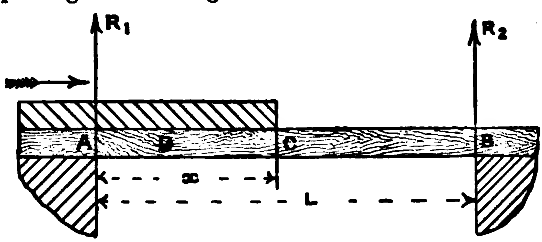


Diagram of Maximum S.F. and B.M. for a Travelling Crane.

Distributed Travelling Load.—The last case we shall consider is that in which a continuous load of uniform density, and long enough to completely cover it, comes on to a girder and moves off at the other end, such as a long train of uniform weight passing over a bridge.



Illustrating Travelling Load of Uniform Intensity.

In the figure, let w denote the load per unit of length. When the load is in the position shown, it is clear that the S.F., at all points to the right of C, will be equal to R_2; and that at any section D, to the left of C, the S.F. will be less than R_2 by the weight of the portion of the load covering C D. It at once

follows that the S.F. is greatest at C, the front of the load, and this is true for all positions of C.

Hence, $\quad$ S.F. $= R_2$.

Taking moments about A, we have :—

$$R_2 \times L = wx \times \tfrac{1}{2}x.$$

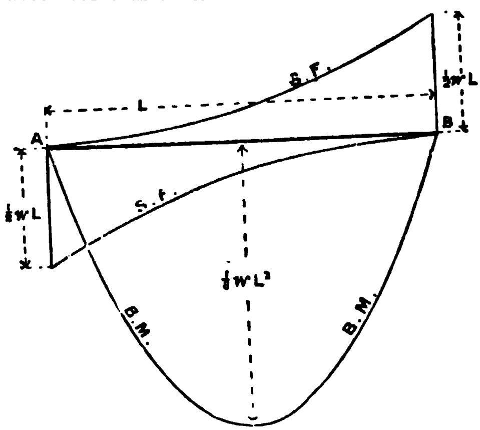
$$\therefore \qquad R_2 = \frac{wx^2}{2L}.$$

That is, $\quad$ S.F. $= \dfrac{wx^2}{2L}$ . . . . (XX)

The shearing force is, therefore, proportional to the square of the length of the part of the load resting on the girder.

The curve of the maximum bending moment is very easily deduced in this case. We have only to remember that the B.M. at *any fixed* section in the girder will get greater and greater for every additional part of the load that comes upon it; so that when the girder is wholly covered by the load the B.M. at every position will then be a maximum. The B.M. diagram is therefore identical with that given for a beam loaded uniformly, whilst the S.F. diagram becomes a parabola instead of a straight line.

The following figure shows how the S.F. and B.M. diagrams are constructed for this case.



S.F. and B.M. Diagrams for Travelling Continuous Load of Uniform Intensity.

Lecture XXXI.—Questions.

1. Define "*bending moment*" and "*shearing force.*" A uniform beam weighing 15 cwts. rests on supports at its ends 20 feet apart. The beam is loaded with three weights of 4, 6, and 10 cwts. at distances of 2, 7, and 12 feet respectively from one of the supports. Find the B.M. and S.F. at a point 8 feet from the same support. *Ans.* B.M. = 98 ft.-cwts.; S.F. = 3 cwts.

2. A bar of pine 44 inches long rests on props at its extremities, and just supports 10 weights, of 14 lbs. each, hung at equal intervals of 4 inches along the rod. Find the amount of a single weight, which, if hung at the centre of the bar, would strain it to the same extent. *Ans.* 43·27 lbs.

3. A batten of fir, 6 feet in length and supported at its extremities, will just sustain a load of 520 lbs. when hung at the centre. If this weight be removed, and two weights, each equal to P lbs., be hung at distances of 2 and 4 feet along the bar, what is the greatest value which may be assigned to P? *Ans.* 390 lbs.

4. A beam, 20 feet long, whose weight is neglected, is supported at both ends and loaded with 1 ton evenly distributed along its length. Find the bending moment at a distance of 7 feet from one end. *Ans.* 5,096 ft.-lbs.

5. A beam, whose weight may be neglected, rests on supports at its ends 15 feet apart. Weights of 10, 6, 5, and 12 cwts. rest on the beam at intervals of 3 feet apart, the weight of 10 cwts. being 3 feet from one support. Find the points where the maximum bending moment and shearing force occur, and obtain their values. Construct the diagrams of bending moments and shearing force for the whole beam. *Ans.* The *max.* B.M. = 66 ft.-cwts., and occurs at all points between the weights 6 and 5 cwts.; the *max.* S.F. = 17 cwts., and occurs at the point where the weight of 12 cwts. rests.

6. A uniform cantilever, or beam fixed at one end and free at the other, 10 feet long, weighs 6 cwts., and carries two loads, one of 2 cwts. at the free end, and the other 4 cwts. at its middle point. Construct the shearing force diagram for the whole cantilever, and find the shearing forces at points $2\frac{1}{2}$ feet and 6 feet from fixed end. *Ans.* 10·5 cwts.; 4·4 cwts.

7. A block of wood weighing 800 lbs., 20 feet long and 12 inches square, floats in water, and is loaded—

(1) By a weight of 200 lbs., placed at each extremity;
(2) By a weight of 400 lbs. at the centre.

Show what forces act on the beam, and draw the curves of shearing force and bending moment for each case.

8. A girder is supported at both ends, and has a clear span of 30 feet. Show by means of a curve the position and magnitude of the greatest bending moment produced by a load of 20 tons as it rolls from one end to the other of the girder. Obtain the numerical results for distances respectively of 10 and 15 feet from one end. (S. & A. Hons. Exam., 1895.)

9. Prove an algebraic formula to show that, with a continuous load of uniform intensity passing over a beam A B such as when a long train passes over a bridge A to B, the maximum shearing stress to any point K of the beam occurs when the part A K is fully loaded while the part

K B is entirely unloaded, and that the magnitude of the stress is proportional to the square of the distance of K from the point A. A train of 1 ton per foot run, and upwards of 100 feet in length, passes over a bridge of 100 feet span; what would be the maximum shearing stresses at distances of 25 and 50 feet respectively from one end of the bridge? Show how to determine graphically the shearing stresses in the beam. (S. & A. Hons. Exam., 1896.)

10. What occurs at the cross-section of a horizontal beam, carrying vertical loads? Where is the neutral line? What is the value of the stress at any place? What is meant by *bending moment?* Describe any model which illustrates, however roughly, what occurs at a section of the beam. (Adv. S. & A. Exam., 1898.)

11. Prove the ordinary formula for the deflection of a beam of length l, supported at its ends and loaded in the middle with a load W. A beam of uniform rectangular section supported at the ends is 20 feet between the supports; what should be its depth in order that the deflection may not exceed 25 inch under a maximum stress of 8,000 lbs. per square inch of section? $E = 28 \times 10^6$ lbs. per square inch. (S. and A. H., Part I., 1899.)

12. A symmetrically loaded beam of uniform section; given the diagram of bending moment when supported at its ends, what is the easy rule for obtaining the diagram when the beam is fixed at the ends? Prove the rule to be correct. (S. and A. H., Part I., 1899.)

13. A wall has to be carried over a gap of 16 feet, and it is proposed to use a rolled steel joist for the purpose. What size of joist would you adopt from the following data: (*a*) the weight of wall is equivalent to a uniformly distributed load of 2,600 lbs. per foot run of joist; (*b*) the greatest intensity of stress per square inch is not to exceed 5 tons? (C. & G., 1900, H., Sec. A.)

14. Show how to obtain, and sketch the diagrams of maximum possible bending moment and maximum possible shear for a uniform rolling load of a given amount per foot run, as it passes over a girder of given span.
(C. & G., 1900, H., Sec. A.)

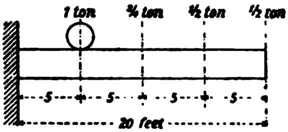
15. A girder of 22 feet span is supported at the two ends, a load of 10 tons rests on a point 2 feet from the left end, and two other loads of 6 and 7 tons respectively, at distances of 7 and 13 feet respectively from the first load: find the bending moment in inch tons under each load, and also the shearing force. (C. & G., 1900, O., Sec. B.)

16. A steel joist is used as a girder on a span of 17 feet 6 inches, and is freely supported at the two ends. It carries a uniformly distributed load of 18 cwts. per foot run, and two concentrated loads, one of 3 tons 3 feet from the left-hand support, and another of $1\frac{3}{4}$ tons 10 feet from the same end. Find the bending moments in inch-pounds under each of the concentrated loads, and also at the centre of the girder, and also the shearing forces in pounds at each of these points. Sketch the bending moment diagram for the whole girder. (C. & G., 1901, O., Sec. B.)

17. A girder, supported at the two ends, is 10 feet long, and is loaded uniformly with a load of $\frac{1}{2}$ ton per foot run, and also carries a weight of 3 tons placed 2 feet from one support. Sketch the curves of shearing force and bending moment, and find their numerical values at the centre of the span and at a section immediately under the concentrated load.
(C. & G., 1902, O., Sec. B.)

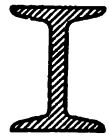
Lecture XXXI.—A.M.Inst.C.E. Questions.

1. A cantilever 20 feet long carries the loads shown in the sketch. Draw



diagrams of bending moment and shearing force. (I.C.E., *Oct.*, 1897.)

2. A rolled steel joist 40 feet in length, and having the section shown in the fig.—which may be taken as equivalent to depth 10 inches, breadth 5 inches, thickness throughout ½ inch—is continuous over three supports, forming two spans of 20 feet each. What uniformly distributed load would produce a maximum stress of 5½ tons per square inch? Sketch the diagrams of bending moments and shearing force. (I.C.E , *Feb.*, 1898.)



3. A rectangular beam of timber, 12 inches square and 15 feet between supports, is subjected to a uniformly distributed load of 200 lbs. per foot run. Illustrate diagrammatically the distribution of shearing and longitudinal stresses over a transverse section 3 feet from one of the supports, and determine the greatest intensity of each stress. (I.C.E., *Feb.*, 1898.)

4. A train, equivalent to a uniform rolling load of 30 cwts. per foot, has entered upon a single-line girder bridge of 100 feet span, and extends over 80 feet of the span (from the first abutment). The dead weight of the structure is a uniform load of 1 ton per foot. Calculate the vertical shearing force at the head of the train, due to dead and live loads, taking both girders together. (I.C.E., *Oct.*, 1898.)

5. If the rolling load, in the last-mentioned case, consisted of any known system of loaded axles, how would you proceed to find, by calculation or by graphic construction, the maximum shearing forces at various points in the span, due to live load only? (I.C.E., *Oct.*, 1898.)

6. In the beam A B C D E, the length (A E) of 24 feet is divided into four equal panels of 6 feet each by the points B, C, D. Draw the diagram of moments for the following conditions of loading, writing their values at each panel-point:—1st Beam supported at A and E, loaded at D with a weight of 10 tons. 2nd, Beam supported at B and D, loaded with 10 tons at C, and with a weight of 2 tons at each end A and E. 3rd, Beam *encastré* from A to B, loaded with a weight of 2 tons at each of the points C, D, and E. (I.C.E., *Oct.*, 1898.)

7. A floating balk of timber, 40 feet long, of straight prismatic form, carries a weight of 6 cwts. slung from its centre. Sketch the diagram of bending moments in the beam, and calculate the shearing force at a point 10 feet from the end. (I.C.E., *Oct.*, 1898.)

8. A horizontal beam span $2a$ is supported at the ends and loaded uniformly with w lbs. per foot run; show that the shearing force and bending moment at any point distant x from the centre are given by the formulæ:—

$$F = wx \qquad M = \tfrac{1}{2} w (a^2 - x^2).$$

Show how these straining actions are represented graphically. (I.C.E., *Oct.*, 1899.)

9. A beam 24 feet span is supported horizontally at the ends, and loaded with weights of 3 and 5 tons concentrated at points distant 8 and 18 feet from one end. Construct roughly to scale diagrams showing the shearing force and bending moment at any point. (I.C.E., *Oct.*, 1899.)

10. Explain carefully the connection which exists between the shearing force and the bending moment at any point of a loaded beam, (1) when the load is continuous, (2) when it is concentrated at various points. Illustrate your remarks by reference to the special cases given in questions 6 and 7. (I.C.E., *Oct.*, 1899.)

11. The platform of a bridge, 40 feet span, rests on a pair of girders below and is loaded uniformly with 2,000 lbs. per foot run. Construct to scale diagrams showing the shearing force and bending moment at any point of one of the girders, and find the maximum values of these straining actions. (I.C.E., *Feb.*, 1900.)

12. A four-wheeled truck weighing 8 tons traverses the bridge of the preceding question; 5 tons resting on the leading pair of wheels and 3 tons on the following pair, axles 8 feet apart. Find the maximum straining actions at the centre of one of the girders. (I.C.E., *Feb.*, 1900.)

13. A rectangular tank 10 feet square is carried by three beams below, each supporting an equal share of the whole weight of 14 tons The section of each beam being rectangular, depth 8 inches, breadth 3 inches, find the maximum stress on the material. (I.C.E., *Feb.*, 1900.)

14. A girder of uniform section is continuous over the two centre piers of a bridge of three equal spans, the ends being simply supported by piers at precisely the same level. Find the points of contrary flexure, and construct diagrams showing the shearing force and bending moment at any point when uniformly loaded throughout. (I.C.E., *Feb.*, 1900.)

15. A rigid body of unit width is rectangular in vertical section, which is 30 feet long and 10 feet high. A force of 20 tons is applied vertically to its upper surface, one-third the length from one vertical face, on the centre of which a normal force of 10 tons acts. If the specific gravity of the block is $2\frac{1}{4}$, find the distribution of pressure on the base. (I.C.E., *Feb.*, 1900.)

16. A beam, A B C D, whose length. A D, is 40 feet, is supported at the two points, A and C, which are 30 feet apart. The beam itself weighs 112 lbs. per lineal foot, and it carries a load of 1 ton at the overhanging end, D Find the reaction at the support, C, and also the bending moment at a point, B, midway between A and C. (I.C.E., *Oct.*, 1900.)

17. A cantilever bridge (of the Kentucky type) consists of three equal spans, A B, B C, and C D, of 300 feet each, the girder being hinged at two points, F and G, which are situated in the side-spans at a distance of 200 feet from each end of the bridge; so that A F is a span of 200 feet carried at F upon the cantilever, F B, 100 feet in length, forming part of the girder, F B C G, which is supported at B and at C. Assume a uniform dead load of 3 tons per foot lineal throughout, and a rolling load equivalent to 2 tons per foot of train. Calculate the bending moment at the centre of each span—first, when the rolling load extends from B to C; and second, when it extends from A to B and from C to D. (I.C.E., *Oct.*, 1900.)

18. A beam is laid horizontally upon two supports which are 12 feet apart, and projects at each end 6 feet beyond the support. A load of 2 tons is carried upon each of the projecting ends, and a load of 1 ton upon the centre of the span. Calculate the bending moment at the centre and also at each support; and sketch the diagram of moments neglecting the weight of the beam. (I.C.E., *Feb.*, 1901.)

19. If a straight prismatic balk of timber floats in a horizontal position with a weight of 4 cwts. hung to each end, what will be the bending moment at the centre of its length? (I.C.E., *Feb.*, 1901.)

20. Two piers in a cantilever bridge stand at a distance of 240 feet apart (between centres of supports) and the span is crossed by a girder 320 feet in length, whose ends project as cantilevers extending 40 feet beyond each support. A uniform load of 15 cwts. per foot is distributed over the whole length of this girder, while each cantilever carries 60 tons at the extreme end. Find the bending moment at each pier and at the centre of the span. (I.C.E., *Feb.*, 1901.)

21. Construct the diagram of moments for the case described in the previous question, and calculate the shearing force immediately to the right and to the left of each support. (I.C.E., *Feb.*, 1901.)

22. Determine by exact computation the position of the points of contrary flexure in the girder described in the two previous questions. (I.C.E., *Feb.*, 1901.)

23. The total length, A E, of a floating pontoon is 80 feet, and the centre-line, A B C D E, is divided into four equal parts by the intermediate points, B, C, and D. The pontoon has a uniform weight per foot lineal and a uniform cross-section throughout, and floats without sensible curvature with a uniform draught of water. Regarding the vessel as a girder, construct the diagram of bending moments for the following loads:—(1) A single load of 240 tons at C. (2) A load of 80 tons at each of the points, B, C, and D. In the latter case find the moments at 10 feet intervals from A to C. (I.C.E., *Feb.*, 1901.)

24. A girder, crossing a span of 120 feet, carries a uniformly distributed load of 10 cwts. per foot and a central load of 20 tons. Find the vertical shearing force at a section 30 feet from either support. (I.C.E , *Feb.*, 1901.)

25. In a certain ship 450 feet in length it is found, that for the central 200-foot length, the weight of the displaced water corresponding to this length exceeds the weight of that portion of the ship by 900 tons, while in each of the 125-foot end sections their weight exceeds that of the water displaced by 450 tons. Find the maximum bending moment in the ship's structure (its total weight is 7,000 tons) when lying in still water. (I.C.E., *Oct.*, 1901.)

26. A rolling load of $1\frac{1}{2}$ tons per foot run crosses a bridge of 120-foot span. Find the maximum bending moments and shearing forces which can occur at sections of the bridge distant 30, 60, and 90 feet respectively from one end. Sketch diagrams of maximum possible bending moments and shearing forces for the whole bridge produced by the above rolling load. The main girders of the bridge are of the bowstring type. Show how you could determine by the method of sections the maximum stresses in the diagonals and verticals produced by the above rolling load. (I.C.E., *Oct.*, 1901.)

27. A continuous girder consists of two equal spans of 45 feet each. Determine the bending moments at the supports, the maximum intermediate bending moments and the reactions, if a uniformly distributed load of 2,000 lbs. per foot run covers both spans. Assume both ends of the girder are free. (I.C.E , *Oct.*, 1901.)

28. A beam supported free at the two ends, bends under its own weight; find the upward force at the centre which will just neutralise this bending action, expressing the answer in terms of the weight of the beam. (I.C.E., *Oct.*, 190 .)

29. A beam of timber 15 feet long, 18 inches by 12 inches cross-section, floats in sea water, and is loaded at the centre with a weight just sufficient

to totally immerse it. Find the maximum values of the bending moment and shearing force on the beam, and state approximately how they vary. (Specific gravity of oak 0·73, of sea-water 1·026.) (I.C.E., *Feb.*, 1902.)

30. A steel rail is 32 feet long and weighs 100 lbs. per yard. It rests on two supports, one at one end of the rail, the other at a point 10 feet from the other end of the rail. Find the position and amount of the maximum bending moment and shear when a weight of 200 lbs. hangs from the free end. Construct the bending moment and shearing force diagrams for this rail. (I.C.E., *Feb.*, 1902.)

31. A girder has a span of 40 feet, and two rolling loads of 10 tons and 15 tons respectively 10 feet apart pass over it. Find the maximum bending moment which can occur at any section and the maximum shear, and construct diagrams of maximum possible bending moment and shear. (I.C.E., *Feb.*, 1902.)

32. In a bridge, 120 feet span, with eight bays, the main girders are connected together by trough flooring, on which a uniform live load of 2 tons per foot moves. Draw the positions of the moving load which would give a maximum stress in the braces. (I.C.E., *Oct.*, 1902.)

33. Compare the loads which can be safely carried at the centre of a bar 12 feet long, 6 inches deep, and 2 inches broad, and of a girder of the same length and cubic contents whose overall depth is 14 inches and breadth of flange 4½ inches, thickness of web ½ inch. (I.C.E., *Oct.*, 1902.)

34. A girder 70 feet long carries a uniform load of 2 tons per lineal foot from one end to the middle, and a load of 20 tons at 20 feet from each end; draw the bending-moment and shearing-force diagrams. (I.C.E., *Feb.*, 1903.)

35. In a plate girder the maximum intensity of stress at right angles to the vertical cross-section of the web is 5 tons per square inch, and the intensity of shearing stress there is 2 tons per square inch; find the position of the planes of the principal stresses at that point and their intensities. (I.C.E., *Feb.*, 1903.)

36. Would two similar girders of the same length carry more weight if placed alongside each other, or if one is placed over the other, and the lower flange of one is riveted to the upper flange of the other? Give reasons for your answer. (I.C.E., *Feb.*, 1903.)

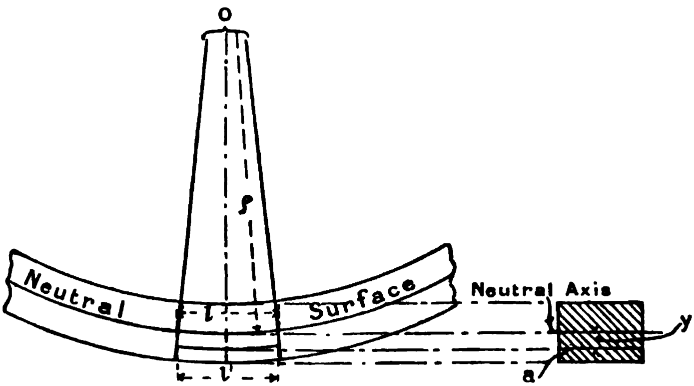
LECTURE XXXII.

CONTENTS.—Resistance of Beams to Flexure—Examples I., II., III., and IV.—Thin Wrought-Iron Girders—Example V.—Curvature and Deflection of Beams—Example VI.—Uniform Beam on Three Supports—Uniform Beam fixed at one end and supported at the other—Beams fixed at both ends and loaded at centre—Beams fixed at both ends and loaded uniformly—Tables—Questions.

Resistance of Beams to Flexure.—In the previous Lecture we saw that the effect of loading a beam was to give rise to both shearing and bending.

From the theory of couples set forth in Vol. I. we know that nothing but a couple can balance a couple. The resistance which a beam offers to bending must be of this nature, and therefore a couple of equal magnitude to that of the applied load, but of opposite tendency. The tendency of the applied couple is to bend or curve the beam, whilst the tendency of the induced couple is to oppose this curving action.

When a beam is curved the longitudinal fibres on the convex side of it are stretched beyond their normal length, and consequently they are in tension. On the concave side the fibres



ILLUSTRATING FLEXURE OF BEAMS.

are shortened, and, therefore, they are in compression. Somewhere within the beam there must be a layer of fibres that are neither lengthened nor shortened, and are therefore unstressed.

This layer is termed the *neutral surface* of the beam, and the intersection of this surface with any cross-section of the beam is termed the *neutral axis* of that section. The neutral axis is of fundamental importance in the theory of beams, because it is the fulcrum about which both the bending and resisting couples act.

We shall now find the position of the neutral axis of any given section of a beam.

Let l be the length of a small portion of the neutral surface; l' that of a parallel layer of fibres on the stretched side of the beam, and at a distance y, from the neutral surface. If $l' = l$ when the beam is straight, it is evident that the amount of stretch in the fibres at distance, y, from the neutral surface will be $l' - l$, and the *strain* $\frac{l' - l}{l}$. Let ρ denote the radius of curvature of the neutral surface at the cross-section bisecting l. Then the radius of curvature corresponding to l' will be $= \rho + y$.

Hence, $$\frac{\rho + y}{\rho} = \frac{l'}{l},$$

Or, $$\frac{y}{\rho} = \frac{l' - l}{l}.$$

If f be the tensile stress at distance y, from the neutral axis, and E the modulus of elasticity of the material, we already know that:—

$$\frac{\text{stress}}{\text{strain}} = \text{E},$$

Or, $$\frac{f}{\frac{l' - l}{l}} = \text{E}.$$

Substituting $\frac{y}{\rho}$ for $\frac{l' - l}{l}$, and inverting, we get:—

$$\frac{1}{\rho} = \frac{f}{\text{E}y} \quad \text{. . . . . (I)}$$

If we had considered in the same way a layer of fibres at a distance y', to the concave side of the neutral surface, and denoted the stress there as $-f'$ (the minus sign indicating compressive stress), we should have arrived at the equation:—

$$\frac{1}{\rho} = \frac{-f'}{\text{E}y'} \quad \text{. . . . . } (\text{I}_a)$$

Let a be a small element of the cross-sectional area at a distance y, then on the one side of the neutral axis we have for the total resistance to tension :—

$$\Sigma a f = \frac{E}{\rho} \Sigma a y. \quad . \quad . \quad . \quad . \quad . \quad \text{(II)}$$

On the other side of the neutral axis the total resistance to compression is :—

$$-\Sigma a f' = \frac{E}{\rho} \Sigma a y'. \quad . \quad . \quad . \quad . \quad . \quad (\text{II}_a)$$

But these forces constitute a couple, and are therefore equal. Hence, equating the right hand members, we have, neglecting the common factor, $\frac{E}{\rho}$:—$\Sigma a y = \Sigma a y'$.

The neutral axis, therefore, passes through the centre of area of each cross-section. If, however, E be not the same for Tensile and Compressive stresses, then the N.A. will not pass through the centre of the area, but will lie to the side having the greater value of E.

To obtain the magnitude of the resisting couple, we multiply the resistances, af and af', by their respective distances, y and y', from the neutral axis, and sum up these products for the whole section. Thus, from equation (II) the total moment of resistance on the convex side of the neutral axis is :—

$$\Sigma a f y = \frac{E}{\rho} \Sigma a y^2;$$

And on the concave side :—

$$-\Sigma a f' y' = \frac{E}{\rho} \Sigma a y'^2.$$

The sum of these results constitutes the total Resisting Moment, R.M., for the section.

Hence, $$\text{R.M.} = \frac{E}{\rho} \Sigma a y^2 + \frac{E}{\rho} \Sigma a y'^2.$$

There is now no longer any need for distinguishing between y and y', since the process of summation is the same all over the cross-section. We, therefore, finally get :—

$$\text{R.M.} = \frac{E}{\rho} \Sigma a y^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III)}$$

The quantity $\Sigma a y^2$, being a purely geometrical function, depending only on the *form* of the section, is termed its

Moment of Inertia, and is usually denoted by the symbol I, and sometimes by the product $A k^2$ (see Lecture XXII). Table II., Lecture XXII., gives the values of k^2 for most of the sections required in the following examples. These multiplied by A will give the required values of I.

Writing I for $\Sigma a y^2$, our equations become :—

$$\text{B.M.} = \text{R.M.} = \frac{\text{E I}}{\rho};$$

Or, the curvature, $\frac{1}{\rho} = \frac{\text{M}}{\text{E I}}$ . . . . (IV)

Where M stands for either the B.M. or R.M.

Again, from equations (I) and (I_a), we get :—

$$\left.\begin{aligned} \frac{f}{y} &= \frac{\text{E}}{\rho} = \frac{\text{M}}{\text{I}} \\ \therefore \quad \text{M} &= \frac{f}{y}\,\text{I} \\ \text{Or,} \quad f &= \frac{\text{M}}{\text{I}}\,y \end{aligned}\right\} \quad \ldots\ldots \quad (\text{V})$$

Formulæ (IV) and (V) are the fundamental equations of the theory of the strength of beams and girders. In applying the latter equation, it must always be borne in mind that f stands for either the tensile or compressive stress at any distance y, above or below the neutral axis.

The greatest stress comes on the fibres farthest from the neutral axis, and is the principal effect to be considered in questions of strength. If this is amply provided for, the beam will be safe. Let y now denote the distance of the fibres farthest from the neutral axis :—

Then, $$f_{max.} = \frac{\text{M}}{\text{I}} \times y = \text{M} \div \frac{\text{I}}{y}.$$

The ratio $\frac{\text{I}}{y}$ is usually denoted by Z, and is called the **Modulus of the Section.***

* See Prof. Unwin's *Elements of Machine Design*, Pages 56 to 59, for a table of the moduli of sections of beams, where $\text{Z} = \frac{\text{I}}{y}$. Also, in Seaton & Rounthwaite's *Pocket Book of Marine Engineering Rules and Tables*, from which we extracted the tables at the end of this lecture, we also find $\text{Z} = \frac{\text{I}}{y}$ as the modulus of the section.

Hence, writing Z for $\frac{I}{y}$, we have:—

$$\left.\begin{aligned} f_{max.} &= \frac{M}{Z} \\ \text{Or,} \quad M &= Z f_{max.} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{VI})$$

In applying this equation the student must be careful to remember that in those cases where the section of the beam or girder is not symmetrical about the neutral axis, there will be two values of y to be taken into account, and therefore two values of Z. On the whole, we think it safer to adhere to the general formula (V) as being less likely to lead to confusion; at the same time, it is very convenient to use equation (VI) in taking out quantities in the drawing office by aid of tables since it reduces the work of calculation.

EXAMPLE I.—A floor joist, 12 inches deep and 3 inches broad, has a span of 15 feet, and carries a uniformly distributed load of 1 cwt. per foot-run. Find the greatest intensity of stress within the timber. (S. and A. Adv. Exam., 1891.)

ANSWER.—In problems involving the calculation of stress within the beam, the student will find it best to express all dimensions in *inches*, and, therefore, bending moments in *inch-lbs.* or *inch-tons* as the case may be.

In this problem the greatest stresses will occur at the middle of the joist where the bending moment attains its maximum value, which, in this case, is:—

$$\text{Max. B.M.} = \tfrac{1}{8}\, w\, L^2 \text{ inch-lbs.}$$

Here, $\qquad w = \dfrac{112}{12}$ lbs.

And, $\qquad L = 15 \times 12$ inches.

$$\therefore \qquad \text{B.M.} = \frac{1}{8} \times \left(\frac{112}{12}\right) \times (15 \times 12)^2 \text{ inch-lbs.}$$

$$\text{Or,} \qquad \text{B.M.} = 14 \times 15 \times 15 \times 12 \qquad ,,$$

The value of I for a rectangular section is:—

$$I = \frac{1}{12}(\text{breadth}) \times (\text{depth})^3,$$

$$\text{Or,} \qquad I = \frac{1}{12} \times 3 \times 12^3 = 3 \times 12 \times 12.$$

The greatest stress at the middle section of the joist will occur in the fibres farthest away from the neutral axis. Hence, y = 6 inches.

Applying equation (V) we have:—

$$f = \frac{\text{B.M.}}{\text{I}} y,$$

$$,, = \frac{14 \times 15 \times 15 \times 12}{3 \times 12 \times 12} \times 6 = \mathbf{525 \text{ lbs. per sq. in.}}$$

EXAMPLE II.—A uniform beam of oak, 10 feet long, 15 inches deep and 10 inches wide, sustains, in addition to its own weight, a load of 5,000 lbs. placed at the centre. Find the greatest bending moment and the greatest stress in the fibres.

The specific gravity of oak is 0·934. (S. and A. Adv. Exam., 1894.)

ANSWER.—Here the greatest bending moment takes place at the centre of the beam and is made up of two parts: (1) that due to the beam's own weight which is uniformly distributed along its length; and (2) that due to the 5,000 lbs. concentrated at its middle.

For (1), $\text{B.M.}_1 = \frac{1}{8} w \text{L}^2$ inch-lbs.

And for (2), $\text{B.M.}_2 = \frac{1}{4} \text{W L}$,,

∴ Total, $\text{B.M.} = \frac{1}{8} w \text{L}^2 + \frac{1}{4} \text{W L}$ inch-lbs.

Taking the weight of a cubic inch of water as 0·036 lb., then a cubic inch of oak will weigh $0·934 \times 0·036 = 0·0336$ lb.

$$\therefore \quad w = 0·0336 \times 15 \times 10 = 5·04 \text{ lbs.}$$

And, $\text{B.M.} = \frac{1}{8} \times 5·04 \times (10 \times 12)^2 + \frac{1}{4} \times 5,000 \times (10 \times 12)$

$$,, \quad = 9,072 + 150,000 = \mathbf{159,072 \text{ inch-lbs.}}$$

Here, $\text{I} = \frac{1}{12} \times 10 \times 15^3 = \frac{1}{2} \times 5 \times 5 \times 15 \times 15$

And, $y = \frac{1}{2} \times 15$ inches.

$$\therefore \quad f = \frac{\text{B.M.}}{\text{I}} y = \frac{159,072}{\frac{1}{2} \times 5 \times 5 \times 15 \times 15} \times \frac{1}{2} \times 15$$

$$,, = \mathbf{424·1 \text{ lbs. per sq. inch.}}$$

EXAMPLE III.—A round steel spindle 10 inches long, and held at one end, revolves at the rate of 150 revolutions per minute round a vertical axis, to which the axis of the spindle is parallel

and from which it is 2 feet distant. The spindle has a uniformly distributed load, the whole revolving weight being 30 lbs. What should be the diameter of the spindle when the safe working stress of the material in tension or compression is taken at 25,000 lbs. per square inch? (S. & A. Hons. Exam., 1891.)

ANSWER.—The spindle in this problem may be likened to a beam fixed at one end and carrying a uniformly distributed load. The load being not the revolving weight of 30 lbs., but the centrifugal force of that weight due to its being whirled round at the rate of 150 revolutions per minute.

$$\text{Velocity of spindle,} = \frac{150 \times 2\pi \times 2}{60} = 10\,\pi \text{ ft. per sec.}$$

$$\text{Centrifugal force,} = \frac{30 \times (10 \times \pi)^2}{32 \times 2} = \frac{1500 \times \pi^2}{32} \text{ lbs.}$$

This force, multiplied by half the length of the spindle, gives us the bending moment at the fixed end of the spindle :—

$$\text{That is,} \qquad \text{B.M.} = \frac{1500 \times \pi^2}{32} \times \frac{10}{2} \text{ inch-lbs.}$$

If D be the diameter of the spindle in inches, then from Lecture XXII. we get :—

$$\text{The Moment of Inertia, I} = \frac{\pi}{64} D^4$$

$$\text{And the Modulus of Section, Z} = \frac{I}{\frac{D}{2}} = \frac{\pi}{32} D^3.$$

Now, fZ = B.M. ; and, f = 25,000 lbs. per sq. inch.

$$\therefore \qquad 25{,}000 \times \frac{\pi}{32} D^3 = \frac{1500\,\pi^2}{32} \times \frac{10}{2}$$

$$\text{Or,} \qquad D^3 = 0{\cdot}3 \times \pi = 0{\cdot}94248$$

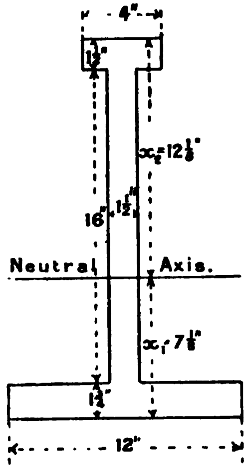
$$\text{Hence,} \qquad D = \sqrt[3]{0{\cdot}94248} = 0{\cdot}98 \text{ inch.}$$

EXAMPLE IV.—The section of a cast-iron girder, and the maximum safe tensile and compressive stresses being given, explain how to determine its moment of resistance to bending. The dimensions of the section of a cast-iron girder are the

following :—Top flange, 4 by $1\frac{1}{2}$ inches; bottom flange, 12 by $1\frac{3}{4}$ inches; web, 16 by $1\frac{1}{2}$ inches. Determine the moment of resistance, the greatest permissible tensile and compressive stresses being $2\frac{1}{2}$ and $7\frac{1}{2}$ tons per square inch respectively. If the girder be 20 feet long, and is supported at its two ends, find the greatest safe load which it will carry when uniformly distributed along its length. (S. and A. Hons. Exam., 1895.)

ANSWER.—As this is an excellent example for showing the student how the R.M. of a girder section is calculated, we shall go into the matter in detail. Let the accompanying figure represent the cross-section in question.

We have first to find the position of the neutral axis N A, by writing down the sectional areas of the parts composing the figure, and taking moments about the lower edge.



SECTION OF GIRDER.

$$
\begin{array}{llll}
\text{Area of top} & \text{flange} = 4 \times 1\frac{1}{2} & = & 6 \text{ sq. in.} \\
\quad ,, \;\; \text{bottom} & ,, \quad = 12 \times 1\frac{3}{4} & = & 21 \quad ,, \\
\quad ,, \;\; \text{web} & \quad\;\; = 16 \times 1\frac{1}{2} & = & 24 \quad ,, \\
\therefore \quad \text{Total area of section} & & = & 51 \quad ,,
\end{array}
$$

Then, since N A passes through the centre of area of the section, we have :—

$$51 \times x_1 = 6 \times 18\frac{1}{2} + 24 \times 9\frac{3}{4} + 21 \times \frac{7}{8} = 363\frac{3}{8}.$$

$$\therefore \qquad x_1 = \frac{363\frac{3}{8}}{51} = 7\frac{1}{8}.$$

And $\qquad x_2 = 19\frac{1}{4} - 7\frac{1}{8} = 12\frac{1}{8}.$

We calculate the value of I, the moment of inertia of the section about the neutral axis, by finding that for each of the parts into which the section is divided and taking their sum. As the neutral axis does not pass through the centre of any of

those parts, we shall have to employ Proposition I. of Lecture XXII., to which we again refer the student.

Remembering that the moment of inertia of a rectangular area about an axis through its centre of gravity is:—

$$\tfrac{1}{12}\,(\text{breadth}) \times (\text{depth})^3, \text{ we have:—}$$

For top flange, $I_t = \frac{1}{12} \times 4 \times (1\frac{1}{2})^3 + 6 \times (11\frac{3}{8})^2$.

" " " $= 1{\cdot}125 + 776{\cdot}343 = 777{\cdot}468$.

For bottom flange, $I_b = \frac{1}{12} \times 12 \times (1\frac{3}{4})^3 + 21 \times (6\frac{1}{4})^2$.

" " " $= 5{\cdot}359 + 820{\cdot}312 = 825{\cdot}671$.

For web, $I_w = \frac{1}{12} \times 1\frac{1}{2} \times (16)^3 + 24 \times (2\frac{5}{8})^2$.

" " $= 512{\cdot}0 + 165{\cdot}375 = 677{\cdot}375$.

$\therefore$ For whole section, $I = 777{\cdot}468 + 825{\cdot}671 + 677{\cdot}375 = 2280{\cdot}5$.*

To illustrate what we said about the moduli of unsymmetrical sections, we shall find both moduli for this example:—

$$\text{For tension, } Z_t = \frac{I}{x_1} = \frac{2280{\cdot}5}{7{\cdot}125} = 320{\cdot}0.$$

$$\text{For compression, } Z_c = \frac{I}{x_2} = \frac{2280{\cdot}5}{12{\cdot}125} = 188{\cdot}0.$$

The question gives as the greatest permissible values for:—

Tensile stress, $f_t = 2{\cdot}5$ tons per sqr. inch.

Compressive stress, $f_c = 7{\cdot}5$ " "

Since, R.M. $= Z f_{max.}$, we must take the lower of the two values of R.M. in fixing the load to be carried by the girder. These are:—

$$Z_t \times f_t = 320 \times 2{\cdot}5 = 800 \text{ inch-tons.}$$

And, $$Z_c \times f_c = 188 \times 7{\cdot}5 = 1410 \text{ inch-tons.}$$

$\therefore$ B.M. = R.M. = 800 inch-tons.

* Another and rather shorter method of finding I for this form of section is to (1) produce the sides of top and bottom flanges to meet the neutral axis N A, (2) calculate the moments of inertia of the two full rectangles thus formed, (3) subtract from their sum the moments of inertia of the four rectangular areas which are in excess of the section of the beam. All these moments may be found by the formula $I = \frac{1}{12} B \times D^3$, which will only require to be used four times as the blank rectangles on each side of the web are equal in pairs.

The girder will, therefore, safely carry a uniformly distributed load, given by the equation:—

$$\tfrac{1}{8}\, w\, L^2 = 800.$$

$$\therefore \qquad W = w\,L = \frac{8 \times 800}{20 \times 12} = 26\tfrac{2}{3} \text{ tons.}$$

This will make the maximum compressive stress

$$f_{c\,max.} = \frac{800}{188} = 4{\cdot}255 \text{ tons,}$$

instead of 7·5 as given; showing that the girder is not well designed.

In a properly proportioned section we should have:—

$$Z_t \times f_t = Z_c \times f_c.$$

Thin Wrought-Iron Girders.—In the case of wrought-iron girders where the flanges are thin compared with their distance apart, and where the bending resistance of the web is disregarded as a provision against the shearing force acting at the section, the formulæ for the moment of resistance are very simple.

Let A_t = Area of flange in tension.
" A_c = Area of flange in compression.
" H = Distance between centres of flanges.
" f_t = Mean stress in tension flange.
" f_c = Mean stress in compression flange.

Distance between centre of tension flange and the neutral axis is:—

$$y_t = \left(\frac{A_c}{A_t + A_c}\right) H;$$

and the distance between centre of compression flange and the neutral axis is:—

$$y_c = \left(\frac{A}{A_t + A_c}\right) H.$$

The moment of inertia of the flanges, with respect to the neutral axis, is:—

$$I = A_t \left(\frac{A_c}{A_t + A_c}\right)^2 H^2 + A_c \left(\frac{A_t}{A_t + A_c}\right)^2 H^2$$

Or, $\quad I = \{A_t \times A_c^2 + A_c \times A_t^2\} \times \left(\dfrac{H}{A_t + A_c}\right)^2$

Since, $f = \dfrac{M}{I} y$.

$$\therefore \quad f_t = \frac{M}{\{A_t \times A_c^2 + A_c \times A_t^2\}\left(\dfrac{H}{A_t + A_c}\right)^2} \times \left(\frac{A_c}{A_t + A_c}\right)H$$

$$\left.\begin{aligned} &\text{Hence,} \qquad & f_t &= \frac{M}{A_t \times H} \\ &\text{Similarly,} \qquad & f_c &= \frac{M}{A_c \times H} \end{aligned}\right\} \quad \ldots\ldots \quad \text{(VII)}$$

Example V.—A wrought-iron riveted girder of I section has a top flange of 9 square inches in sectional area, and a bottom flange of 8 square inches. The distance between the centres of gravity of the flanges is 12 inches, and the ends of the beam rest on abutments, 16 feet apart. The girder being loaded uniformly with a load equal to 1 ton per lineal foot (including the weight of the beam). What would be the mean stress per square inch on the metal in each flange at the dangerous section? The resistance of the web to bending is neglected. (S. & A. Hons. Exam., 1892.)

Answer.—By "dangerous section" is here meant the middle section of the girder, where the maximum bending moment occurs. (See equation (XI_a) of Lecture XXXI.)

$$\text{Max. B.M.} = \frac{1}{8}\left(\frac{1}{12}\right) \times (16 \times 12)^2 = 32 \times 12 \text{ inch-tons.}$$

Hence, mean stress in tension flange,

$$f_t = \frac{32 \times 12}{8 \times 12} = \textbf{4 tons per square inch.}$$

And, mean stress in compression flange,

$$f_c = \frac{32 \times 12}{9 \times 12} = \textbf{3·55 tons per square inch.}$$

Curvature and Deflection of Beams.—When we speak of the curvature or the deflection of a beam we mean that of its neutral surface.

If the beam is fixed at one end, we take the origin of co-ordinates at that end; but if supported or fixed symmetrically at both ends, we take it at the middle.

Let the co-ordinates of the neutral surface curve be denoted, as usual, by x and y, then the deflection of the beam at any distance x, from the origin will be measured by y and the tangent of its inclination to the horizon by $\frac{dy}{dx}$.

The equation of the curve into which the beam is bent will be :—

$$y = \varphi(x),$$

Where $\varphi(x)$ is a function of x to be determined for each particular case.

In treatises on the analytical geometry of plane curves it is shown that the general expression for radius of curvature is :—

$$\rho = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}.$$

Although of great importance, in the theory of beams $\frac{dy}{dx}$ is always such a small fraction, that its square becomes a perfectly negligible quantity in comparison with unity. We may, therefore, safely disregard the value of $\left(\frac{dy}{dx}\right)^2$ in the above formula, and write $\frac{d^2y}{dx^2} = \frac{1}{\rho}$.

But by equation (IV) of this Lecture we know that :—

$$\frac{1}{\rho} = \frac{M}{EI}. \quad \therefore \frac{d^2y}{dx^2} = \frac{M}{EI}. \quad . \quad . \quad . \quad . \quad (\text{VIII})$$

In what follows we shall assume that the beam or girder is of uniform section so that I is constant, the more general cases where I varies being rather beyond the scope of this treatise.

We shall begin by working out the following example, which will form a good introduction to this rather mathematical part of our subject.

Example VI.—Investigate a formula for calculating the amount of deflection of a beam supported at its ends and loaded uniformly. Find the deflection in a beam of timber of uniform rectangular section, 6 inches wide and 12 inches deep, the beam

being supported at its ends in a horizontal position on two walls 12 feet apart. There is to be taken into account a single concentrated load of 4,000 lbs. at the centre, and a uniformly distributed load of 2,500 lbs., the modulus of elasticity being 1,750,000 lbs. per square inch. (S. & A. Hons. Exam., 1891.)

ANSWER.—Taking the middle of the beam as the origin of co-ordinates, we have already proved (see equation (XI) in Lecture XXXI.) that the bending moment at x inches from this point, in the case of a beam L inches between supports, and loaded uniformly with w lbs. per inch-run, is :—

$$\text{B.M.} = \tfrac{1}{2} w \left(\tfrac{1}{4} L^2 - x^2\right) \text{ inch-lbs.}$$

Substituting this in formula (VIII) we get :—

$$\frac{d^2y}{dx^2} = \frac{w}{2 E I} \left(\tfrac{1}{4} L^2 - x^2\right).$$

Now, multiplying both sides by dx, and integrating, we have :—

$$\frac{dy}{dx} = \frac{w}{2 E I} \int \left(\tfrac{1}{4} L^2 - x^2\right) dx.$$

Or, $$\frac{dy}{dx} = \frac{w}{2 E I} \left(\tfrac{1}{4} L^2 x - \tfrac{1}{3} x^3\right).$$

This needs no correction because $\frac{dy}{dx} = 0$, when $x = 0$.

Integrating a second time, we get :—

$$y = \frac{w}{2 E I} \int \left(\tfrac{1}{4} L^2 x - \tfrac{1}{3} x^3\right) dx.$$

Or, $$y = \frac{w}{8 E I} \left(\tfrac{1}{2} L^2 x^2 - \tfrac{1}{3} x^4\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX)}$$

This also requires no correction, as x and y vanish together. Now, let Δ_1 denote the deflection of the beam for the distributed load :—

$$\Delta_1 = y, \text{ when } x = \tfrac{1}{2} L.$$

Or, $$\Delta_1 = \frac{w}{8 E I} \left\{\tfrac{1}{2} L^2 \left(\tfrac{1}{2} L\right)^2 - \tfrac{1}{3} \left(\tfrac{1}{2} L\right)^4\right\}.$$

$\therefore$ $$\Delta_1 = \frac{5 w L^4}{384 E I} \text{ inches.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(X)}$$

In the case of a beam carrying a single load of W lbs. at its

middle point, the bending moment due to that load at x inches from the middle point (Equation (VII) Lecture XXXI.) is :—

$$B.M. = \tfrac{1}{2} W (\tfrac{1}{2} L - x) \text{ inch-lbs.}$$

$$\therefore \quad \frac{d^2 y}{d x^2} = \frac{W}{2 E I} (\tfrac{1}{2} L - x).$$

The first integration of this equation gives :—

$$\frac{d y}{d x} = \frac{W}{2 E I} (\tfrac{1}{2} L x - \tfrac{1}{2} x^2) = \frac{W}{4 E I} (L x - x^2).$$

And the second integration :—

$$y = \frac{W}{4 E I} (\tfrac{1}{2} L x^2 - \tfrac{1}{3} x^3). \quad . \quad . \quad . \quad . \quad . \quad (XI)$$

If Δ_2 be the total deflection in this case, then Δ_2 is the value of y when $x = \frac{1}{2} L$.

$$\therefore \quad \Delta_2 = \frac{W}{4 E I} \{\tfrac{1}{2} L (\tfrac{1}{2} L)^2 - \tfrac{1}{3} (\tfrac{1}{2} L)^3\}.$$

$$\text{Or,} \quad \Delta_2 = \frac{W L^3}{48 E I} \text{ inches.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (XII)$$

If the beam carry both loads at the same time, as given in the question, then the total deflection due to the two loads will be :—

$$\Delta = \Delta_1 + \Delta_2.$$

$$\therefore \quad \Delta = \frac{5 w L^4}{384 E I} + \frac{W L^3}{48 E I}$$

$$\text{Or,} \quad \Delta = \frac{L^3}{48 E I} (\tfrac{5}{8} w L + W).$$

The numerical data given are :—

$$L = 12 \times 12 \text{ inches.}$$

$$I = \tfrac{1}{12} b d^3 = \tfrac{1}{12} \times 6 \times 12^3 = 6 \times 12^2.$$

$$W = 4{,}000 \text{ lbs.}$$

$$w L = 2{,}500.$$

$$\text{And,} \quad E = 1{,}750{,}000.$$

$$\therefore \quad \Delta = \frac{(12 \times 12)^3}{48 \times 1{,}750{,}000 \times 6 \times 12^2} (\tfrac{5}{8} \times 2{,}500 + 4{,}000)$$

$$\Delta = 0{\cdot}2288 \text{ inches.}$$

Uniform Beam on Three Supports.—Suppose we are given a uniform beam resting on three supports all on the same level, to find the pressure on the middle support.

It is clear that if the middle support were taken away, the weight of the beam would cause it to bend down at the middle [as found above by equation (X)] through a distance

$$\Delta_1 = \frac{5\,w\,L^4}{384\,E\,I} \text{ inches.}$$

We have also seen by equation (XII) that a single concentrated load of W lbs. applied at the middle of the beam would produce an amount of deflection, $\Delta_2 = \dfrac{W\,L^3}{48\,E\,I}$ inches.

This gives us the upward deflection caused by the reaction of the central support if we put its value, P, instead of W in the equation.

The total deflection will be zero if all three supports are on the same level.

Then,
$$\frac{P\,L^3}{48\,E\,I} = \frac{5\,w\,L^4}{384\,E\,I}.$$

Or,
$$P = \tfrac{5}{8}\,w\,L.$$

The pressure on the middle support is thus seen to be $\frac{5}{8}$ of the weight of the beam; whilst the end supports each carry $\frac{3}{16}$ of the weight.

Uniform Beam Fixed at one End and Supported at the other.—If w be the weight of the beam in lbs. per inch-run and L its length in inches, then, we already know that at x inches from the fixed end, the

$$\text{B.M.} = \tfrac{1}{2}\,w\,(L - x)^2 \text{ inch-lbs.}$$

Putting this value of the B.M. in equation (VIII), we get:—

$$\frac{d^2 y}{d x^2} = \frac{w}{2\,E\,I}(L - x)^2 = \frac{w}{2\,E\,I}(L^2 - 2\,L\,x + x^2).$$

$$\therefore \quad \frac{d\,y}{d\,x} = \frac{w}{2\,E\,I}\int(L^2 - 2\,L\,x + x^2)\,d\,x$$

$$\quad " \quad = \frac{w}{2\,E\,I}\{L^2 x - L\,x^2 + \tfrac{1}{3}\,x^3\}.$$

And,
$$y = \frac{w}{2\,E\,I}\int\{L^2 x - L\,x^2 + \tfrac{1}{3}\,x^3\}\,d\,x$$

$$y = \frac{w}{2\,E\,I}\{\tfrac{1}{2}\,L^2 x^2 - \tfrac{1}{3}\,L\,x^3 + \tfrac{1}{12}\,x^4\}. \quad . \quad . \quad \text{(XIII)}$$

This last equation gives the droop of the beam at any distance x, from the fixed end. At the free end let Δ_1 be the value of y when $x = L$.

Then, $$\Delta_1 = \frac{w L^4}{8 E I} \text{ inches.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (XIV)$$

Let P be the upward pressure in lbs., between the beam and a support placed under its free end. The bending moment, due to P, at x inches from the fixed end is $P(L - x)$ inch-lbs. Hence, the curvature produced by P will be :—

$$\frac{d^2 y}{d x^2} = \frac{P}{E I}(L - x).$$

$$\therefore \quad \frac{dy}{dx} = \frac{P}{E I}\int (L - x)\, dx = \frac{P}{E I}(L x - \tfrac{1}{2} x^2).$$

$$\text{And,} \quad y = \frac{P}{E I}\int (L x - \tfrac{1}{2} x^2)\, dx = \frac{P}{2 E I}(L x^2 - \tfrac{1}{3} x^3). \quad . \quad . \quad (XV)$$

When $x = L$, let $y = \Delta_2$.

$$\therefore \qquad \Delta_2 = \frac{P L^3}{3 E I} \text{ inches.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (XVI)$$

If $\Delta_2 = \Delta_1$, the supported end will be raised to the same level as the fixed end.

Then, $$\frac{P L^3}{3 E I} = \frac{w L^4}{8 E I}.$$

Or, $$P = \tfrac{3}{8} w L \text{ lbs.}$$

This result shows that the pressure on the prop is equal to $\frac{3}{8}$ of the weight of the beam.

It will be instructive for the student to observe that this result might easily have been inferred from the previous case of a beam resting on three props.

In that case the part of the beam immediately over the middle support is in exactly the same condition as the fixed end of the beam in this case; so that whatever is true of each half of the beam in the former case will here hold good for the whole beam. The pressure on the end supports is, therefore, identical in magnitude in each case; because $\frac{3}{16}$ of the weight of the whole beam is the same thing as $\frac{3}{8}$ of the weight of each half.

Beam Fixed at Both Ends and Loaded at the Centre.—When a beam is fixed, or built horizontally into a wall at both ends, the fixing causes a bending moment which is constant all over the

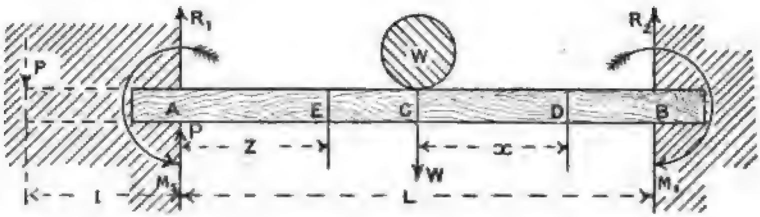
beam. For the reaction of the left support in keeping the beam horizontal is equivalent to a force P, acting downwards at some distance l, to the left of that support, and an upward force P, at the support. The bending moment at the support is then :—

$$M_s = P \times l.$$

And, at any other point, E, of the beam, at a distance, z (less than half the span from the support), the B.M. caused by this reaction at the support is :—

$$\text{B.M.} = P(z + l) - Pz = Pl = M_s.$$

Consequently, the fixing at the ends causes a constant B.M. all over the beam, equal to that at the supports, in addition to that caused by the load (*but in the opposite direction*).



BEAM FIXED AT ENDS AND LOADED AT CENTRE.

Taking our origin of co-ordinates at C, the centre, and the undeflected axis, or neutral line of the beam, as our axis of x, we have, at a section D, distant x from C :—

$$\text{B.M.} = R_2(\tfrac{1}{2}L - x) - M_s = \tfrac{1}{2}W(\tfrac{1}{2}L - x) - M_s$$

Hence, from eqn. (VIII), $$\frac{d^2y}{dx^2} = \frac{1}{EI}\{\tfrac{1}{2}W(\tfrac{1}{2}L - x) - M_s\}$$

∴ $$\frac{dy}{dx} = \frac{1}{EI}\{\tfrac{1}{4}W(Lx - x^2) - M_s x\}$$

The beam is horizontal at the centre and at the ends, therefore $\frac{dy}{dx}$ is zero when x is zero, and when $x = \tfrac{1}{2}L$.

∴ $$0 = \frac{1}{EI}\{\tfrac{1}{4}W(\tfrac{1}{2}L^2 - \tfrac{1}{4}L^2) - \tfrac{1}{2}M_s L\}$$

Or, $$M_s = \tfrac{1}{8}WL.$$

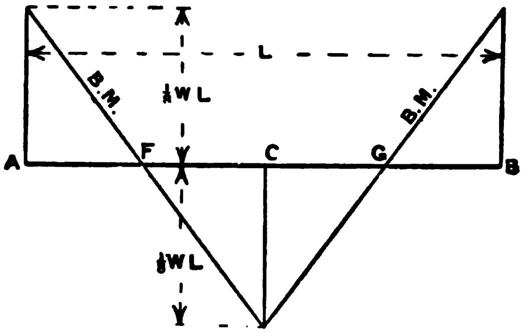
Inserting this value in the above equation for the B.M. we get :—

$$\text{B.M.} = \tfrac{1}{2}\text{W}\left(\tfrac{1}{2}\text{L} - x\right) - \tfrac{1}{8}\text{W L}$$

Or, $$\text{B.M.} = \tfrac{1}{2}\text{W}\left(\tfrac{1}{4}\text{L} - x\right) \quad . \quad . \quad . \quad . \quad (\text{XVII})$$

At the centre, $$\text{B.M.} = \tfrac{1}{2}\text{W} \cdot \tfrac{1}{4}\text{L} = \tfrac{1}{8}\text{W L} = \text{M}_s$$

$\therefore$ $$\text{Maximum B.M.} = \text{M}_s = \tfrac{1}{8}\text{W L} \quad . \quad . \quad . \quad . \quad . \quad (\text{XVIII})$$



B.M. DIAGRAM FOR BEAM FIXED AT ENDS AND LOADED AT CENTRE.

We thus see that, in this case the fixing of the ends reduces the maximum B.M. to half what it would be with free ends, and that this maximum B.M. occurs both at the centre and the ends.

The B.M. diagram is similar to what we had for a beam simply supported, but the base line is shifted half way down the diagram, so that it is crossed at F and G by the lines representing the B.M. It will be seen from equation (XVII) that the B.M. is zero where $x = \frac{1}{4}$ L, and that it is positive on one side of this point, and negative on the other. This is one of the points where the B.M. curve cuts the base line, and it is called a *point of inflection*, because the beam is straight just at that point and the curvature changes sign. There is, of course, another point of inflection at the distance $\frac{1}{4}$ L on the other side of the centre.

In large girder bridges that part of the span between the two points of inflection is made separate from the remainder and rests on rollers at these points. This allows freedom of expansion without reducing the strength of the bridge.

Integrating the value of $\dfrac{dy}{dx}$ we get :—

$$y = \frac{1}{\text{E I}}\left\{\tfrac{1}{4}\text{W}\left(\tfrac{1}{2}\text{L}x^2 - \tfrac{1}{3}x^3\right) - \tfrac{1}{16}\text{W L}x^2\right\}.$$

Therefore, at the ends, where $x = \frac{1}{2}L$:—

$$y = \frac{1}{EI}\left\{\frac{1}{4}W\left(\frac{L^3}{8} - \frac{L^3}{24}\right) - \frac{1}{16}W\frac{L^3}{4}\right\} = \frac{W L^3}{192 E I}.$$

Hence, the difference of level between the centre and the ends is :—

$$\Delta = \frac{W L^3}{192 E I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (XIX)$$

This is only one-fourth of the deflection when the beam simply rested on its supports (Equation XII), so that the beam is now four times as stiff.

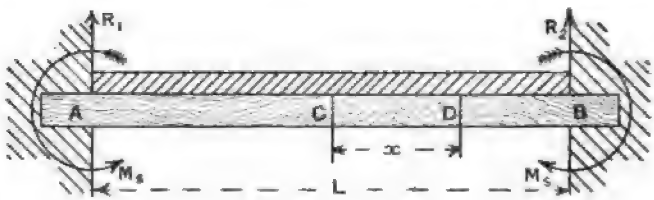
Beam Fixed at Both Ends and Loaded Uniformly.—Taking axes as before, the B.M. at any section, D, is :—

$$B.M. = R_2\left(\tfrac{1}{2}L - x\right) - \tfrac{1}{2}w\left(\tfrac{1}{2}L - x\right)^2 - M_s.$$

Or, $$B.M. = \tfrac{1}{2}wL\left(\tfrac{1}{2}L - x\right) - \tfrac{1}{2}w\left(\tfrac{1}{4}L^2 - Lx + x^2\right) - M_s.$$

$\therefore$ $$\frac{d^2 y}{d x^2} = \frac{1}{EI}\left\{\tfrac{1}{2}w\left(\tfrac{1}{4}L^2 - x^2\right) - M_s\right\}.$$

Or, $$\frac{d y}{d x} = \frac{1}{EI}\left\{\tfrac{1}{2}w\left(\tfrac{1}{4}L^2 x - \tfrac{1}{3}x^3\right) - M_s x\right\}.$$



BEAM FIXED AT BOTH ENDS AND LOADED UNIFORMLY.

In this case also, $\frac{d y}{d x}$ is zero when x is zero, and when $x = \frac{1}{2}L$.

$\therefore$ $$0 = \tfrac{1}{2}wL^3\left(\tfrac{1}{8} - \tfrac{1}{24}\right) - \tfrac{1}{2}M_s L.$$

Or, $$M_s = \frac{wL^2}{12} = \frac{W L}{12}.$$

Hence, $$B.M. = \tfrac{1}{2}w\left(\tfrac{1}{4}L^2 - x^2\right) - \tfrac{1}{12}wL^2.$$

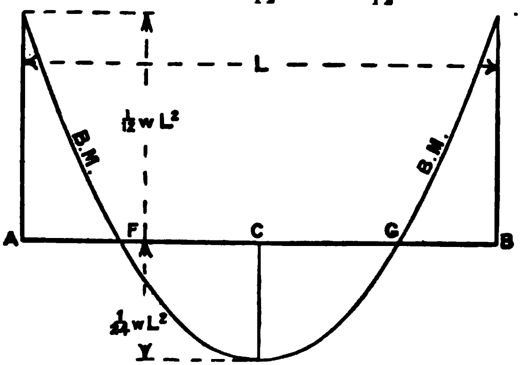
Or, $$B.M. = \tfrac{1}{2}w\left(\tfrac{1}{12}L^2 - x^2\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (XX)$$

At the centre, where $x = 0$, the B.M. is :—

$$M_c = \frac{wL^2}{24} = \frac{W L}{24} = \tfrac{1}{2}M_s.$$

This is only half of that at the support. Hence, the greatest bending moment occurs at the support, and its value is :—

$$\text{Maximum B.M.} = \tfrac{1}{12} w \, L^2 = \tfrac{1}{12} \, W \, L. \quad . \quad . \text{(XXI)}$$



B.M. Diagram for Beam Fixed at both Ends and Loaded Uniformly.

The points of inflection occur where $x^2 = \frac{1}{12} L^2$ or $x = \pm \dfrac{L}{2\sqrt{3}}$.

By integrating the above value of $\dfrac{dy}{dx}$:—

$$y = \frac{1}{EI}\left\{ \tfrac{1}{2} w \left(\tfrac{1}{8} L^2 x^2 - \tfrac{1}{12} x^4\right) - \tfrac{1}{12} w \, L^2 \frac{x^2}{2} \right\} = \frac{w}{48\,EI}(L^2 x^2 - 2\,x^4).$$

Putting $x = \dfrac{L}{2}$ we obtain the amount by which the centre of the beam is deflected by the load, viz. :—

$$\Delta = \frac{w}{48\,EI}\left(\frac{L^4}{4} - \frac{L^4}{8}\right) = \frac{w\,L^4}{384\,EI} = \frac{W\,L^3}{384\,EI} \quad . \quad . \quad \text{(XXII)}$$

We thus see that, by fixing the ends horizontally for this manner of loading, the strength of the beam is increased in the ratio 3 : 2, and its stiffness in the ratio 5 : 1.

When the span of the beam is small, it may be designed wholly from considerations of strength; but when the span is great a beam may be strong enough, and yet not suitable, because it yields too much when the load is put on it. It then becomes necessary to take the stiffness into account by using one of the formulæ we have found for the deflection. The greatest deflection usually allowed in beams is 1 inch in 100 feet, or $\frac{1}{1200}$ of the span.

In the tables below we give a summary of these results, showing the relation between them.

TABLE III.—STRENGTH AND STIFFNESS OF BEAMS UNDER A TOTAL LOAD OF W LBS.

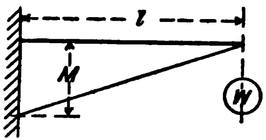
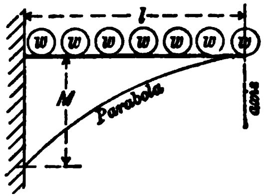
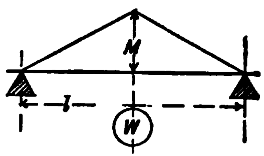
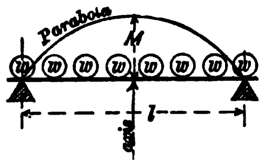
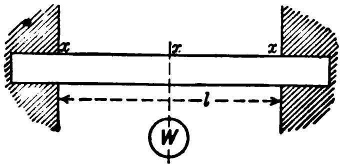
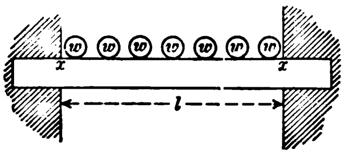
No.	Manner of Supporting and Loading.	Maximum Bending Moment = M.	Relative Strength.	Deflection in Terms of W.	Deflection in Terms of M.	Deflection in Terms of Stress.	Relative Stiffness under same Load.
I.	 CANTILEVER LOADED AT END.	Wl	$\frac{1}{4}$	$\frac{1}{3} \cdot \frac{Wl^3}{EI}$	$\frac{1}{3} \cdot \frac{Ml^2}{EI}$	$\frac{1}{3} \cdot \frac{fl^2}{Ey}$	$\frac{1}{16}$
II.	 CANTILEVER LOADED UNIFORMLY.	$\frac{Wl}{2}$	$\frac{1}{2}$	$\frac{1}{8} \cdot \frac{Wl^3}{EI}$	$\frac{1}{4} \cdot \frac{Ml^2}{EI}$	$\frac{1}{4} \cdot \frac{fl^2}{Ey}$	$\frac{1}{6}$
III	 SUPPORTED AT BOTH ENDS. LOADED AT CENTRE.	$\frac{Wl}{4}$	1	$\frac{1}{48} \cdot \frac{Wl^3}{EI}$	$\frac{1}{12} \cdot \frac{Ml^2}{EI}$	$\frac{1}{12} \cdot \frac{fl^2}{Ey}$	1

TABLE III.—STRENGTH AND STIFFNESS OF BEAMS UNDER A TOTAL LOAD OF W LBS. (*continued*).

No.	Manner of Supporting and Loading.	Maximum Bending Moment = M	Relative Strength.	Deflection in Terms of W.	Deflection in Terms of M.	Deflection in Terms of Stress.	Relative Stiffness under same Load.
IV.	 SUPPORTED AT BOTH ENDS. LOADED UNIFORMLY.	$\frac{Wl}{8}$	2	$\frac{5}{384} \cdot \frac{Wl^3}{EI}$	$\frac{5}{48} \cdot \frac{Ml^2}{EI}$	$\frac{5}{48} \cdot \frac{fl^2}{Ey}$	$\frac{8}{5}$
V.	 FIXED AT ENDS. LOADED AT CENTRE.	$\frac{Wl}{8}$	2	$\frac{1}{192} \cdot \frac{Wl^3}{EI}$	$\frac{1}{24} \cdot \frac{Ml^2}{EI}$	$\frac{1}{24} \cdot \frac{fl^2}{Ey}$	4
VI.	 FIXED AT ENDS. LOADED UNIFORMLY.	$\frac{Wl}{12}$	3	$\frac{1}{384} \cdot \frac{Wl^3}{EI}$	$\frac{1}{32} \cdot \frac{Ml^2}{EI}$	$\frac{1}{32} \cdot \frac{fl^2}{Ey}$	8

The quantities in the sixth column are obtained by substituting the value of the maximum B.M. given by the third column in the fifth, and for those in the seventh we have put the value of M $\left(\text{viz., } \frac{fI}{y}\right)$ found in equation (V).

We also print for reference* a table of the strengths of materials and of the moduli of different sections.

* From Seaton & Rounthwaite's *Pocket Book of Marine Engineering Rules and Tables*, which may be consulted for other cases of beams; or Unwin's *Machine Design*, Part I.

Note.—Students who are specially interested in this subject, should procure a copy of the "British Standard Sections," issued by The Engineering Standards Committee, consisting of The Institution of Civil Engineers, Mechanical Engineers, &c. It comprises the following Lists of Standard Sections :—

List 1. Equal Angles.	List 5. Bulb Plates.
,, 2. Unequal Angles.	,, 6. Zed Bars.
,, 3. Bulb Angles.	,, 7. Channels.
,, 4. Bulb Tees.	,, 8. Beams.

List 9. Tee Bars.

Printed and published by William Clowes & Sons, Ltd., Charing Cross, London, S.W., in February, 1903.

These sections are now universally approved of, and it is impossible, even if I got liberty, to include the whole Report and figures in this Volume.

The following books on "Theory of Structures" and on the "Strength and Elasticity of Materials" may be consulted with advantage when solving the questions to be found in Parts IV. and V., as well as Appendix D. of this volume :—

Experimental Engineering, by Rollo C. Carpenter, C.E. (Chapman & Hall, London, 1895.)

Theory of Structures and Strength of Materials, by Henry T. Bovey, M A., D.C.L. (John Wiley & Sons, New York.)

"The Practical Strength of Beams." Paper by B. Baker. (*Proc. Inst. C. E.*, vol. lxii., p. 251.)

Mechanical Engineering Materials, by E. C. R. Marks. (Technical Publishing Co., Manchester.)

Strength and Properties of Materials, by W. G. Kirkcaldy, London.

Strength and Determination of the Dimensions of Structures of Iron and Steel, by J. J. Weyrauch, translated by A. J. Du Bois. (New York, 1891.)

Strength of Materials and Structures, by Sir J. Anderson. (Longmans, Green & Co., 1892.)

Civil Engineering and Applied Mechanics (latest Editions), by Prof. Rankine. (Chas. Griffin & Co.)

Design of Structures, by S. Anglin, C.E. (Chas. Griffin & Co.)

Bridge Construction, by Prof. Fidler, M.Inst.C.E. (Chas. Griffin & Co.)

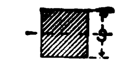
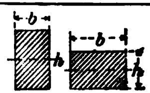
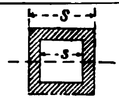
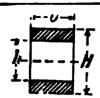
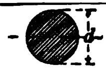
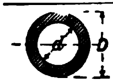
Strength of Materials, by Prof. Ewing, F.R.S., Cambridge.

STRENGTHS, &c., OF MATERIALS (SUMMARY).

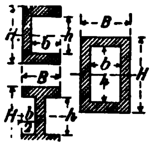
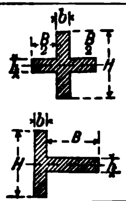
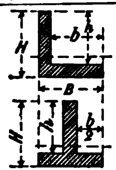
Material.	Ultimate Tensile Strength. lbs. per square inch.	Elastic Strength. lbs. per square inch.	Elongation per cent., when broken by Tensile Stress.
Cast-iron (ordinary good)	18,000	11,000	...
,, (Admiralty), .	not less than 20,160	...	...
Wrought-iron bars (ordinary good), . .	54,000	29,000	15 °/o in 8 ins.
Yorkshire plate—			
With grain, . .	54,000	26,000	20 °/o ,,
Across ,, . .	49,000	...	14 °/o ,,
Staffordshire plate—			
With grain, . .	50,000	24,000	12 °/o ,,
Across ,, . .	41,000	...	8 °/o ,,
Iron forgings—			
Large, . . .	45,000	...	9 °/o ,,
Small, . . .	50,000	...	13 °/o ,,
Steel castings (ordinary good), . . .	67,000	35,000	10 °/o ,,
Steel castings (Admiralty)	not less than 63,000	...	not less than 13½-18½ °/o in 2 ins.
,, (Lloyd's), .	not exceeding 67,000	...	not less than 10 °/o in 8 ins.
Steel boiler plate—			
(Ordinary good), .	65,000	36,000	20 °/o ,,
(Admiralty) internal,	not exceeding 60,480	...	20 °/o ,,
,, shell, .	60,480-67,200	not less than 31,360	...
(B. of T.) internal, .	58,240-67,200	...	...
,, shell, .	60,480-71,680	...	18 °/o in 10 ins.
Lloyd's, . . .	58,240-67,200	...	not less than 20 °/o in 8 ins.
Steel forgings (Admiralty)	62,720-78,400	34,500 to 43,120	28 °/o to 24 °/o in 2 ins.
Sheet copper, . .	30,000	5,600	35 °/o in 8 ins.
Copper wire (annealed),	40,000	...	...
Gun-metal (ordinary good). . . .	27,000	6,500	10 °/o in 2 ins.
Gun-metal (Admiralty),	31,000	...	...
Phosphor bronze (cast),	35,000	19,000	12 °/o in 2 ins.
Manganese bronze ,, .	55,000	...	10 °/o ,,
,, (rolled),	67,000	...	20 °/o ,,
Muntz metal, . .	50,000	30,000	30 °/o ,,
Naval brass, . . .	54,000	24,000	25 °/o in 8 ins.

Moment of Inertia, Modulus, &c., of some Sections.

The plane of bending is supposed perpendicular to plane of paper, and parallel to side of page.

Form of Section.	Area of Section. A	Moment of Inertia of Section about Axis through Centre of Gravity. I	Square of rads. of gyration of Section. $\rho^2 = \frac{I}{A}$	"Modulus" of Section. $Z = \frac{I}{y}$
	s^2	$\frac{s^4}{12}$	$\frac{s^2}{12}$	$\frac{s^3}{6}$
	bh	$\frac{bh^3}{12}$	$\frac{h^2}{12}$	$\frac{bh^2}{6}$
	$S^2 - s^2$	$\frac{S^4 - s^4}{12}$	$\frac{S^2 + s^2}{12}$	$\frac{1}{6}\left(\frac{S^4 - s^4}{S}\right)$
	$b(H - h)$	$\frac{b(H^3 - h^3)}{12}$	$\frac{H^2 + hH + h^2}{12}$	$\frac{b(H^3 - h^3)}{6H}$
	$\cdot 7854d^2$	$\cdot 0491d^4$	$\frac{d^2}{16}$	$\cdot 0982d^3$
	$\cdot 7854(D^2 - d^2)$	$\cdot 0491(D^4 - d^4)$	$\frac{D^2 + d^2}{16}$	$\cdot 0982\left(\frac{D^4 - d^4}{D}\right)$

MOMENT OF INERTIA, MODULUS, &c., OF SOME SECTIONS—*Continued.*

Form of Section.	Area of Section. A	Moment of Inertia of Section about Axis through Centre of Gravity. I	Square of rads. of gyration of Section. $\rho^2 = \frac{I}{A}$	"Modulus" of Section. $Z = \frac{I}{y}$
	$BH - bh$	$\frac{BH^3 - bh^3}{12}$	$\frac{1}{12}\left(\frac{BH^3 - bh^3}{BH - bh}\right)$	$\frac{BH^3 - bh^3}{6H}$
	$Bh + bH$	$\frac{Bh^3 + bH^3}{12}$	...	$\frac{Bh^3 + bH^3}{6H}$
	$BH - bh$	$\frac{(BH^2 - bh^2)^2 - 4BHbh(H-h)^2}{12(BH - bh)}$	...	$\frac{(BH^2 - bh^2)^2 - 4BHbh(H-h)^2}{6(BH^2 + bh^2 - 2bHh)}$

Lecture XXXII.—Questions.

1. A wrought-iron flanged girder is required to support a travelling load of 50 tons, the distance between the supports being 40 feet. The stress comes upon the girder at two points, the wheels on the traveller being 10 feet apart. What section of girder will be required to afford the necessary strength, presuming that the ultimate strength of the girder is six times that of the greatest stress to which it will be subjected?

2. Prove the law which governs the transverse strength of a beam of timber when supported at both ends and loaded at the centre. How are the constants required for applying this law arrived at?

3. A bar of wood, 7 feet long and 2 inches square, is supported at both ends, and is broken by a weight of 500 lbs. suspended at the centre. What weight in pounds will a rectangular bar of the same material, supported and loaded in like manner, sustain, when its length is 8 feet, its breadth $2\frac{1}{2}$ inches, and its depth 4 inches? *Ans.* 2187·5 lbs.

4. A rectangular beam of fir, of uniform section throughout, is supported horizontally on two walls 15 feet apart, and has to carry a load of $1\frac{1}{4}$ tons at 5 feet from one of the walls. The width of the beam is 5 inches; find its depth, taking the breaking load at four times the safe load. How much should the depth of the beam be increased, the breadth remaining constant, if the load were shifted from its original position to the centre of the beam, the breaking weight of a beam of fir 15 inches long, 1 inch broad, and 1 inch deep, supported at both ends and loaded in the middle, being taken at 360 lbs.? *Ans.* 8·9 inches; $\frac{1}{2}$ inch.

5. A solid rectangular girder, 3 inches deep and 2 inches broad, is supported at both ends on supports 5 feet apart. It is loaded with a uniformly distributed load, including its own weight, of 10 cwts. per foot run. What is the maximum intensity of stress at the outer fibres?

6. If two cast-iron beams—one circular in section and 2·73 inches in diameter, the other of rectangular section, 3 inches broad and 2 inches deep—be each supported at two points 20 inches apart, and loaded at the centre with a load of 2 tons; what will be the maximum intensity of stress produced in each case?

7. A beam of fir is built into a wall at one end, and projects 6 feet from the wall. The width of the beam is 4 inches; find its depth to bear safely a load of 1,200 lbs. uniformly distributed along its length. Assume that a bar of fir 1 foot long, 1 inch broad, and 1 inch deep, will break under a load of 125 lbs. when fixed at one end and loaded at the other end, and that the safe load is $\frac{1}{4}$ the breaking load. *Ans.* 6 inches.

8. What must be the breadth in inches of an oak cantilever or overhanging beam, 6 feet long and 9 inches deep, in order to carry a load of $\frac{1}{2}$ ton at its extremity, and how much must its breadth be increased in order that it may carry an additional load of $\frac{1}{2}$ ton uniformly distributed over its length? The actual stress is not to exceed $\frac{1}{4}$ of the breaking stress, and the breaking weight of an oak cantilever 6 inches long, 1 inch deep, and 1 inch broad, is 280 lbs. *Ans.* 2·37 inches; 1·18 inches.

9. A beam of fir supported at each end is inclined at an angle of 60° to the horizon, and is loaded at the centre of its length with a weight of 1 ton. The length of the beam is 10 feet, and its breadth is 2 inches, find the depth; the breaking load on the centre of a beam 1 foot long, 1 inch

broad, and 1 inch deep, and supported at the ends in a horizontal position, being 450 lbs. *Ans.* 3·527 inches.

10. A cast-iron cantilever or overhanging beam of T-section is 6 feet long, and 9 inches deep, the top flange being 6 inches wide. The beam has to carry, with safety, at its end a load of 1 ton, together with a distributed load of 1 ton over its length. Find the thickness of the top flange, the tensile breaking strength of cast-iron being 8 tons per square inch, and the admissible load for a safe stress being one-fourth the breaking load. *Ans.* 1 inch.

11. Find the greatest load that may be uniformly distributed on a cast-iron girder having top and bottom flanges united by a web of the following dimensions—width of upper flange 3 inches, of lower flange 9 inches, total depth 12 inches, thickness of each flange and of the web being 1 inch, distance between the points of support 10 feet—when the greatest admissible stress in the compression flange is 3 tons per square inch, and that in the tension flange is $1\frac{1}{2}$ tons per square inch. *Ans.* Maximum compressive stress = 2·5 tons; 8·8 tons.

12. Make a diagram of a flanged cast-iron girder to carry a load of 12 tons in the centre, the distance between the points of support being 20 feet. What should you make the depth of the beam, and what should be the sectional area of the top and bottom flanges respectively?

13. A rolled steel girder has a mean depth of 10 inches, the top and bottom flanges are each 6 inches wide, and the metal in the flanges and webs is $\frac{1}{2}$ inch in thickness throughout. If the breaking strength of the material be taken as 40 tons to the square inch of section for both tension and compression, then (using 4 as a factor for safety) what would be the maximum safe load uniformly distributed over such a girder, supposing it to be supported at each end, the supports being 12 feet apart? Also make a diagram showing the distribution of the shearing stress in the middle transverse section.

14. A rectangular beam of timber is supported at both ends, and loaded by a weight in the centre. Make the necessary calculations for measuring the strength of the beam to resist breaking. For a bar of larch 6 feet long by 2 inches square, supported as above, the breaking weight is 515 lbs.; taking this datum, you are required to solve the following question:—A cistern containing 2 tons of water rests on two cantilevers of larch, each 4 feet long and 5 inches in depth; find the breadth of each cantilever. *Ans.* 1·85 inches.

15. A cast-iron beam of rectangular section, and having its lowest side horizontal, is supported at both ends. What difference should you make in the upper outline according as the load is evenly distributed or collected in the centre?

16. A beam will safely carry a stationary load of 5 tons with a deflection of 2 inches, from what height may a weight of 200 lbs. be let drop upon the same beam without deflecting it to a greater extent? *Ans.* 56 inches.

17. A steady load of 10 tons, suspended at the centre of a beam, deflects it through $\frac{5}{8}$ inch. From what height would a weight of 300 lbs. require to fall in order to produce a like deflection when dropping on the beam? *Ans.* 22·7 inches.

18. A cylindrical iron beam is 15 inches in its external diameter, and the metal is $1\frac{1}{2}$ inches in thickness. The beam is fixed at the two ends, and is 35 feet between the supports; find the greatest load uniformly distributed that the beam will bear, the greatest safe stress on the metal being 9,000 lbs. per square inch.

19. Compare the resistance to bending of a wrought-iron I section beam

when the beam is placed like this I, and like this ⟝. The flanges of the beam are each 6 inches wide and 1 inch thick, and the web is $\frac{3}{8}$ inch thick and measures 8 inches between the flanges. (Adv. S. & A. Exam., 1897.)

20. A horizontal bar of round iron, 1 inch diameter, 6 feet long, hinged at the ends, is subjected to equal and opposite pushing forces of 1,000 lbs. at its ends, and a load of 10 lbs. is hung at the middle so that it is both a beam and a strut. Find the greatest stress anywhere. $E = 29 \times 10^6$ lbs. per square inch. (S. & A. Hons. Exam., 1897.)

21. Draw the bending moment diagrams, and state the maximum bending moments for the six standard cases of loading and supporting a beam of the same length, same load. (1) Fixed at one end, loaded at the other. (2) Fixed at one end, loaded uniformly. (3) Supported at the ends, loaded in the middle. (4) Supported at the ends, loaded uniformly. (5) Fixed at the ends, loaded in the middle. (6) Fixed at the ends, loaded uniformly. (Adv. S. & A. Exam., 1897.)

22. A uniform beam is fixed at its ends, which are 20 ft. apart. A load of 5 tons in the middle; loads of 2 tons each at 5 ft. from the ends. Find the diagram of bending moment and prove your rule. State what the maximum bending moment is, and where are the points of inflexion. (Hons. S. & A. Exam., 1897.)

23. A rectangular beam, loaded in the middle, supported at the ends; find the shear stress at any point in any section. Find the deflection at the middle, and distinguish between the parts due to ordinary bending and to shear. (S. & A. Hons. Exam., Part II., 1898.)

24. What occurs at the cross-section of a horizontal beam, carrying vertical loads? Where is the neutral line? What is the value of the stress at any place? What is meant by *bending moment?* Describe any model which illustrates, however roughly, what occurs at a section of the beam. (S. & A. Adv. Exam., 1898.)

25. A symmetrically loaded beam of uniform section; given the diagram of bending moment when supported at the ends, what is the easy rule for finding the diagram when the beam is fixed at the ends? Prove the rule to be correct. (B. of E. H., Part I., 1900.)

26. A beam of timber 2 feet long, 3 inches square, supported at the ends and loaded at the middle, breaks with a load of 7,500 lbs. What load may be expected to break a beam of the same material, fixed at one end and loaded at the other, length 10 feet, breadth 5 inches, depth 9 inches. If the specimen beam deflected 0·034 inch for a load of 1,000 lbs., what would be the deflection of the second beam for a load of 200 lbs.? (B. of E., Adv., 1901.)

27. Suppose the vertical loads and supporting forces of a horizontal beam to be known, show how we find (1) the shearing force at a section, (2) the position of the neutral line, (3) the compressive stress at any part of the section, and (4) the curvature of the beam. Prove your statements. (B. of E. H., Part I., 1901.)

28. Suppose the vertical loads and supporting forces of a horizontal beam to be known, show how we find (1) the position of the neutral line of a cross-section; (2) the compressive stress at any part of the section. Prove your statements. (B. of E. Adv., 1902.)

29. What are the functions of the top and bottom booms and of the diagonal pieces of a railway girder? Why are the booms usually larger in section towards the middle of the girder, and the diagonal pieces larger towards the ends of the girder? (B. of E. Adv., 1902.)

30. A rolled joist 12 inches deep, with flanges 4 inches wide and 1 inch thick, and a web $\frac{1}{2}$ an inch thick, carries on a 14-foot span a distributed

load of 1 ton per foot run, and a single concentrated load in the centre of $2\frac{1}{4}$ tons. Determine:—(*a*) The maximum bending moment and shearing force; (*b*) the maximum tensile and compressive stresses per square inch at the centre section; (*c*) the maximum shearing stress per square inch at a section close to the supports. (C. & G., 1901, H., Sec. A.)

31. Obtain a formula for the maximum deflection in a girder of uniform cross-section, freely supported at the two ends and loaded with a uniformly distributed load. Apply your formula to find how much a cast-iron bar of rectangular cross-section (2 inches deep and 1 inch thick) would deflect in the centre of a 3-foot span under a uniformly distributed load sufficient to produce a maximum stress per square inch of 8 tons. ($E = 12{,}500{,}000$ lbs. per square inch.) (C. & G., 1901, H., Sec. A.)

32. Describe, with sketches, any hydraulic lift you are acquainted with, explaining carefully any arrangements for counterbalancing the deadweight of cage and ram. (C. & G., 1901, H., Sec. C.)

33. If a beam, 12 feet long and 5 inches square section, be just strong enough to carry 1 ton at its centre, what weight may be placed at the centre of a beam of the same material 20 feet long, 6 inches broad, and 11 inches deep? (C. & G., 1902, O., Sec. B.)

34. State precisely the assumptions made in dealing with beams, and obtain an expression for the moment of resistance of a beam of known cross-section in terms of the dimensions of the section and of the stress induced. Explain in what respects the assumptions fail to hold when the stress exceeds the elastic limit. (C. & G., 1902, H., Sec. A.)

35. A cast-iron test bar is 1 inch wide and 2 inches deep, and is tested on 3-feet centres. The elastic deflection per ton load at the centre was found to be ·23 inch, and the bar broke when the load was $1\frac{1}{4}$ tons. Calculate the values of the modulus of rupture and of the coefficient of elasticity. (C. & G., 1902, H., Sec. A.)

LECTURE XXXII.—A.M.INST.C.E. EXAM. QUESTIONS.

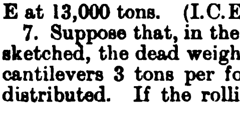
1. Assuming the relation $d^2y/dx^2 = M/EI$, find the slope at the ends and deflection at the centre of a beam of span l supported at the ends and loaded with W at the centre. (I.C.E., *Oct.*, 1897.)

2. Show the relations between the curves of loads, shearing force, and bending moment for a beam. Illustrate your answer by descriptions and sketches of the forms of these curves in the case of a beam supported at each end, and carrying a distributed load, varying in intensity at a uniform rate from zero at one end to w per foot at the other. (I.C.E., *Feb.*, 1898.)

3. A suspension bridge has a span of 800 feet, a dip (or sag) of cord of 60 feet, and carries by means of four cables, a total fixed load of 250 tons uniformly distributed along the length of the platform. Assuming the hanging rods to be very numerous, determine the tension in each cable at the lowest point and at the piers. Assuming the cables to be attached to saddles resting upon rollers on the tops of the piers and the anchor cables to make an angle of 45° with the vertical, determine the maximum stress in the anchor cables and the total pressure on each of the piers. (I.C.E., *Feb.*, 1898.)

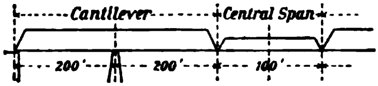
4. Prove the formulæ $\frac{M}{I} = \frac{f}{Y}$ and $\frac{M}{IE} = \frac{1}{r}$, with respect to beams, and explain the assumptions on which they are based. (I.C.E., *Feb.*, 1898.)

5. Deduce a formula for the maximum bending moment in a beam built into a wall at one end, supported at the other, and carrying a uniformly distributed load—assuming that when unloaded the beam touches but does not rest upon the outer support. You may assume any formulæ relating to cantilevers. (I.C.E., *Feb.*, 1898.)



6. In a rolled steel beam the section (symmetrical about the neutral axis) is such that the moment of inertia works out at 72 inch-units. The beam is 8 inches deep, and is laid across an opening of 10 feet, and carries a distributed load of 9 tons. Find the maximum fibre stress—also the central deflection, taking E at 13,000 tons. (I.C.E., *Oct.*, 1898.)

7. Suppose that, in the cantilever bridge which is here diagrammatically sketched, the dead weight of the central span is 1 ton per foot, and of the cantilevers 3 tons per foot; and that in each case the load is uniformly distributed. If the rolling load is taken at 2 tons per foot, what disposition of that rolling load would produce a reverse (sagging) moment in the cantilever? At what part of the bridge would the reverse bending moment take place? How far could it extend? What would be its greatest magnitude? (I.C.E., *Oct.*, 1898.)



8. As between two girder-bridges of the same span, and tested under the same load, what would you expect to be their relative deflections at the centre of the span—(*a*) When the two bridges are designed with the same sectional area of flange, but with different depths? (*b*) When the two bridges are designed with different depths of girder, but with the same working stress per square inch of flange section? (In each case assume the girders to have parallel flanges.) (I.C.E., *Oct.*, 1898.)

9. A beam of uniform section is used as a cantilever fixed at one end, A, and covered with a uniform load over the free length, A B. Give a hand sketch of the curve of deflection; prove that the curve is parabolic of the fourth order, and find the intersection of the two tangents drawn to the curve at A and B. (I.C.E., *Oct.*, 1898.)

10. A beam of uniform section is supported at the two ends, 30 feet apart, and is found to bend 6 inches downwards in the middle under its own weight. Determine the slope of the beam at the points of support. Find also what the slope would be if the same deflection of 6 inches were produced by the imposition of a central load instead of a uniform load. (I.C.E., *Oct.*, 1898.)

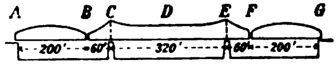
11. A horizontal beam of uniform section, whose moment of inertia is I, and whose total length is $2l$, is supported at the centre, one end being anchored down to a fixed abutment. Neglecting the weight of the beam, suppose it to be loaded at the other end with a single weight, W, and write an expression for the vertical deflection of that end below its unstrained position. (I.C.E., *Oct.*. 1898.)

12. A tee-bar, 6 inches wide by 3 inches in height and $\frac{1}{2}$ inch thick, is used as a beam, with the top table of the T downwards. Find the position of the neutral axis, and the moment of resistance when the maximum compressive stress is 4 tons per square inch. (I.C.E., *Feb.*, 1899.)

13. The chains of a suspension bridge, of 360 feet span, have a dip of 30 feet, and lie in a parabolic curve. Each of the two side spans has a width of 180 feet, the chains forming a parabolic arc similar to one-half of the central span. Under a total uniform load of 2 tons per lineal foot of roadway calculate the vertical and horizontal forces acting upon piers and abutments; also the tensile stress in the chain at the centre and at each end of the main span. (I.C.E., *Feb.*, 1899.)

14. In the suspension bridge above described, suppose the saddles to be fixed to the tops of the towers, or that the roller-bearings refuse to move, and that a load of 2 tons per foot covers the central span, while the load on each side-span is $1\frac{1}{2}$ tons per foot. Find the direction and the magnitude of the resultant force upon the top of each tower, and show how you would trace the line of pressure down the tower. (I.C.E., *Feb.*, 1899.)

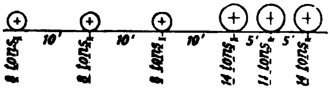
15. The cantilever bridge, whose outline is sketched below, is virtually hinged at B and at F. The dead load is $2\frac{1}{2}$ tons per foot throughout the



whole length; the live load, 2 tons per foot. Assume the live load to cover all the spans, and find the bending moments at the pier C and at the centre D of the middle span. (I.C.E., *Feb.*, 1899.)

16 In the structure above described, assume next that the live load extends from C to E, and lastly that it covers the two spans A C and E G; and for each of these cases find the bending moments at C and at D. (I.C.E., *Feb.*, 1899.)

17. Suppose a train to be made up as here sketched, a six-coupled tank-engine of 42 tons weight being followed by a train of wagons whose axles are at uniform distances of 10 feet, and with 8 tons on each axle. In



traversing a girder bridge of 50 feet span suppose the trailing axle of the engine to have reached the centre of the span; and find the greatest bending moment at any point in the girder due to the rolling load in that position. (I C.E., *Feb.*, 1899.)

18. When the bridge referred to in the previous question is traversed by the train therein described, find the greatest shearing force that takes effect at the right abutment during the transit, and also the greatest shearing force at the centre of the span. (I.C.E., *Feb.*, 1899.)

19. An iron wire hangs across an opening of 700 feet between the towers of a suspension bridge, with a dip of 50 feet. The length of the wire is therefore, approximately, 709½ feet. Calculate the elongation of the wire under the stress due to its own weight. (Assume the same modulus, and take the mean stress as being very nearly the arithmetical mean between the stresses at the centre and at each end.) (I.C.E., *Feb.*, 1899.)

20. What will be the ratio between the deflection of a girder of uniform I section, and one of uniform strength, both girders being designed for the same span, with the same uniform depth, and with the same maximum working stress per square inch of flange section under a given central load? (I.C.E., *Feb.*, 1899.)

21. What will be the relative deflections of the two girders above referred to if, in another case, they are both designed with the same maximum working stress under a given distributed load?

(I.C.E., *Feb.*, 1899.)

22. A continuous girder, built for crossing two equal spans, has a uniform section whose moment of inertia is I, while its uniform weight per foot lineal is w. The girder is launched across the spans from one end, and when its centre comes nearly over the central pier, the leading end will droop downwards under its unsupported weight. Write the expression for the extreme deflection of the leading end, the length of each span being denoted by L. (I.C.E., *Feb.*, 1899.)

23. Timber beams of a given rectangular section are employed to carry a uniform distributed load over two adjacent spans of equal width with a central support between the two spans. If we decide to make the beams continuous over the two spans, what advantage shall we obtain in respect of (a) strength and (b) rigidity, as compared with two discontinuous beams of the same section? (I.C.E., *Feb.*, 1899.)

24. If the beams referred to in the last question are made continuous, how much of the total load will be borne upon the central support, and how much at the extreme ends of the beams? (I.C.E , *Feb.*, 1899.)

25. Explain the term "neutral axis" used in the theory of the bending of a beam, and find its position for a beam of given section when the limit of elasticity is not overpassed. For example, take a tube of rectangular section 20 feet deep, 10 feet wide, thickness of top side ⅞ inch, of bottom side 1⅛ inch, and of the vertical sides ½ inch: find the neutral axis.

(I.C.E., *Oct.*, 1899.)

26. A beam of rectangular section, depth 8 inches, breadth 3 inches, 10 feet long, is supported horizontally at the ends and loaded with a weight

of 5 tons distributed uniformly. Employing the usual theory of transverse flexure: find the maximum stress produced at any point of a transverse section at the centre. (I.C.E., *Oct.*, 1899.)

27. Distinguish between the kinds of elasticity called into play when a bar is (*a*) stretched, (*b*) twisted, and (*c*) bent. Find the deflection of the beam of the last question assuming a modulus of 12,000 in ton-inch units. (I.C.E., *Oct.*, 1899.)

28. A horizontal beam of uniform section originally straight rests on immovable supports to which it is rigidly fixed, so that its ends remain horizontally when the beam is loaded. Find the shearing force and bending moment at any point due to a uniformly distributed load, and state in what ratio (*a*) the strength (*b*) the stiffness is increased by fixing the ends. (I.C.E., *Oct.*, 1899.)

29. Explain what are meant by the "booms" and "web" of a girder, and state the part played by each in supporting the load which it carries. If the booms be parallel and the web consist of a single triangulation of verticals combined with diagonals inclined at 45°, show how the diagonals must be arranged so as to be all in tension and find the stress on each, when the girder is loaded with 2 tons at each of the lower joints. Span = 120 feet. Number of divisions 8. (I.C.E., *Feb.*, 1900.)

30. In the last question find the maximum stress, both tensile and compressive, on each diagonal during the passage of an additional travelling load of 5 tons. (I.C.E., *Feb.*, 1900.)

31. Assuming that the deflection of a beam should not exceed a certain given fraction of the span, show that the least permissible ratio of depth to span can be found. What other considerations determine the ratio of depth to span in practice? (I.C.E., *Feb.*, 1900.)

32. Show that the moment of resistance to bending in beams of a given type is proportional to the sectional area multiplied by the depth Compare the strength of an I section, of depth 8 inches, breadth of each flange 3 inches, thickness of both flanges and web 1 inch, with that of a rectangular section of the same area and depth. (I.C.E., *Feb.*, 1900.)

33. The span, A C, of an independent girder supported at each end is 120 feet, and the uniform depth from centre to centre of booms is 12 feet. The web consists of a single system of diagonal ties and vertical posts which divide the span into ten equal panels. A uniform dead load of 16 cwts. per foot is assumed to be equally divided between the upper and lower joints. Under this load, calculate the compressive stress in post No. 2, which is 24 feet from abutment, A. (I.C.E., *Oct.*, 1900.)

34. The girder described in the last question carries a deck attached to the upper boom, and when a live load of 25 cwts. per foot (for each girder) extends along the deck from post No. 2 to the abutment, C, what will be the total compressive stress in post No. 2 due to dead and live load? (I.C.E., *Oct.*, 1900.)

35. In the railway bridge above described, the end-posts are vertical and the girders are supported beneath them upon rocker-bearings at A and roller-bearings at C, while the lower chords are united by wind-bracing. Under ordinary working conditions, what external forces can you discover that would impose stress of any kind upon the first bar of the lower chord at either end of the span? (I.C.E., *Oct.*, 1900.)

36. At the centre of a plate-webbed cross girder the section is made up as follows:—Web-plate 17 inches deep and $\frac{1}{2}$ inch thick; upper and lower flanges each formed of a plate $7\frac{1}{2}$ inches $\times$ $\frac{1}{2}$ inch, united to the web by a pair of angle-bars $3\frac{1}{2}$ inches $\times$ $3\frac{1}{2}$ inches $\times$ $\frac{1}{2}$ inch. Calculate the moment of inertia, I, of the section. (I.C.E., *Oct.*, 1900.)

37. If the bending moment at the centre of this cross girder is 60 foot-tons, what will be the maximum stress in the extreme fibres?
(I.C.E., *Oct.*, 1900.)

38. Prove that the "curvature" of an elastic beam of uniform section is everywhere proportional to the bending moment. (I.C.E., *Oct.*, 1900.)

39. In a girder of 150 feet span, supported at each end, the uniform depth from centre to centre of booms is 15 feet, and the sectional area of each boom is at all points adapted to a uniform working stress of 5 tons per square inch of gross section under the maximum load. What will be the deflection of the girder under the maximum load? (I.C.E., *Oct.*, 1900.)

40. When a beam of uniform section is supported at each end and loaded at the centre, prove that the deflection-curve is a cubic parabola.
(I.C.E., *Oct.*, 1900.)

41. A square bar of steel 3 inches by 3 inches, weighing about 30 lbs. per lineal foot, and 32 feet in length, is laid in a horizontal position and supported at each end. What will be the deflection of the bar under its own weight? Assume E = 28,000,000 lbs. (I C.E., *Oct*, 1900.)

42. Laying the same bar across a central support so that each arm becomes a cantilever 16 feet long, by how much will the ends droop below the level of the central support? (I.C.E., *Oct.*, 1900.)

43. Taking a similar bar of steel 36 feet in length, and laying it as a continuous beam across three equal openings, A B, B C, C D, of 12 feet each, find the bending moments at the two intermediate supports, B and C, and at the centre of the span, B C. Make also a sketch of the diagram of moments for the three spans. (I.C.E., *Oct.*, 1900.)

44. A beam of cast iron, 1 inch broad and 2 inches deep, is tested upon supports 3 feet apart, and shows a deflection of $\frac{1}{4}$ inch under a central load of 1 ton Calculate the modulus, E. (I.C.E., *Feb.*, 1901.)

45. A rolled steel beam of uniform section is carried at the ends upon supports 20 feet apart. The moment of inertia of its section amounts to 300 inch-units; and the lower edge of the beam is 6 inches below the neutral axis. When a concentrated load of 8 tons is placed upon the centre, what will be the maximum tensile stress in the extreme fibres?
(I.C.E., *Feb.*, 1901.)

46. In the case of the beam described in Question 6 under the central load of 8 tons, calculate the deflection at the middle of the span, assuming the modulus E to be 13,000 tons per square inch. (I.C.E., *Feb.*, 1901.)

47. For the same beam calculate the deflection at the middle of the span under a uniformly distributed load of 16 tons. (I.C.E., *Feb.*, 1901.)

48. If two precisely similar beams of rectangular section, one of cast-iron and the other of wrought-iron, were laid across the same span and loaded with the same load (within the elastic limit), what would be the relative deflections of the two beams? (I.C.E., *Feb.*, 1901.)

49. A straight bar of steel 40 inches in length, 1 inch broad, and $\frac{1}{16}$ inch in thickness, is bent into the form of a bow, having an elastic deflection of 2 inches in the middle, and the ends are united by the bow-string. Taking the modulus, E, at 29,000,000 lbs., what will be the tension on the string? (I.C.E., *Feb.*, 1901.)

50. A beam of timber, rectangular in transverse section, is 2 inches broad, 3 inches deep, and 4 feet in length, and rests upon supports at its ends. The breaking load at the centre is 2,000 lbs. What would have been the breaking load if the beam had been 4 inches deep, 2 inches broad, and 4 feet between the supports, but loaded at a distance of 1 foot from one end? (I.C.E., *Oct.*, 1901.)

51. A rolled-steel joist 16 inches deep, with flanges 6 inches wide and

1 inch thick (the web being $\frac{3}{8}$ inch thick), is used to support a uniformly distributed load of 2 tons per foot run. If the span is 12 feet 6 inches, what is the maximum tensile stress in the metal of the lower flange? (I.C E., *Oct.*, 1901.)

52. In designing a plate girder it is found that the shearing force acting on a particular section is 212 tons. If the mean depth of the girder at that point is 12 feet, find (*a*) the thickness of the web-plate, (*b*) the pitch of the rivets uniting the web-plate to the booms. Assume a working shearing stress of 9,000 lbs. per square inch, and a diameter of $1\frac{1}{8}$ inch for the rivets. (I.C.E., *Oct.*, 1901.)

53. A brick wall 18 inches thick and 30 feet in height has to be carried by a steel plate girder over a span of 30 feet. Assuming that the brick-work weighs 120 lbs. per cubic foot, and that the weight of the floors supported by the wall is equal to an additional distributed load of $\frac{3}{4}$ ton per foot run of the girder, determine suitable cross-sections for the girder at the centre and at the ends. (I.C.E., *Feb.*, 1902.)

54. Find an expression for the maximum deflection of a uniform beam with its ends fixed in a horizontal direction under a uniformly distributed load of W pounds per foot run. If the beam is solid circular section $2\frac{1}{2}$ inches diameter, and the span is 10 feet, find (*a*) the maximum deflection, (*b*) the position of the points where there is no bending moment. Assume $E = 29{,}000{,}000$ lbs. per square inch. (I.C.E., *Feb.*, 1902.)

55. A floor is supported by wooden joists spaced 12 inches apart from centre to centre. If the joists are 9 inches deep and 3 inches wide, and the clear span is 18 feet, what distributed load per square foot could the floor safely carry if the maximum longitudinal stress in the wood is not to exceed 1,200 lbs. per square inch. (I.C.E., *Feb.*, 1902.)

56. In a test of a cast-iron beam on a span of 30 inches the following deflection results were obtained:—

Load in Centre of Span. Lbs.	Deflection in Centre. Inch.
200	0·010
400	0·018
600	0·028
800	0·037
1,000	0·046

Load in centre at rupture 3,800 lbs. If the cross section of the beam at the centre was 1·05 inches in breadth and 2·07 inches in depth, find the modulus of elasticity of the cast iron, and the maximum tensile stress when rupture occurred. (I.C.E., *Feb*, 1902.)

57. A beam 12 feet long is loaded at distances of 1, 5, and 10 feet from one end with weights of 4, 5, and 6 tons. It is supported at the middle and ends, the middle supporting force being 5 tons. Find the other supporting forces and the greatest bending moment. (I.C.E., *Oct.*, 1902.)

58. In a bridge carrying two lines of railway of standard gauge, the cross-girders are 26 feet long, and the maximum load that can come upon each is 60 tons, distributed between the four rails; design the flanges of an intermediate cross-girder if its depth is 3 feet. (I.C.E., *Oct.*, 1902.)

59. A suspension bridge, 120 feet span, is stiffened by two horizontal stiffening girders hinged at their centre; if the live load is 1 ton per foot run and it covers one-half of the bridge, find the horizontal component of the pull in the wire ropes of the bridge. (I.C.E., *Oct.*, 1902.)

60. A mild steel bar, 5 feet long, 3 inches deep, and 1 inch broad, has a load of $\frac{1}{2}$ ton applied at its centre; what would be the deflection if the

modulus of elasticity is 13,000 tons per square inch? Start with the fact that the second differential coefficient of the deflection is equal to the bending moment divided by the product of the modulus of elasticity and the moment of inertia. (I.C.E., *Oct.*, 1902.)

61. A wrought-iron beam 10 feet long, 6 inches deep, and 2 inches broad, has a weight of 5,000 lbs. dropped on it at its centre; what distance must this weight fall to produce a maximum stress of 5 tons per square inch in the beam? What load applied gradually at the centre would produce the same stress, and what would the deflection be under this load?

(I.C.E., *Oct.*, 1902.)

62. A girder of I section, 12 feet long, flanges 4 inches broad by $\frac{3}{4}$ inch thick, depth centre to centre of flanges 6 inches, carries a load of 30 tons at the centre; draw a diagram showing the variation of the maximum normal stress in the flanges from the ends to the centre. What is the total shearing stress between the flanges and web from each end to the centre? (I.C.E., *Oct.*, 1902.)

63. A beam of 80 feet span carries nine cross girders, including one at each end, 10 feet apart, which support a uniform load of 2 tons per foot run; draw the bending-moment diagram for the beam, and show that the ordinate at one-eighth the span is the shearing force to a certain scale in the first one-eighth span; that the difference between the ordinates at the second and first one-eighth of the span is the shearing force to the same scale in the second one-eighth, and so on. Give the scale of the bending-moment diagram, and state to what scale this shearing-force diagram would be drawn. (I.C E., *Oct.*, 1902.)

64. A cast-iron beam is the shape of an inverted T 9 inches deep overall, width of flange 6 inches, thickness of web and flange 1 inch; if its length is 12 feet, find what weight at the centre will cause a tensile stress of 1 ton per square inch in the flange. What would the maximum compressive stress then be? (I.C.E., *Oct.*, 1902.)

65. A tank, which weighs $\frac{1}{2}$ ton and measures 10 feet × 6 feet × 3 feet, is filled with water and carried on three girders placed lengthwise, but so that each girder takes the same weight. If the depth of the girders is 6 inches, find the necessary breadth and thickness of the flanges.

(I C.E., *Feb.*, 1903.)

66. What is meant by resilience? A timber beam 30 feet long and 12 inches square in cross-section rests on a support at each end; if a load of 1 ton is placed in the centre of the beam, find the work done in deflecting it. (I.C.E., *Feb.*, 1903.)

PART VI.—HYDRAULICS AND HYDRAULIC MACHINERY.

LECTURE XXXIII.

HYDROSTATICS—HYDRAULIC MACHINES.

CONTENTS.—Hydraulics—Fluids—Viscosity—Transmission of Pressure by a Fluid—Pressure of a Heavy Fluid—Head—Pressure on an Immersed Surface—Examples I., II., III, and IV.—Centre of Pressure—Centre of Pressure on a Rectangle—Triangle—Circle—Example V.—Energy of Still Water—Common Suction Pump—Belt-driven Suction Pump—Example VI.—Air Pump—Single-acting Force Pump—Single-acting Force Pump with Ball Valves—Force Pump with Air Vessel—Continuous Delivery Pumps without Air Vessels—Double-acting Force Pump—Double-acting Circulating Pump—Worthington Steam Pump—Pulsometer Pumps—Roots' Blower—Bramah's Hydraulic Press—Examples VII. and VIII.—Hydraulic Flanging Press—Hydraulic Jack—Examples IX., X., and XI.—Hydraulic Bear—Lead-covering Cable Press—Hydraulic Accumulator—Example XII.—Hydraulic Cranes—Hydraulic Wall Crane—Movable Jigger Crane—Double Power Hydraulic Crane—Hydraulic Capstan—Questions.

Hydraulics.—In its widest sense, the term "Hydraulics" is given to the study of the mechanical properties of fluids and their application to practical purposes. In a more restricted sense, it refers to the science of the pressure and flow of water and their applications in engineering. It is divided into two sections:—**Hydrostatics**, the science of fluids at rest; and **Hydrokinetics**, the science of fluids in motion.

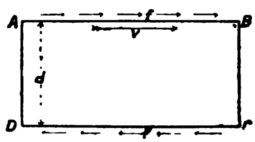
Fluids.—In many investigations it is necessary for simplicity to assume that we are dealing with a *perfect fluid;* that is, one which possesses the following property:—

DEFINITION.—**A fluid is a substance which offers no resistance to a continuous change of shape.**

There are two kinds of fluids—those which are practically incompressible, termed *liquids;* and those which are easily compressed, called *gases* and *vapours.* We know of no substance which completely fulfils the above definition; but water, many other liquids, and all gases, so nearly comply with it, that for many purposes we may, in practice, consider them as perfect fluids.

Viscosity.—Ordinary fluids, however, do offer some resistance to a change of shape, and the property in virtue of which they do so is called the *viscosity* of the fluid. A substance, such as syrup, which offers considerable resistance to a rapid change of form, but which goes on changing its shape so long as any deforming forces are applied to it, however small these forces may be, is usually called a *viscous fluid*. This term, strictly speaking, applies to all fluids, since all have some viscosity. It should, however, be noted that even the most mobile fluid will offer an appreciable resistance to a *sudden* deformation, because parts of it have to be set in motion and their inertia comes into play. This must not be confounded with their viscosity.

A solid body differs from a viscous fluid in that a small force produces in it a definite change of shape in a short time, and thereafter no further deformation takes place. Many solids, however, such as lead, tin, copper, and iron, when subjected to very great stresses, behave like viscous fluids, and keep *flowing* as long as the pressure is kept up. Even with very small forces, such apparently solid bodies as sealing wax and cobbler's wax, which fly to pieces when we subject them to a sudden force, such as a blow from a hammer, will gradually yield when sufficient time is allowed, and consequently they must be considered as very viscous fluids. For instance, a leaden bullet will sink in a thick piece of cobbler's wax and a cork will rise upwards through it, just as they would do through syrup or water, but they may take many months or years to do so.



VISCOSITY OF A FLUID.

The viscosity of a fluid is measured by the shear stress required to deform it at the uniform rate of unit shear strain per unit time. Thus, if the figure represents a small portion of fluid and if a tangential stress f acts along A B and C D, the fluid will change its shape by the part A B moving along with a velocity v, relatively to the part D C.

Then, $\left.\begin{array}{c}\textit{Shear strain produced} \\ \textit{in unit time}\end{array}\right\} = \dfrac{\textit{Velocity of } \mathrm{A}}{\textit{Distance } \mathrm{A\,D}}$

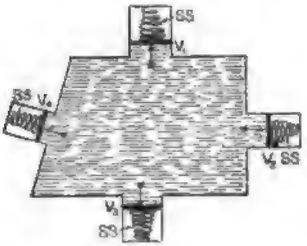
Or, $\quad$ *Rate of shear* $= \dfrac{v}{d} = \omega.$

$\therefore$ $\quad$ **Coefficient of viscosity** $= \mu = \dfrac{f}{\omega}. \quad . \quad . \quad . \quad . \quad$ (I)

When dealing with fluids at rest and in hydraulic machines,

such as presses, cranes, &c., in which the motion of the liquid is comparatively slow, we need not take account of their viscosity; but when considering their flow through pipes and channels it becomes of great importance.

Transmission of Pressure by a Fluid.—Pascal's law, that "fluids transmit pressure equally in all directions," follows at once from our definition of a fluid. Thus, take a vessel filled with a fluid and fitted with several frictionless pistons which all have the same area and are held in place by springs. If we now apply an inward force through the spring to one of the pistons, say V_1, we shall find that each of the other pistons will be pushed outward with the same force. Had the pistons been of different areas we should have found the forces proportional to their areas, showing that the pressure per unit area is the same in all directions.

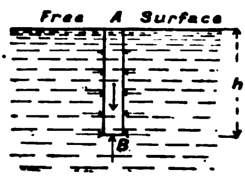


(HORIZONTAL SECTION.)

TRANSMISSION OF PRESSURE BY FLUIDS.

Another property following from our definition is that the pressure on any surface, real or imagined, is everywhere normal to that surface.

Pressure of a Heavy Fluid—Head.—Had the fluid in the above experiment been water or mercury and the pistons placed at different levels, we should have found that the pressure was not the same on all of them, but greatest on the lowest and least on the uppermost piston. This difference is due to the weight of the fluid. For example, if we have a quantity of liquid, the pressure at the bottom end B, of a vertical column A B, would be greater than at A; and, since the pressures round the sides of the column balance one another, the weight and the pressures on the ends must be in equilibrium. The difference of pressure is, therefore, equal to the weight of a cylinder of liquid whose length is A B and whose cross section has unit area. This will obviously be proportional to the length of the column A B; that is, to the difference of level.



PRESSURE OF A HEAVY FLUID.

In the figure, the upper surface is open to the atmosphere, and is, therefore, called the *Free Surface*. The pressure at A is atmospheric, but in connection with hydraulics it is customary to reckon

this as the zero of reference, and when we speak of the pressure of a fluid we mean the excess of its pressure above that of the atmosphere.

Since we can have any pressure by taking A B of suitable length, we very often measure a pressure by the length of the vertical column of liquid which it will support—or, what is the same thing, which will produce the same pressure—and we shall term this length the *Head*. The column itself we shall refer to as the *Pressure Column*. With water each foot of head, and with mercury each inch, corresponds to nearly half a pound per square inch. The exact figures are:—

1 Foot of Water = 0·434 lb. per square inch.

1 Inch of Mercury = 0·491 " "

If h be the height of the free surface above the point we are considering, and w the weight of unit volume of the liquid, the pressure per unit area will be:—

$$p = w\,h. \qquad \text{(II)}$$

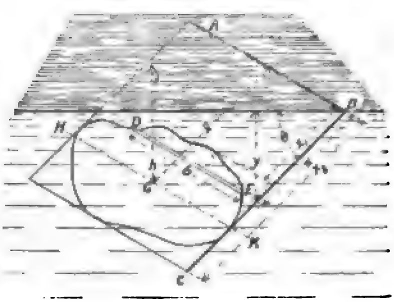
For fresh water, w may be taken as 62·42, or nearly $62\frac{1}{2}$ lbs. (1,000 ounces) per cubic foot, or 0·0361 lb. per cubic inch.

Pressure on an Immersed Surface.—If the above pressure be exerted on every unit of a surface whose area is a, the *total pressure* on that surface will be:—

$$P = a \times p = a\,w\,h. \qquad \text{(III)}$$

This is also frequently called "the pressure," but it is usually quite clear whether the total pressure or the intensity of pressure is meant, although they are commonly denoted by the same term.

On a plane surface which is not level, the intensity of pressure is not the same for all parts. In such a case, we may find the total pressure as follows:—



PRESSURE ON A SUBMERGED SURFACE.

Let A B be the intersection of the plane of the submerged surface with the free surface of the water. Draw the line B C in that plane perpendicular to A B, and take D E, a very narrow strip or element of the surface, at right angles to B C

and distant x from A B. A B and D E will be parallel since they are in one plane and both perpendicular to B C. Consequently, D E will be horizontal, and, therefore, the intensity of pressure over it will be uniform and equal to $w\,y$, y being the depth of the strip below the surface. Hence, if b be the breadth D E of the surface (*i.e.*, the length of the strip), and $d\,x$ that of the strip, the total pressure on the element will be $b\,d\,x \times w\,y$, or $b\,d\,x\,w\,x \sin\theta$ since $y = x \sin\theta$, if θ be the inclination of the plane to the horizontal. Now, we can split up the whole surface into a very large number of such elements and the total pressure on it will be the sum of all those on the elements:—

$$\therefore \qquad P = w \sin\theta \int_{x_1}^{x_2} b\,x\,d\,x.$$

But $b\,x\,d\,x$ is the area of an element multiplied by its distance from A B, and, therefore, from the definition of the centre of gravity of a lamina, the integral is equal to the whole area a multiplied by the distance of its centre from A B. Let this distance be $\bar{x}$, and let $h = \bar{x} \sin\theta$, be the depth of the centre below the surface:—

$$\text{Then,} \qquad P = w \sin\theta \times a\,\bar{x} = a\,w\,h. \qquad . \quad . \quad . \quad (\mathrm{III}_a)$$

This is evidently the same pressure as if the surface were level and immersed at the same depth as its centre of gravity.

Example I.—A cylindrical tank, 6 feet in diameter and 10 feet deep, is filled with water; find the bursting pressure round the base of the tank, and the pressure on its base.

Answer.—The bursting pressure round the base is measured by the intensity of the fluid pressure on any small area of the curved surface infinitely near to the base. This pressure will be exactly equal to that on the base. Hence, the question resolves itself into finding the intensity of the pressure on the base.

$\therefore$ **Bursting pressure round the base of tank** $\Big\}$ = *Pressure per square inch on base.*

„ „ = $h\,w$.

„ „ = $\frac{1}{144} \times 10 \times 62{\cdot}5 =$ **4·34 lbs. per sq. in.**

Again, **Total pressure on base** $\Big\}$ = $\Big\{$ *Area of base in sq. ins.* $\times$ *pressure per sq. in.*

Or, „ „ = $\frac{22}{7} \times 36 \times 36 \times 4{\cdot}34 =$ **17,678 lbs.**

EXAMPLE II.—A circular water tank is 20 feet in diameter and 25 feet deep. It is constructed of 6 rings of cast-iron plates. Find the *total* stress on any vertical section of the bottom row of plates made by a plane passing through the axis of the cylinder, neglecting any assistance afforded by the flanges or connection with the bottom plate. (S. and A. Exam., 1888.)

ANSWER.—It has been proved in Lecture XXIX. that when a cylindrical shell is subjected to internal fluid pressure, the *total* stress in the material along any section made by a plane containing the axis of the cylinder is equal to the total fluid pressure on either side of that part of the plane intercepted within the cylinder.

Hence, total stress in material of bottom row of plates = total fluid pressure on vertical plane through the axis of the cylinder at the bottom row of plates.

Since the breadth of each ring $= \frac{25}{6} = 4\frac{1}{6}$ ft.; therefore, depth of *c. g.* of bottom ring $= h = 25 - \frac{1}{2} \times 4\frac{1}{6} = 22 \cdot 9\dot{6}$ ft.

$\therefore$ **Total stress in material along section at bottom row of plates** $\Big\} = a\,h\,w,$

" " $= (20 \times 4\frac{1}{6}) \times 22 \cdot 9\dot{6} \times 62 \cdot 5$ lbs.

" " $=$ **119,357 lbs.**

EXAMPLE III.—How is the pressure of water on a given area immersed in it ascertained? A water tank, 8 feet long and 8 feet wide, with an inclined base, is 12 feet deep at the front and 6 feet deep at the back, and is filled with water. Find the pressure in lbs. on each of the four sides, and on the base; water weighing $62\frac{1}{2}$ lbs. per cubic foot.

ANSWER.—The total fluid pressure on any area immersed in the fluid is given by the formula—$P = a\,h\,w$.

Where a = Area of surface exposed to the fluid pressure,

h = Depth of centre of gravity of immersed area below free surface of fluid,

w = Weight of a cubic unit of fluid.

The shape and dimensions of the tank will be readily seen from the figure.

(*a*) *To find the total pressure on the front* A B C D.

Here, a = A D $\times$ D C = 8 $\times$ 12 = 96 sq. ft.

$h = \frac{1}{2}$ depth D C = 6 ft. $w = 62\frac{1}{2}$ lbs. per cubic ft.

$\therefore$ **Pressure on front A B C D** $= a\,h\,w,$

" " $= 96 \times 6 \times 62\frac{1}{2} =$ **36,000 lbs.**

(*b*) *To find the total pressure on the back* E F M N.

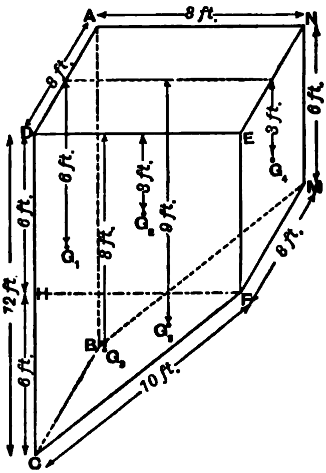
Here, a = F M × M N = 8 × 6 = 48 sq. ft.; $h = \frac{1}{2}$ E F = 3 ft.

∴ **Pressure on back E F M N** = $a\,h\,w$,

„ „ = 48 × 3 × $62\frac{1}{2}$ = **9,000 lbs.**

(*c*) *To find the total pressure on base* C B M F.

Before we can find the area of the base, we must know its length C F. From F draw F H parallel to E D, and therefore perpendicular to D C. Then C H F is a right-angled triangle



PRESSURE ON SIDES OF TANK.

whose sides are F H = E D = 8 ft., and H C = D C − D H = D C − E F = 6 ft.

$$\therefore \qquad CF = \sqrt{HF^2 + HC^2} = \sqrt{8^2 + 6^2} = 10 \text{ ft.}$$

$$\therefore \qquad a = CF \times CB = 10 \times 8 = 80 \text{ sq. ft.}$$

Again, the depth of the c. ⋅. of the base C B M F is clearly—

$$h = \tfrac{1}{2}(DC + EF) = 9 \text{ ft.}$$

∴ **Pressure on base C B M F** = $a\,h\,w$,

„ „ = 80 × 9 × $62\frac{1}{2}$ = **45,000 lbs.**

(*d*) *To find the total pressure on either side* C D E F or A B M N.

In this case it is perhaps best to divide the trapezoidal area C D E F into two figures whose centres of gravity can be easily determined. Thus, the line F H divides the side C D E F into a rectangle D E F H, and a triangle F H C. Then the total pressure on C D E F is equal to the sum of the pressure on D E F H and F H C.

Area of D E F H = 8 × 6 = 48 sq. ft.

And, *Depth of c. g. of area* D E F H $= \frac{1}{2}$ E F = 3 ft.

∴ *Pressure on* D E F H = *a h w*

" " = 48 × 3 × $62\frac{1}{2}$ lbs.

Again, *Area of* F H C = $\frac{1}{2}$ H F × H C = $\frac{1}{2}$ × 8 × 6 = 24 sq. ft.

The *c. g.* of triangle F H C is at a distance of $\frac{1}{3}$ of H C below the horizontal F H, and therefore at a distance of 6 + $\frac{1}{3}$ of 6 or 8 feet below D E.

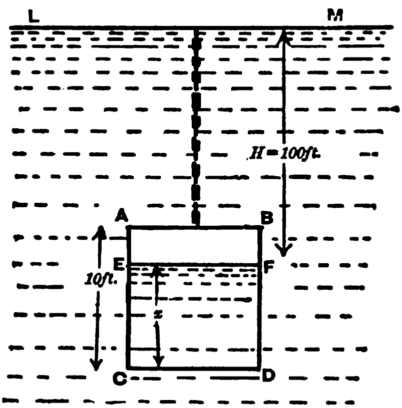
∴ *Pressure on* F H C = *a h w*,

" " = 24 × 8 × $62\frac{1}{2}$ lbs.

∴ **Pressure on side C D E F or A B M N** = 48 × 3 × $62\frac{1}{2}$ + 24 × 8 × $62\frac{1}{2}$ lbs.,

" " = 48 × $62\frac{1}{2}$ × (3 + 4) = **21,000 lbs.**

Example IV.—A cylindrical vessel, 10 feet long, open at one



PRESSURE IN DIVING BELL.

end and closed at the other, forms a diving bell. It is lowered

into water with its open end downwards until the surface of the water in the cylinder is at a depth of 100 feet. Find how far the water has risen in the cylinder, and the pressure of the contained air. (Take the height of the water barometer as 34 feet.)

Answer.—

Let H = Depth of surface of water in bell = 100 feet.
" h = Height of water barometer = 34 feet.
" l = Length of cylinder forming bell = 10 feet.
" x = Height that water rises in bell.

Before the bell is immersed in the water the pressure of the contained air is simply that due to the atmosphere. After immersion the pressure will be greater than that of the atmosphere by an amount due to a head of water of H feet.

Assuming, then, that the air in the bell has been compressed according to Boyle's Law ($p v$ = a const.), we get:—

$$\left.\begin{array}{c}\textit{Press. of compressed air}\\ \times\ \textit{Vol. of compressed air}\end{array}\right\} = \left\{\begin{array}{c}\textit{Press. of atmosphere}\\ \times\ \textit{Vol. of bell.}\end{array}\right.$$

$$\therefore \qquad \frac{\textit{Vol. of compressed air}}{\textit{Vol. of bell}} = \frac{\textit{Press. of atmosphere}}{\textit{Press. of compressed air}}.$$

Since the bell is of uniform cross sectional area throughout, we get:—

$$\frac{\textit{Vol. of compressed air}}{\textit{Vol. of bell}} = \frac{l - x}{l}.$$

$$\therefore \qquad \frac{l - x}{l} = \frac{h}{\mathrm{H} + h}.$$

$$\therefore \qquad x = \frac{\mathrm{H}\,l}{\mathrm{H} + h} = \frac{100 \times 10}{100 + 34} = 7{\cdot}46 \text{ feet.}$$

The pressure of the air in the bell when immersed is equal to the pressure due to a depth of (H + h) feet of water.

$$\therefore \quad \text{Pressure of air in bell} = a\,(\mathrm{H} + h)\,\mathrm{W} = \frac{1}{144} \times 134 \times 62{\cdot}5$$

$$\text{,,} \qquad \text{,,} \qquad = 58{\cdot}16 \text{ lbs. per sq. inch.}$$

Centre of Pressure.—We could balance the pressure on an immersed surface by a single force—the reverse of the resultant of the pressure—acting through a certain point in the plane of the surface, and this point is called the *Centre of Pressure.*

To find the depth of the centre of pressure we may proceed as follows :—

Referring to our former figure we see that the moment about A B of the pressure on the elementary strip D E is $w\,y \times b\,d\,x \times x$, or $w \sin\theta\, b\, x^2\, d\,x$. Hence the total moment is:—

$$M = w \sin\theta \int_{x_1}^{x_2} b\,x^2\,d\,x.$$

Now, $b\,x^2\,d\,x$ is the product of the area of an element into the square of its distance from A B, and consequently $\int_{x_1}^{x_2} b\,x^2\,d\,x$ is the second moment, or moment of inertia, of the area about A B. As shown in equation (III) of Lecture XXII., it is, therefore, equal to $I + a\,\bar{x}^2$, where I is the moment of inertia about an axis H K, through the centre of gravity parallel to A B :—

$$\therefore \qquad M = w \sin\theta\,(I + a\,\bar{x}^2).$$

Again, the moment of the resultant must be the sum of the moments of its components. Let X be the distance of the centre of pressure from A B :—

$$\text{Then,} \qquad P\,X = M = w \sin\theta\,(I + a\,\bar{x}^2).$$

$$\therefore \qquad X = \frac{w \sin\theta\,(I + a\,\bar{x}^2)}{P} = \frac{w \sin\theta\,(I + a\,\bar{x}^2)}{w \sin\theta\, a\,\bar{x}}.$$

$$\therefore \qquad X = \frac{I + a\,\bar{x}^2}{a\,\bar{x}}. \quad \ldots\ldots\ldots \quad \text{(IV)}$$

That is, the distance of the centre of pressure from A B is the ratio of the second moment of the surface about A B to its first moment about the same axis.

If for I we write $a\,k^2$, k being the radius of gyration about the axis H K, and h for $\bar{x} \sin\theta$, we get :—

$$X = \frac{a\,k^2 + a\,\bar{x}^2}{a\,\bar{x}} = \frac{k^2 + \bar{x}^2}{\bar{x}} = \frac{k^2 \sin^2\theta + h^2}{h \sin\theta}. \quad \ldots \quad \text{(V)}$$

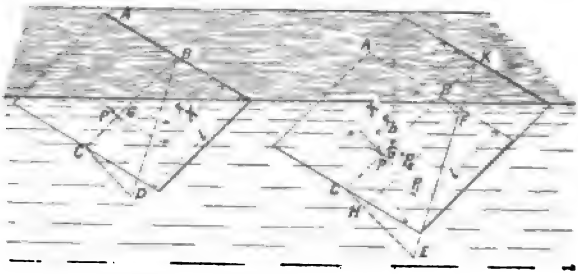
We shall now show how to apply these results to a few simple cases, and will also explain an easier method for special cases.

Centre of Pressure on a Rectangle.—First, consider a rectangle immersed with one edge in the surface. We find from Table II. in Lecture XXII., that the value of k^2 for a rectangle is $\frac{1}{12}\,l^2$,

where l is the length of the rectangle at right angles to the axis. Also, $\bar{x}$ will be $\frac{1}{2}\,l$:—

$$\therefore \qquad \mathbf{X} = \frac{\frac{1}{12}\,l^2 + \frac{1}{4}\,l^2}{\frac{1}{2}\,l} = \left(\tfrac{1}{6} + \tfrac{1}{2}\right) l = \tfrac{2}{3}\,l. \quad . \quad . \quad . \qquad \text{(VI)}$$

If the rectangle be immersed further, until its centre is at a depth h, the top edge being kept horizontal, we do not get such a simple result, but it can be at once obtained for any given case from equation (V).



CENTRE OF PRESSURE ON A RECTANGLE.

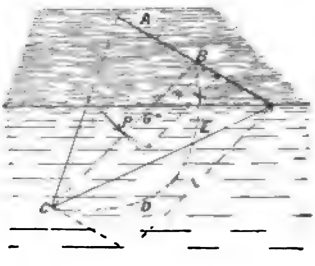
We can also obtain our result in the following manner:—At every point in B C—the central line of the rectangle—draw a line at right angles to it of such a length as to represent the whole pressure on a horizontal strip at that level. When the ends of these are joined we will have a triangle B C D, whose area represents the total pressure on the rectangle. The resultant pressure will pass through the centre of gravity of this triangle, and will, therefore, be two-thirds down from the vertex. Hence, the centre of pressure is distant two-thirds of the length of the rectangle from the top.

When the upper edge of the rectangle is below the surface, we obtain, instead of a triangle to represent the pressure, a trapezium B F E C, whose inclined sides, when produced, meet in the surface of the liquid. We ascertain the centre of pressure by finding the resultant of two forces, P_1 and P_2, the former of which is proportional to the area of the triangle F E H, and is two-thirds down from F, while the latter passes through the centre of the rectangle B F H C, and is proportional to its area. This may be done graphically as explained in Lecture XXVIII.

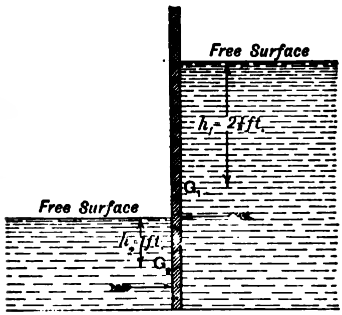
Centre of Pressure on a Triangle.—For a triangle with its base in the surface, $k^2 = \frac{1}{18}\,l^2$, and $\bar{x} = \frac{1}{3}\,l$:—

$$\therefore \qquad \mathbf{X} = \frac{\frac{1}{18}\,l^2 + \frac{1}{9}\,l^2}{\frac{1}{3}\,l} = \left(\tfrac{1}{6} + \tfrac{1}{3}\right) l = \tfrac{1}{2}\,l. \quad . \quad . \qquad \text{(VII)}$$

This may also be proved geometrically. The intensity of pressure on a horizontal strip is proportional to its depth below the surface, while the length of the strip, and, therefore, its area, is proportional to its distance from the vertex C. Consequently, the whole pressure on each horizontal element will be proportional to the product $x\ (l - x)$ for elements of the same width. The area representing the pressure will, therefore, be a parabola, B E D C, passing through B and C, with its axis perpendicular to B C, and, consequently, the centre of pressure must be half way down.



CENTRE OF PRESSURE ON A TRIANGLE.



PRESSURE ON SLUICE GATE.

If the vertex is in the surface, and the base horizontal, then $\bar{x} = \frac{2}{3} l$:—

$$\therefore \qquad X = \frac{\frac{1}{18} l^2 + \frac{4}{9} l^2}{\frac{2}{3} l} = \left(\tfrac{1}{12} + \tfrac{2}{3}\right) l = \tfrac{3}{4} l. \quad . \quad . \quad \text{(VIII)}$$

Centre of Pressure on a Circle.—The only other case we shall consider is that of a circle immersed vertically, with its centre at a depth h. Here $k^2 = \frac{1}{4} r^2$, and $\bar{x} = h$:—

$$\therefore \qquad X = \frac{\frac{1}{4} r^2 + h^2}{h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX)}$$

When the circumference just touches the surface, $h = r$, and this becomes :—

$$X = \tfrac{5}{4} r = \tfrac{5}{8} d. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(X)}$$

Where r is the radius, and d the diameter of the circle.

EXAMPLE V.—A sluice gate is 4 feet broad and 6 feet deep, and the water rises to a height of 5 feet on one side, and 2 feet on the other side. Find the pressure on the gate, and the centres of pressure.

ANSWER.—The net pressure on the sluice gate is evidently equal to the difference of the pressures on the two sides.

Total Pressure on Back $= a_1 h_1 w = (4 \times 5) \times 2{\cdot}5 \times 62{\cdot}5 = 3{,}125$ lbs.

" " *Front* $= a_2 h_2 w = (4 \times 2) \times 1 \times 62{\cdot}5 = 500$ "

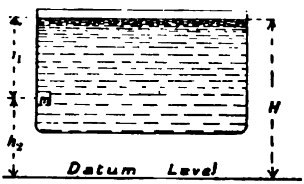
$\therefore$ Net pressure on gate $= 2{,}625$ "

The centre of pressure on the upper side is one-third of 5 feet, or 1 foot 8 inches, up from the bottom, and on the lower side, a third of 2 feet, or 8 inches. To find the resultant centre of pressure take moments about the bottom of the gate. Then, if this centre be distant x inches from the bottom :—

$$2{,}625 \times x = 3{,}125 \times 20 - 500 \times 8 = 62{,}500 - 4{,}000.$$

$$\therefore \quad x = \frac{58{,}500}{2{,}625} = 22{\cdot}3 \text{ ins.} = 1 \text{ ft. } 10{\cdot}3 \text{ ins.}$$

Energy of Still Water.—When water is at rest it possesses potential energy in virtue of its position and of its pressure. Consider a tank filled with water, and imagine a small mass m of the water to escape from the tank. This mass will not only lose potential energy through falling to a lower level, but it could also do work because of the pressure of the rest of the water pushing it away.



ENERGY OF STILL WATER.

It is convenient to assume some datum level at which we take the energy of position as zero.

Let H = Height of free surface above the datum level.
" h_1 = Height of free surface above m.
" h_2 = Height of m above datum level.
" g = Acceleration due to gravity.
" ρ = Density of fluid = mass of unit volume.
And, w = Weight of unit volume of fluid $= \rho g$.

Then the work done in forcing out the mass m is :—

Energy of Pressure = *Volume* × *Pressure.*

$$\text{" \quad "} \quad = \frac{m}{\rho} \times w h_1 = m g h_1.$$

And, *Energy of Position* $= m g h_2$.

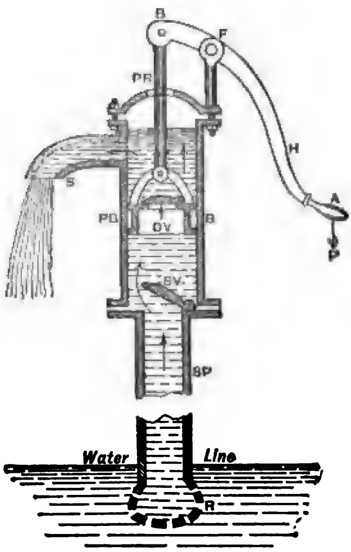
$\therefore$ *Total Energy* $= m g (h_1 + h_2) = m g \text{H}$.

Or, for a unit mass :—

$$\textbf{Energy per unit mass} = g (h_1 + h_2) = g \text{ H.} \quad . \quad \text{(XI)}$$

This is constant for all parts of a homogeneous fluid at rest, and H may be called the *total head* of the water in the tank.

Common Suction Pump.—This consists of a bored cast-iron barrel P B, terminating in a suction pipe, S P, fitted with a perforated end or rose R, which dips into the well from which the water is to be drawn. The object of the rose is to prevent leaves or other matter getting into the pump, that might clog and spoil the action of the valves. At the junction between the barrel and suction pipe there is fitted a suction valve S V, of the hinged clack type faced with leather. The piston or bucket B is worked up and down in the barrel of the pump by a force P, applied to the end of the handle H. This force is communicated to it through the connecting link of the hinged piston-rod, P R. In the centre and at the top of the bucket is fixed the clack delivery valve D V, which is also faced with leather in order to make it water-tight. The bucket is sometimes packed with leather; but, in the present instance, a coil of tightly woven flax rope is wrapped round the packing groove.



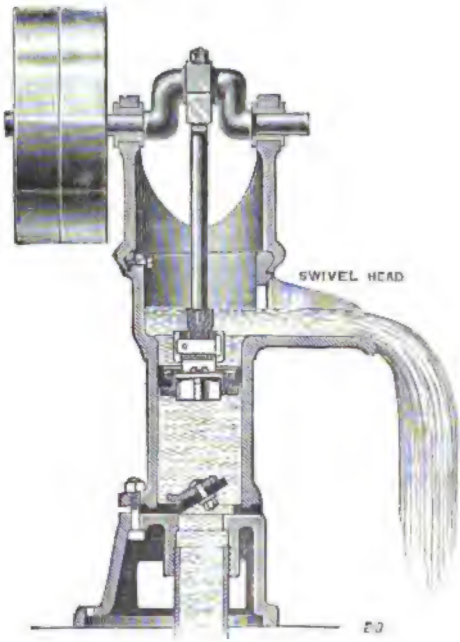
Common Suction Pump.

Action of the Suction Pump.—(1) Let the barrel and the suction pipe be filled with air down to the water-line, and let the bucket be at the end of the down stroke. Now raise the bucket to the end of the up stroke by depressing the pump handle. This tends to create a vacuum below the delivery valve; therefore, the air which filled the suction pipe opens the suction valve, expands, and fills the whole volume of the barrel. Consequently, according to Boyle's law, its pressure must be diminished in the *inverse ratio* to the enlargement of its volume. This enables the pressure of the atmosphere to force a certain quantity of water up the suction pipe, until the weight of this column of water and the pressure of the air between the suction and delivery valve, balance the pressure of the outside atmosphere.

(2) In pressing the bucket to the bottom of the barrel by ele-

vating the handle, the suction valve closes and the delivery valve opens, thereby permitting the compressed air in the barrel to escape through the delivery valve into the atmosphere.

(3) Raise and depress the piston several times so as to produce the above actions over again, and thus gradually diminish the volume of the air in the pump to a minimum. Then water will have been forced by the pressure of the atmosphere up the suction pipe and into the pump, if the bucket and the valves are tight, and if the delivery valve when at the top of its stroke be not more than the height of the hydro-barometric column above the water line of the well.*



Belt-driven Suction Pump.

(4) The bucket now works in water instead of in air; in fact, the machine passes from being an air-pump to being a water one. During the down stroke of the piston water is forced through the delivery valve, and during its up stroke, this water is ejected through the spout; at the same time, more water is forced up through the suction pipe and valve to fill the vacuum created by the receding piston. In the case of a common suction pump water is therefore discharged *only* during the up stroke of its piston.

* Theoretically, such a pump should be able to lift water from a depth of 34 feet below the highest part of the stroke of the delivery valve, but practically, owing to the imperfectly air-tight fitting of the piston and the valves, it is not used for withdrawing water from wells more than 20 to 25 feet below this position of the delivery valve. In fact such a pump frequently requires a bucket or two of water to be poured into it above the delivery valve in order to make it work at all, if it should have been left standing for some time without being worked.

Belt-driven Suction Pump.—When we have to raise a considerable quantity of water during a long time, then it becomes advisable to apply power derived from some prime mover. The foregoing figure illustrates an ordinary suction pump driven by a belt, with fast and loose pulleys, crank shaft, and connecting-rod. Here the suction valve and the piston are faced with leather or india-rubber, whilst the bucket valve is made of brass and ground to fit its seat. The upper portion of the pump is fitted with a swivel head, so that the pulleys may be placed fair in line with the driving pulleys.

LEATHER PACKING FOR PISTON OF SUCTION PUMP.

EXAMPLE VI.—Given two simple bucket pumps, each having a stroke of 1 foot, and cross area of bucket, 20 square inches. Suppose everything perfectly air tight, and the supply pipe 20 square inches area in one case, and 10 square inches area in the other. Neglecting friction, you are to compare the tensions on the pump rods at ends of first up stroke in each case, supposing the bucket to be 24 feet above free surface of the water in the well when at the bottom of its stroke. The supply pipe reaches to the under surface of bucket when the latter is at the bottom of its stroke.

ANSWER.—

Let p = Pressure in lbs. per *sq. ft.* on *under* surface of bucket at end of first up stroke.

" a = Area of bucket $= \frac{20}{144}$ sq. ft.

" h = Height of water barometer = 34 ft.

" w = Weight of 1 cub. ft. of water = $62\frac{1}{2}$ lbs.

" x = Height water rises in suction pipe for first up stroke.

FIRST CASE.—*Lifting or suction pipe having an area equal to that of the bucket.*

Then at end of first up stroke, we get:—

Pressure of air on upper surface of bucket, $\left.\right\}$ = *Atmospheric Pressure*

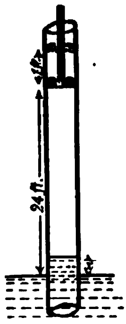
" " $= a\,h\,w$ *lbs.* . . . . (1)

Pressure on under surface of bucket, $= p\ a = \left\{ \begin{array}{l} \textit{Atmos. pressure} \\ - \textit{pressure of} \\ \textit{small column, } x, \\ \textit{of water} \end{array} \right.$

" " $= a\,(h - x)\,w.$

And, $p = (h - x)\,w.$ . . . . . (2)

Since there is exactly the same quantity of air between the bucket and the surface of the water in the pipe at the end of the stroke as there was before the stroke commenced, we may apply Boyle's Law to determine the *volume* of air under the bucket at end of stroke.



FIRST CASE.

$$\therefore \quad p\,a \times (25 - x) = a\,h\,w \times 24.$$

Substituting, $p = (h - x)\,w$, from equation (2), we get:—

$$(h - x)(25 - x) = 24\,h.$$

Since, $h = 34$ ft., we get, by substitution, and multiplication:—

$$x^2 - 59\,x + 34 = 0.$$

$$\therefore \quad x = \frac{59 \pm 57{\cdot}83}{2} \text{ ft.}$$

The minus sign in the numerator of the fraction on the right-hand side of this equation is the only one admissible.

$$\therefore \quad x = \frac{1{\cdot}17}{2} = {\cdot}58 \text{ ft. } or = 7 \text{ inches, nearly.}$$

Hence,

$$\text{Tension in pump rod} = \left\{ \begin{array}{l} \textit{Press. on upper surface of bucket} \\ \quad - \textit{Press. on under surface.} \end{array} \right.$$

$$\text{,,} \qquad \text{,,} \qquad = a\,h\,w - a\,(h - x)w = a\,x\,w.$$

$$\text{,,} \qquad \text{,,} \qquad = \frac{20}{144} \times {\cdot}58 \times 62{\cdot}5 = 5{\cdot}04 \text{ lbs.}$$

SECOND CASE.—*Lifting or suction pipe having an area equal to half that of the bucket.*

The symbols denoting the same quantities as before, we get:—

Pressure of air on upper surface of bucket $= a\,h\,w$ lbs. . . (3)

Pressure on under surface of bucket $= p\,a$

,, ,, $= a\,(h - x)\,w$. . . (4)

Vol. of air between bucket and surface of water at beginning of stroke . . $= \frac{1}{2}\,a \times 24$

,, ,, $= 12\,a$ *cub. ft.*

Vol. of air between bucket and surface of water at end of up stroke . . . $= \frac{1}{2}\,a \times (24 - x) + a \times 1$

,, ,, $= \frac{1}{2}\,(26 - x)\,a$ *cub. ft.*

∴ By Boyle's Law, we get :—

$$(h - x)w \times \tfrac{1}{2}(26 - x)a = hw \times 12a.$$

Substituting $h = 34$, and simplifying, we get:—

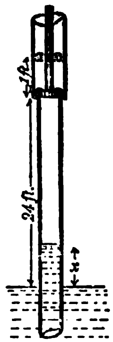
$$x^2 - 60x + 68 = 0$$

$$\therefore \qquad x = 1{\cdot}15 \text{ ft. } or = 13{\cdot}8 \text{ inches, nearly.}$$

Hence,

Tension in pump rod $= axw.$

$$\text{,,} \qquad \text{,,} \qquad = \frac{20}{144} \times 1{\cdot}15 \times 62{\cdot}5 = 10 \text{ lbs.}$$



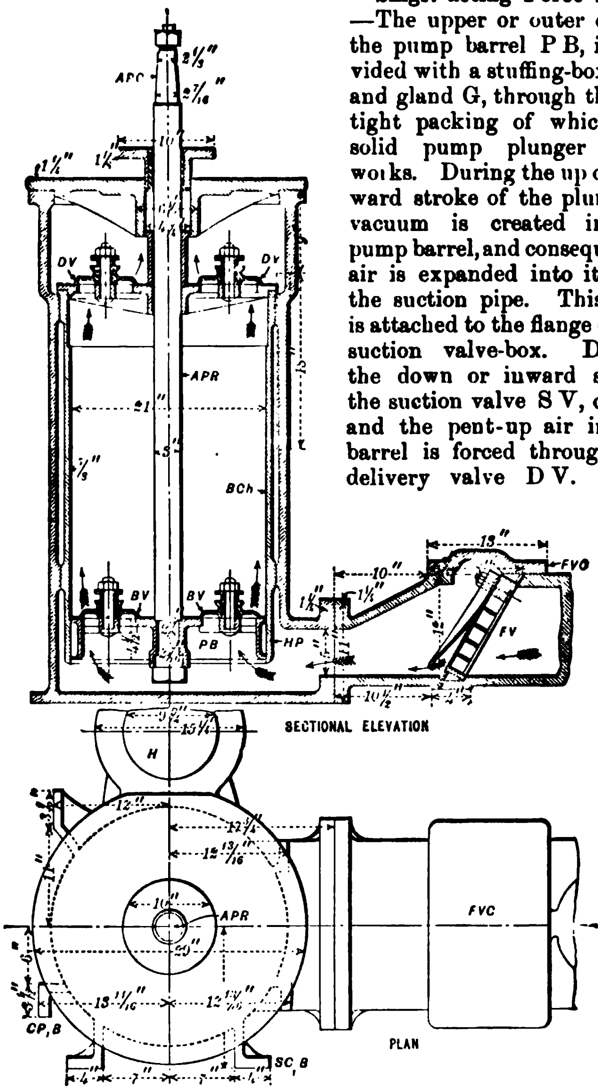
SECOND CASE.

Air Pump.—The figure on next page is a sectional elevation and outside plan of the air pump for the 1500 horse-power compound engines of the S.S. "St. Rognvald," which are fully described in the Author's *Text-Book on Steam and Steam Engines.* During the up stroke of the pump bucket P B, condensed steam and vapour are drawn from the surface condenser through the foot valve F V, into the space below the bucket, whilst any water and vapour that may have been lying above it, are forced through the delivery valves D V, into the hot well H. During the down stroke of the bucket, the water and vapour below it pass upwards through the bucket valves B V, into the space left by the descending bucket ; at the same time, the foot and delivery valves automatically close on their seats. These actions take place in succession during each up and down stroke of the air pump-rod A P R, which passes through an air-tight stuffing box, and is linked to the piston-rod crosshead of the high-pressure cylinder by short connecting-rods and side levers.

The object of placing the delivery valves on the top of the air-pump barrel in addition to the ordinary bucket valves, is to cause a vacuum to be produced above the latter during the down stroke of the bucket, and thus facilitate their opening, as well as to give the vapour from the condenser a free space between these two sets of valves into which it can expand.

The cast-iron barrel of the air pump is lined with a truly bored brass chamber B Ch, the pump bucket is rendered tight by hemp rope packing H P, the foot valve is readily inspected or removed by unbolting or lifting the foot valve cover F V C. whilst the whole is bolted securely to the surface condenser bracket $S\,C_1\,B$, and to the circulating pump bracket $C\,P_1\,B$.

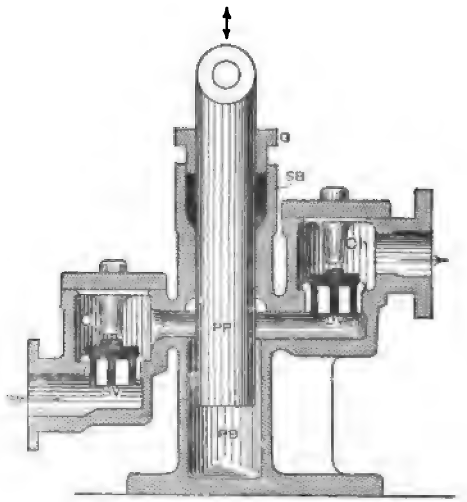
Single-acting Force Pump.—The upper or outer end of the pump barrel P B, is provided with a stuffing-box S B, and gland G, through the air-tight packing of which the solid pump plunger P P, works. During the up or outward stroke of the plunger a vacuum is created in the pump barrel, and consequently air is expanded into it from the suction pipe. This pipe is attached to the flange of the suction valve-box. During the down or inward stroke the suction valve S V, closes, and the pent-up air in the barrel is forced through the delivery valve D V. This



AIR PUMP FOR A MARINE ENGINE.

action goes on precisely in the manner just explained in the case of the suction pump, until the water rises into the barrel. Then the inward stroke of the plunger drives through the delivery valve to any desired height or against any reasonable back pressure, as in the case of a feed pump for a steam boiler.

Both the suction and the delivery valves are made of brass, and fit accurately into their brass seats. The covers to the valve chests are provided with checks Ch, to prevent the valves from rising more than the distance required to pass the water freely through them.*



SINGLE-ACTING FORCE PUMP.

The eye of the plunger may be attached to a connecting-rod actuated by a hand lever, as in the case of the common suction pump, or it may be worked from an eccentric or crank revolved by a steam engine or other motor. By whichever way it is worked, the force applied to the plunger must be sufficient to overcome the friction between the plunger and the packing, the resistance due to sucking the water from the source of supply, and of driving the same up to the place where it is delivered.

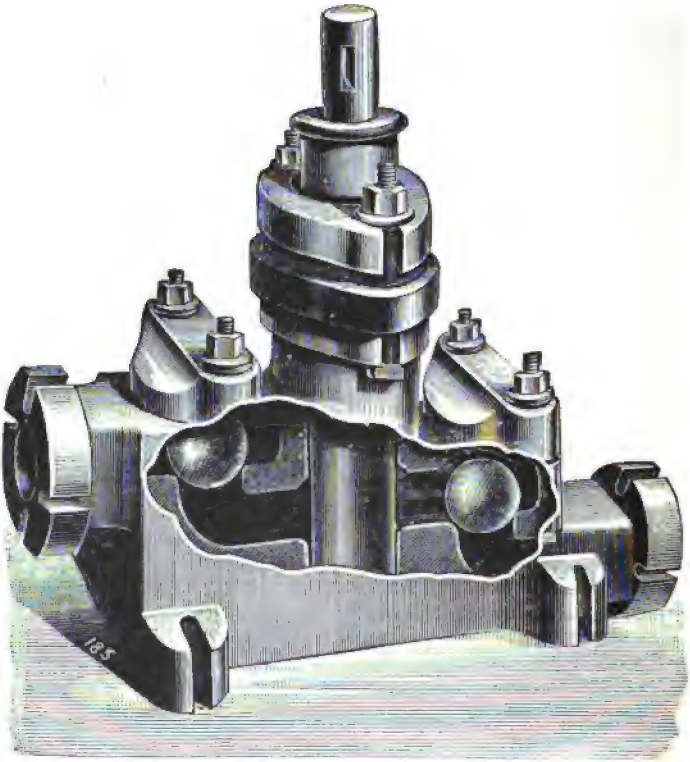
As in the case of the suction pump, the water is only delivered during each alternate stroke of the plunger, and, consequently, in an intermittent or pulsating fashion.

Very often three pumps of this kind are combined in one, each plunger being driven by a separate crank on a common shaft, and the cranks making angles of 120° with each other. Such an

* If d be the diameter of the bore of the valve seat, and h the required lift of the valve to give an opening equal in area to that bore, then h must be quarter of d. For the area of bore $= \frac{\pi d^2}{4}$ and the equivalent area of the valve opening $= \pi d h$. $\therefore \frac{\pi d^2}{4} = \pi d h$. Or, $h = \frac{d}{4}$.

arrangement is called a *three-throw pump*, and gives a very steady stream.

Single-acting Force Pump with Ball Valves.—The following illustration is a simple modification of the previous one, wherein ball valves are substituted for the common-circular three feathered type. The right-hand side forms the suction and the left-hand the delivery side. All the parts are made extra thick and strong



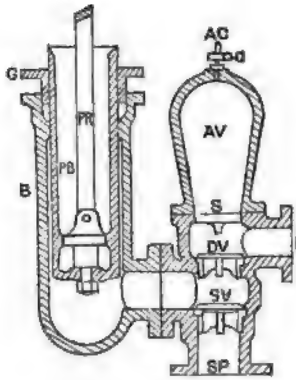
SINGLE-ACTING FORCE PUMP WITH BALL VALVES.

to resist shocks and vibrations, and most of the bolt holes have been cored to the outside of the flanges for the purpose of facilitating rapid connection and disconnection. This form of pump is much used for forcing feed-water into steam boilers, &c.

Force Pump with Air Vessel.—In the accompanying figure we

have an illustration of a force pump with both the suction and the delivery valves placed on one side of the pump barrel and then surmounted by an air vessel. The plunger, instead of being solid, as in the previous cases, is made up of a hollow trunk or barrel, with a connecting-rod fixed to an eye-bolt at its lower end.

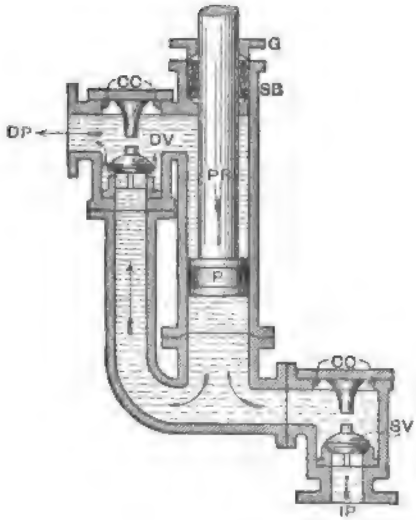
Action of the Air Vessel.—During the inward or delivery stroke of the plunger barrel P B, part of the water, which is forced from the barrel B, goes up through the delivery valve D V, into the delivery pipe D P, and the remainder enters the air vessel A V, and consequently compresses the air therein. During the outward or non-delivery stroke of the plunger the compressed air in the air vessel presses the rest of the water into the delivery pipe. In this simple way a continuous flow of water is maintained in the delivery pipe, and with far less shock, jar, and noise than in the previous cases. Where very smooth working is required, an air vessel is also put on to the suction pipe S P. Should the air in the air vessel become entirely absorbed by the water, the fact will be noticed at once, by the noise and the intermittent delivery. Then, the pump should be stopped, the air cock A C opened, and the water run out. When the air vessel is again full of air, the air cock should be shut and the pump restarted.



FORCE PUMP WITH AIR VESSEL.

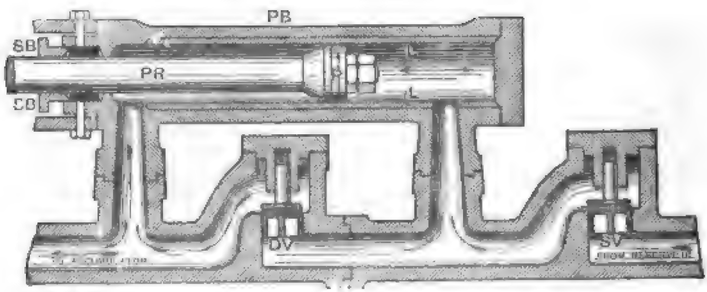
Continuous-delivery Pumps without Air Vessels.—A fairly continuous delivery of water may be obtained by making the plunger of the piston form, and the pump-rod exactly half its area; for here, during the down stroke, half the water expelled by the piston P, from the under side of the pump barrel goes up the delivery pipe D P, and the other half is lodged above the piston, to be in turn sent up the delivery pipe during the up stroke. Where very high pressures are required, such as in the filling of an accumulator ram, pumps working on this principle, but of the following form, are frequently used. The action is precisely the same as in the one just described, and the same index letters have been used, so that the student will have no difficulty in understanding the figure. The directions of motion of the piston and of the ingoing and outflowing water have been marked by straight and feathered arrows respectively.

With accumulators, and for other kinds of high-pressure work, it is not advisable to use air vessels, because you cannot prevent the water which enters them absorbing air and carrying the same with it to the hydraulic machines where its presence would be most objectionable. If 750 to 1000 or more lbs. pressure per square inch be generated, then you would require a very large and strong air vessel before it could be of any service. If a pressure of only 750 lbs. per square inch were used, then, since the normal pressure of the atmosphere is 15 lbs. per square inch, the air in the air vessel would be compressed, in accordance with Boyle's law, to $\frac{15}{750}$, or $\frac{1}{50}$ of its original volume. Consequently, with an air vessel of 50 cubic feet internal capacity, there would be only 1 cubic foot of air in it, when the pump was in full action.



CONTINUOUS-DELIVERY FORCE PUMP WITHOUT AN AIR VESSEL.

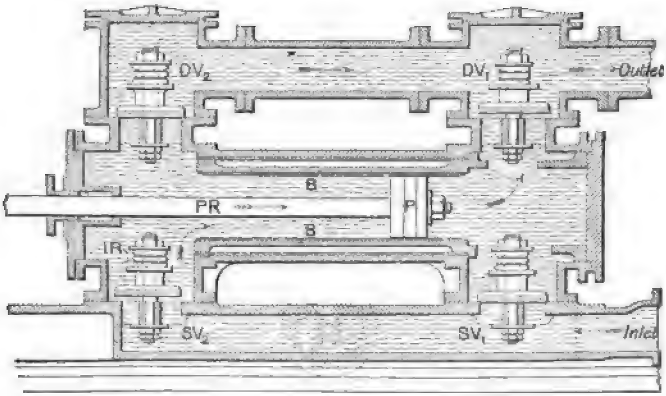
Double-acting Force Pump.—The pumps which we have hitherto considered are all single-acting, in the



CONTINUOUS-DELIVERY FORCE PUMP AS USED IN CONNECTION WITH THE ARMSTRONG ACCUMULATOR.

sense that they do not both suck and discharge water during

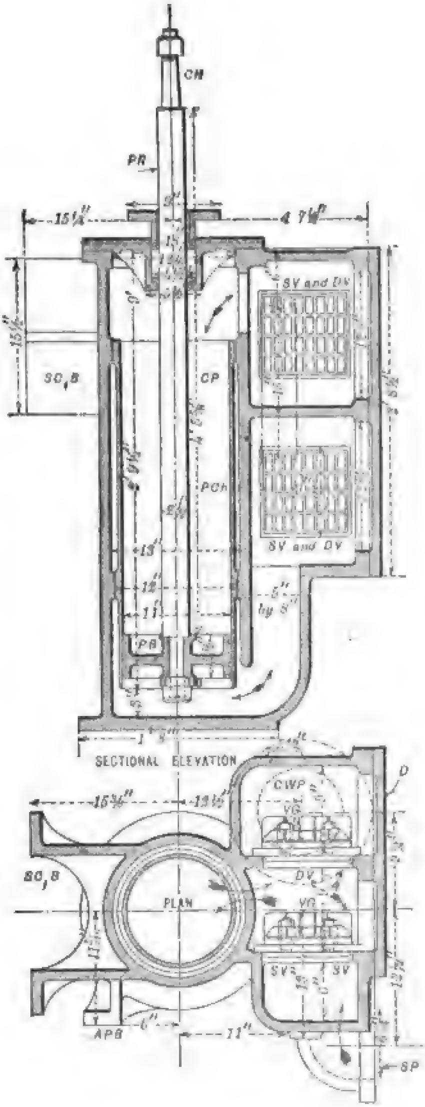
each stroke. This can, however, be accomplished by having two sets of suction and delivery valves placed at each end of the pump barrel, as shown by the accompanying figure. Here, during the outward stroke of the piston the pump draws water from the source of supply through the inlet pipe and suction valve S V_1, while, at the same time, the piston forces water in front of it through the delivery valve D V_2, and outlet pipe. During the inward stroke, suction takes place through S V_2 and discharge through D V_1, all as clearly shown by arrows in the drawing.



Double-acting Force Pump.

The valves are provided with india-rubber cushions I R, to ease the shock and minimise the jarring noise due to their reaction and natural reverberation when they are suddenly opened and closed.

Double-acting Circulating Pump.—The following figure is a sectional elevation and plan of the circulating pump for the same marine engines as the previously described air pump. During the upstroke of the piston or pump bucket P B, water is drawn from the sea through the suction pipe S P, and the lower suction valves S V, into the lower part of the pump chamber P Ch. At the same time, the water from the top part of the chamber is forced up through the upper delivery valves D V, along the circulating water pipe C W P, into the surface condenser tubes, and from thence into the sea. During the down-stroke of the piston, the water which had previously entered by the bottom of the pump chamber is forced through the lower delivery valves D V, into the condenser tubes and sea, and at the same time, more water is taken into the



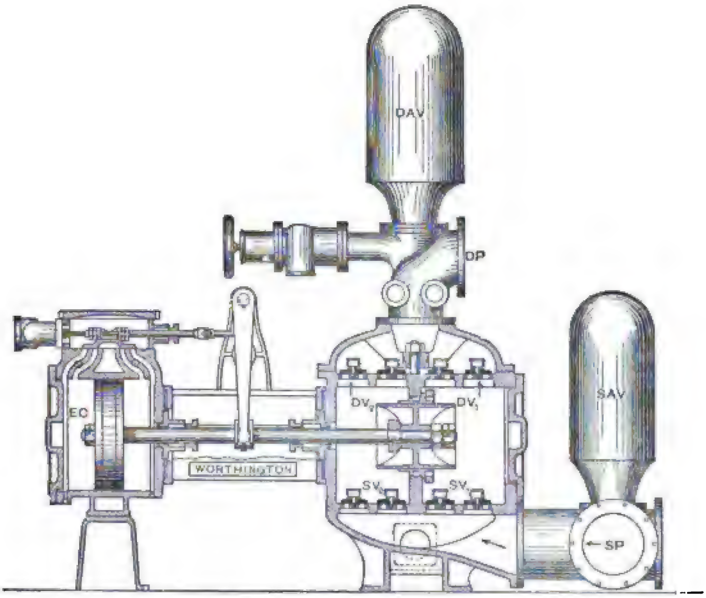
DOUBLE-ACTING CIRCULATING PUMP FOR A MARINE ENGINE.

top part of the pump chamber from the sea through the upper suction valves S V. These double actions take place in the manner described during each stroke, so that a stream of cold water is kept flowing through the condenser tubes, in order to maintain a vacuum during the exhaust of the steam from the low-pressure cylinder on to the outside of the cooled condenser tubes.

The cast-iron pump barrel of the circulating pump is lined with a truly bored brass pump chamber. The pump piston is also made of brass, and is rendered sufficiently water-tight by the simple device of turning three grooves in its outer cylindrical surface. The pump-rod passes through a water-tight gland and stuffing-box, and is connected to the reciprocating piston-rod cross-head by links and side levers, in the same way as the air pump-rod. The whole is securely bolted to the surface condenser and air pump brackets.

Worthington Steam Pump.—The perspective and sectional views

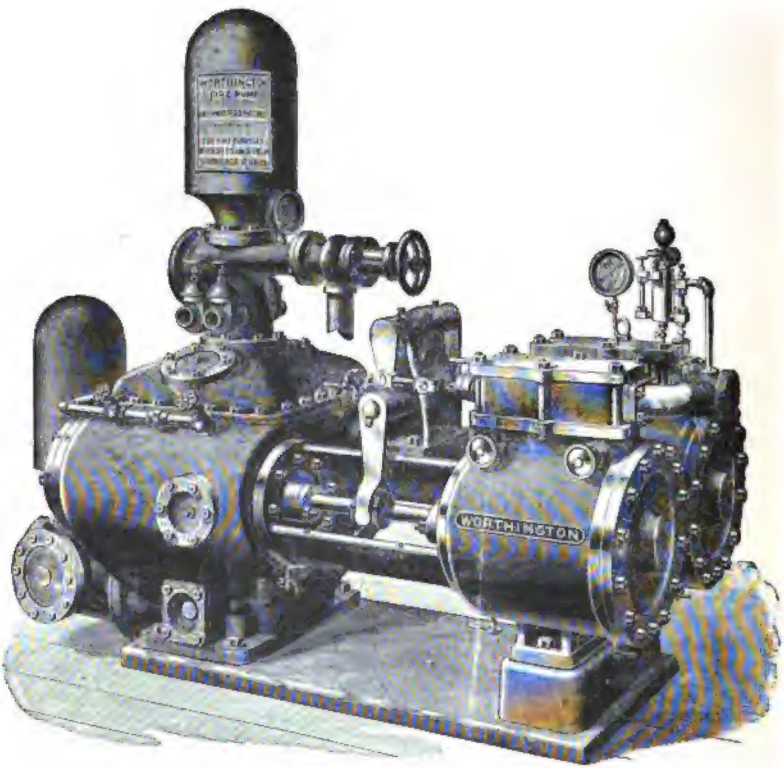
of this well-known pump, together with the following description, will serve to explain the construction and action of one of the best examples of the duplex class of steam pumps for feeding boilers, working accumulators, and hoists. It is termed a duplex pump, from the fact that it consists of two steam and two water cylinders placed side by side. The pumps draw water from the suction pipe S P, and are in this case safeguarded from shock on the suction side by an air vessel S A V. The water is admitted through the suction valves S V_1, S V_2, at each stroke respectively, and delivered by the valves D V_1, D V_2, into the discharge pipe D P, under the smoothing action of the discharge air vessel D A V.



VERTICAL SECTION OF THE WORTHINGTON STEAM PUMP.

The steam pistons and the pump plungers are directly connected together by a piston-rod, and give a swinging motion to the intermediate long levers L, which are attached by two separate spindles to two shorter levers which work the slide valve spindles. Whenever one of the steam pistons moves towards either end of its stroke the other piston is approaching the opposite end of its stroke, and by the combination of levers, piston-rods, and spindles the slide

valve of the one steam cylinder is actuated by its neighbour. The slide valves have neither lap nor lead, but immediately the piston of one cylinder covers one or other of the inner exhaust ports the steam in that cylinder is cushioned, and thus the pistons are prevented from striking their cylinder covers. Each piston as it reaches the end of its stroke automatically waits for its slide valve



Perspective View of the Worthington Steam Pump.

to be moved by the other piston-rod before it makes a return stroke. By this arrangement, the pump valves have time to close properly on their seats, and a natural smooth motion of the whole of the working parts takes place. There are no dead points in this form of duplex pump, consequently it is always ready to be started either

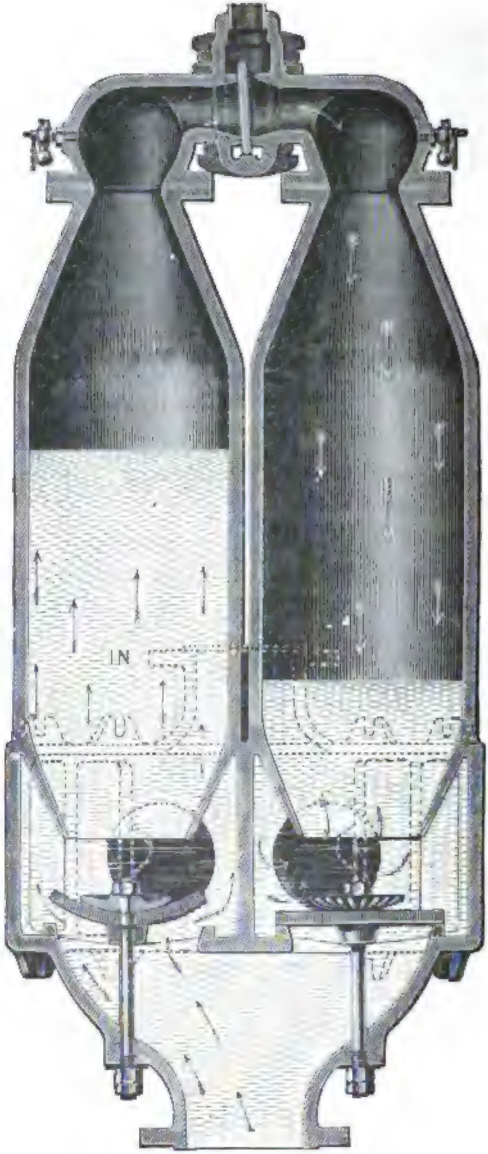
by the opening of the stop valve or by the automatic action of a float connected to the throttle valve by a chain or rope.

Pulsometer Pumps.—The very first steam pump, which was invented by Thomas Savery in 1698, had no working parts except the valves. This type has been revived for certain kinds of work in pumps of the pulsometer class, of which Bailey's "Aqua Thruster" is a good example. It consists of two long chambers, in each of which there is a valve opening upwards at the bottom, and one opening outwards at the side. At the top junction between these two chambers there is a flap valve which can put either in communication with a steam pipe while the other is shut off therefrom.

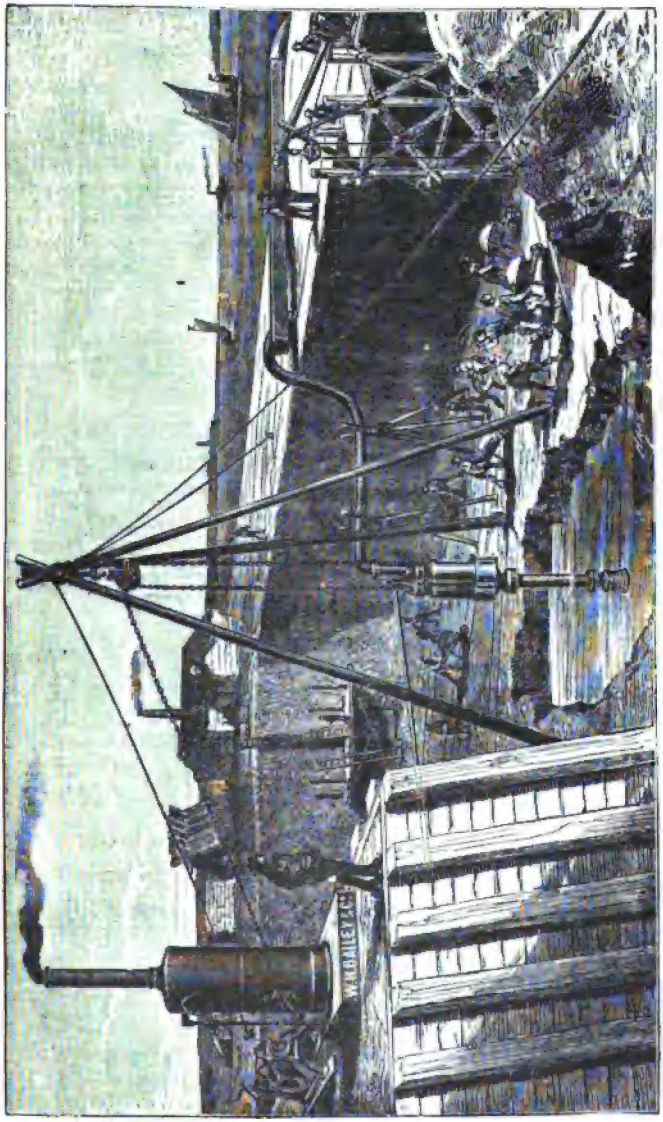
Now, suppose the right-hand chamber to be full of water while the left one is full of steam, and that the upper valve is in the position shown. Steam will enter the right-hand chamber and force the water out through the delivery valve at the side. At the same time the steam in the left compartment will be condensing, and water will therefore rise into it through the bottom valve, provided the apparatus be not too far above the free surface of the water. The inertia of this water will cause it to continue in motion after all the steam is condensed, and it will therefore compress the air that remains to a sufficient extent to shift over the valve to the other side.* If there is no air, then the water itself will strike the valve and knock it over to the other side. The conditions of the chambers are now interchanged. Water will be forced out from the left one, and fresh water will rise into the other, and the process begins again.

A large loss occurs in this kind of pump through the condensation of steam during the down stroke of the water, and also owing to the fact that the steam is used non-expansively. To reduce the former loss little cocks open into the top of the chambers and admit a little air during the time there is a vacuum inside. This air prevents the steam from coming so quickly into contact with the water as it otherwise would do, and thus reduces the loss during admission. A slight escape of steam takes place

* This is not the common explanation of the working of the pulsometer valve. It is—"As soon as the water is lowered below the upper surface of the delivery valve, steam blows through with some violence and causes a commotion and a rapid condensation in the chamber. The valve is then *drawn* to the right-hand side." This is quite wrong. The valve can only be shifted by being *pushed*, owing to the pressure on the closed side becoming greater than that on the other side, and it is difficult to see how the pressure in a chamber in direct communication with the boiler can become less than that in one where the steam is already all condensed, and where the pressure is considerably below that of the atmosphere. Besides, it is probable that in steady working the water never gets as low as the delivery valves.



Pulsometer Pump by W. H. Bailey & Co., Ltd., Manchester.

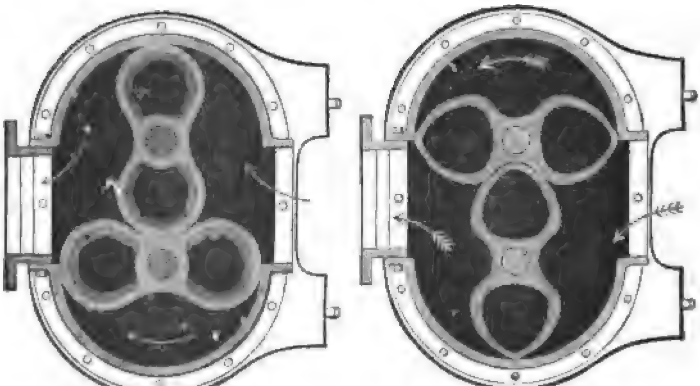


THE "AQUA THRUSTER" IN OPERATION.

through these cocks, as they are always kept open; but as their bore is so very small, the loss is less than the gain. To diminish the latter loss an extra self-acting valve, called the "grel," has been added to some forms of pulsometer with the intention of cutting off the steam earlier, and then using it expansively.

Pumps of this class are exceedingly handy for dealing with dirty water and for temporary purposes owing to their simplicity, few working parts, and the ease with which they can be erected. It is sufficient to suspend them by a chain and connect them by a pipe to a portable boiler. A suction pipe projects down below the water surface, and a flexible hose pipe will carry off the discharged water. The full page illustration shows the "Aqua Thruster" in use for pumping water from a dock during its construction.

Roots' Blower.—A form of rotary pressure pump, known as Roots' Blower, is used for obtaining a blast of air at a moderate



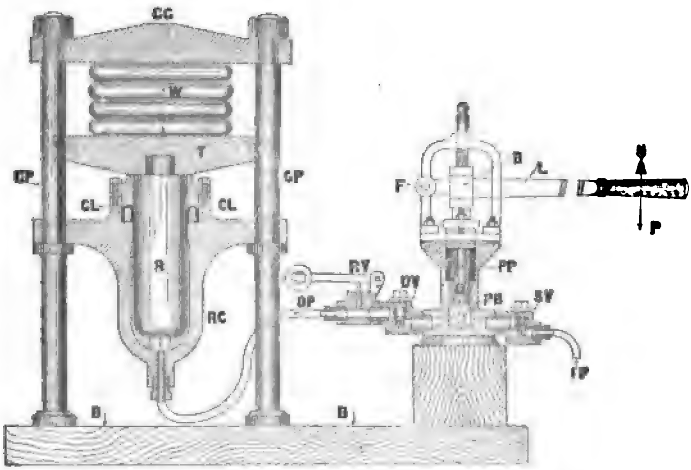
Two Forms of Roots' Blower.

pressure, and for pumping liquids. Two vanes rotate inside a closed casing, and sweep the fluid round with them. They are connected by spur wheels outside so as to be always at right angles to each other, and they have such a shape that practically nothing is carried backwards at the central part of the machine. They can produce a higher pressure than an ordinary blowing fan, and are handier for many purposes than a blower of the cylinder and piston type. The student should note that this is not a centrifugal pump or fan, although there are no reciprocating parts, but simply a rotary form of pressure pump.

If a fluid be forced through this machine, then it will cause the vanes or teeth to rotate; hence it will work as a motor, and there-

fore it is a reversible machine. Many ingenious attempts have been made to produce economical steam engines on this principle; but largely owing to the difficulty of keeping them tight, they have not been so successful as their sanguine inventors expected.*

Bramah's Hydraulic Press.—This useful machine was invented by Pascal, but he could not make the moving parts water-tight. Bramah, about the year 1796, discovered a means by which this difficulty was effectually overcome; and thus the instrument has been handed down to us under his name. As may be seen from the following figure, it consists of a single-acting force pump in connection with a strong cylinder containing a plunger or ram, which is forced outwards from the cylinder through a tight collar by the pressure of the water delivered into the cylinder from the force pump.



VERTICAL SECTION OF A SMALL BRAMAH HYDRAULIC PRESS.

After what has been written about force pumps, we need not particularise about this part of the machine, except to say that the suction and delivery valve boxes at S V and D V can be disconnected from the pump, and the valve cover-checks removed at any time for the purpose of examining the parts, or of regrinding the valves into their seats. The pump plunger P P, extends through a stuffing-box and gland filled with hemp packing, and is guided by a centrally bored bracket bolted to the top flange of the

* See Lecture XXXV. for Centrifugal Pumps.

pump. The lever L, fits through a slot in this guide-bar, whereby it has an easy free motion, when communicating the force applied through it to the pump plunger. The relief valve R V, has a loaded lever so adjusted as to rise and let the water escape when the pressure exceeds a certain amount. It may also be used for ascertaining the pressure on the object under compression, or for lowering the ram R, by simply lifting the little lever and pressing down the table T, when the water flows easily from the ram cylinder R C, by the delivery pipe D P, and the relief valve. The delivery pipe is made of solid drawn brass, and the ram cylinder is carefully rounded at the bottom end, instead of being flat, in order that it may be of the strongest shape.* The guide pillars G P, are securely bolted to the base B, and to the top cross girder C G, by nuts and washers.

CROSS SECTION OF ORDINARY LEATHER PACKING.

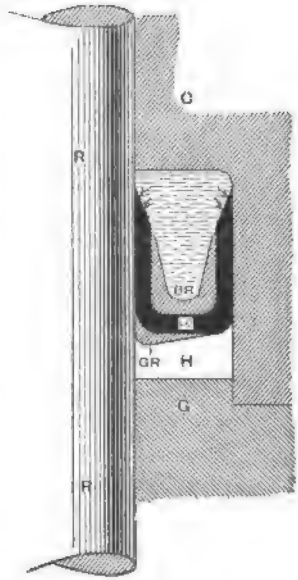
The cup leather packing C L, deserves special attention, because it formed the chief improvement by Bramah on Pascal's press. It consists of a leather collar of ∩ section, placed into a cavity turned out of the neck of the cylinder, and kept there by the gland of the cylinder cover.

This collar is made from a flat piece of new strong well-tanned leather, thoroughly soaked in water, and forced into a metal mould of the requisite size and shape to give it the form of a U collar. The central or disc portion of the leather is then cut out, and the circular edges are trimmed up to a sharp bevel as shown.

The following figure shows an enlarged section of Bramah's packing suitable for a huge press, where the desired shape of the leather collar L C, is maintained by an internal brass ring B R, and an outside metal guard ring G R, resting on a bedding of hemp H. It will be observed at once, from an inspection of this figure, that the water which leaks past the easy fit between the plunger or ram R, and the cylinder C, presses one of the sharp

* In the case of large cylinders for very great pressures, the lower or inner end of the cylinder should be carefully rounded off, both inside and outside. For, if left square, or nearly square, the crystals formed in the casting of the metal naturally arrange themselves whilst cooling in such a manner as to leave an initial stress, and consequent weakness, inviting fracture along the lines joining the inside to the outside corners of the cylinder end. The severe shocks and stresses to which this weak line of division is subjected during the working of the press would sooner or later force out the end of the cylinder, in the shape of the frustum of a cone, unless the cylinder had been made unnecessarily thick and heavy at the bottom end.

edges of the leather collar against the ram, and the other edge against the side of the bored cavity in the neck of the cylinder, with a force directly proportional to the pressure of the water in the cylinder. By this simple automatic action, the greater the pressure in the cylinder the tighter does the leather collar grip the ram and bear on the cylinder's neck.



Leather Collar for a Large Hydraulic Press.

Referring again to the figure of the Bramah press, by taking moments about the fulcrum at F, we obtain the pressure Q, on the plunger of the force pump. Neglecting weight of lever and friction, we get :—

$$P \times AF = Q \times BF.$$

$$\therefore \qquad Q = \frac{P \times AF}{BF}.$$

Further, we know that the statical pressure Q, is transmitted with undiminished force to every corresponding area of the cross section of the ram. Hence,

Q : W : : area of plunger : area of ram.

$$\therefore \qquad W \times \text{area of plunger} = Q \times \text{area of ram}.$$

Or,
$$W \times \pi r^2 = Q \times \pi R^2.$$

Where r = radius of plunger, and R = radius of ram, both in the same unit. Substituting the previous value for Q, and dividing each side of the equation by π, we get :—

$$W \times r^2 = \frac{P \times AF}{BF} \times R^2.$$

$$\therefore \qquad W = \frac{P \times AF}{BF} \times \frac{R^2}{r^2} = \frac{P \times AF}{BF} \times \frac{D^2}{d^2}.$$

Where D and d are the diameters of the ram and plunger respectively.

Example VII.—In a small Bramah press, P = 50 lbs., A F = 20 ins., B F = 2 ins., area of plunger = 1 sq. in., whilst area of

ram = 14 sq. ins. Find W, neglecting friction and weight of lever.

ANSWER.—By the above formula :—

$$W = \frac{P \times A F}{B F} \times \frac{R^2}{r^2}.$$

$$\therefore \qquad W = \frac{50 \times 20}{2} \times \frac{14}{1} = 7{,}000 \text{ lbs.}$$

EXAMPLE VIII.—In Bramah's original press at South Kensington the plunger is 3 ins. in diameter, and it acts at a distance of 6 ins. from the fulcrum, which is at one end of a levor 10 ft. 3 ins. long, carrying a loaded scale-pan at the other end. What should be the pressure of the water in the press in order to lift a weight of 3 cwts. in the scale-pan, neglecting the weight of the lever? Make a diagram of the arrangement. (S. and A. Exam., 1892.)

ANSWER.—Here $d = 3$ ins., consequently the area of the plunger $= \frac{\pi}{4} d^2 = \cdot 7854 \times 3 \times 3 = 7$ sq. ins., and B F = 6 ins.; A F = 10 ft. 3 ins. = 123 ins.; P = 3 cwts. = 3 × 112 = 336 lbs. Now we have to find the pressure per sq. in. on the ram that will balance P, acting with the stated advantage, since the area of the ram is not given.

By the above formula :—

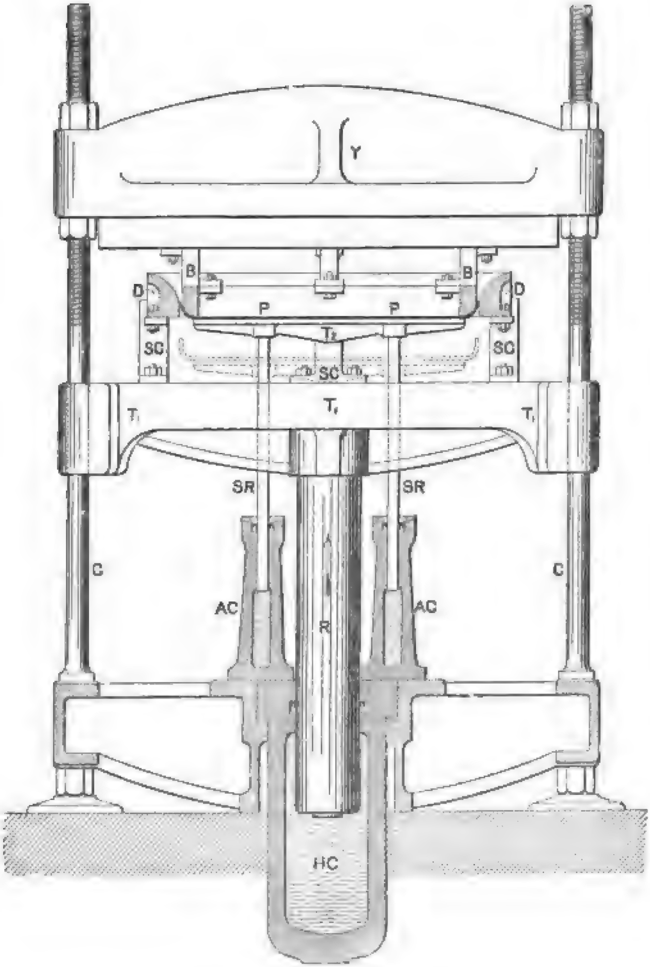
$$W = \frac{P \times A F}{B F} \times \frac{\text{area of 1 sq. in.}}{\text{area of plunger}} = \frac{336 \times 123}{6} \times \frac{1}{7}$$

Or, W = 984 lbs. per sq. in.

Hydraulic Flanging Press.—As an example of the practical application of the Bramah press to modern boiler-making, the accompanying illustration shows the form which it takes when used for flanging. It is worked by a high-pressure water supply derived from a central accumulator, which may at the same time be used to work cranes, punching, riveting, and other similar machine tools.

The operation of flanging the end tube-plates of a locomotive boiler is carried out in the following manner :—The ram R is lowered to near the bottom of the hydraulic cylinder H C, in order to leave room to place the heated boiler plate on the movable table T_2. High-pressure water is then admitted from the central accumulator to the auxiliary cylinders A C, thus forcing the side rams S R, with their table T_2, and the plate P vertically upwards,

until the upper surface of the plate bears hard against the bearers B, or internal part of the dies. Water from the same source is now admitted into the hydraulic cylinder H C, and forces up the ram R with its table T_1, supporting columns S C, and the external



LARGE HYDRAULIC PRESS FOR FLANGING BOILER PLATES.*

* The above figure is a reduced copy of one from Prof. Henry Robinson's book on *Hydraulic Machinery*, published by Messrs. Charles Griffin & Co.

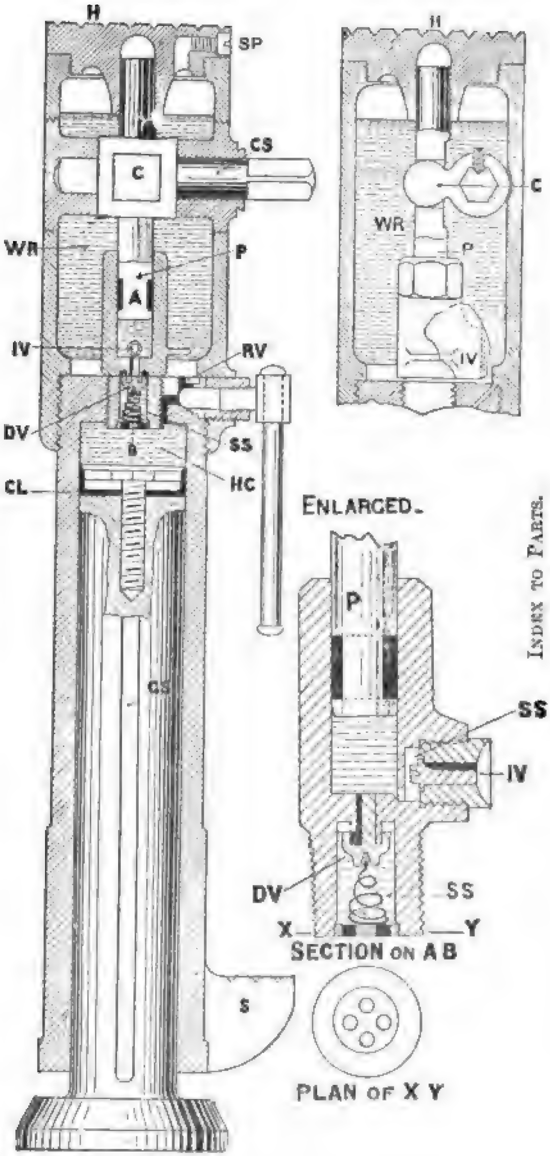
part of the dies D, until the latter has quietly and smoothly bent the heated edge of the plate round the curved corner of the internal bearer B. The ram R is now lowered, carrying with it the table T_1 and dies D, by letting out the water from H C. Then the table T_2 with the flanged plate is lowered by letting out water from A C. The plate is removed from its table, allowed to cool, faced, placed in position in the barrel of the boiler, marked off for the rivet holes, drilled, rimed, and riveted in the usual manner. The student will thus understand what a useful and powerful tool a hydraulic press is to the engineer in the hands of a skilful workman; for, it can be made to do better work in far less time, and with far greater certainty, uniformity, and exactitude, than the boiler-smith can turn out, with any number of hammermen to help him. It is fast replacing the steam-hammer for cressing work, and also steam or belt-driven punching and riveting machines, steam cranes, screw and wheel-gear hoists, as well as the screw press for making up bales of goods.

Hydraulic Jack.—This is a combined force pump and hydraulic press arranged in such a compact form as to be readily portable, and applied to lifting heavy weights through short distances. It therefore effects the same objects as the screw-jack, but with less manual effort and with greater mechanical advantage.

The base on which the jack rests is continued upwards in the form of a cylindrical plunger, so as to constitute the ram of the hydraulic cylinder H C. Along one side of this ram there is cut a grooved parallel guide slot G S, into which fits a steel set pin, screwed through the centre of a nipple cast on the side of the cylinder (not shown in the drawings) for the purpose of guiding the latter up and down without allowing it to turn round. The top of the ram has bolted to it a water-tight cup leather C L, by means of a large washer and screw-bolt.

The action of this cup leather is precisely the same as the leather collar in the cylinder of the Bramah press already described; but it has only to be pressed by the water in one direction —viz., against the sides of the truly-bored cast-steel cylinder, instead of against both the ram and the cylinder neck, as in the former case. The head H and upper portion of the machine is of square section, and is screwed on to the hydraulic cylinder in the manner shown by the figure. It contains a water reservoir W R, which may be filled or emptied through a small hole by taking out the screw plug S P.* In the centre line of the head-

* This screw plug S P is slackened back a little to let the air in or out of the top of the water reservoir when working the jack. There is generally another and separate screw plug opening for filling or emptying the water reservoir, quite independent of the above-mentioned one, which is used in this case for both purposes.



The Hydraulic Jack.

Index to Parts.

H	represents	Head for placing under a load.
SP	"	Screw plug for filling and emptying [W R.
W R	"	Water reservoir.
C	"	Crank.
CS	"	Crank shaft.
P	"	Pump plunger.
IV	"	Inlet valve.
D V	represents	Delivery valve.
S S	"	Spiral springs for I V and D V.
R V	"	Relieve valve.
H C	"	Hydraulic cylinder.
C L	"	Cup leather-packing.
G S	"	Guide slot along the length of arm.
S	"	Step for placing under a load.

piece there is placed a small force pump, the lower end of which is screwed into the centre of the upper end of the hydraulic cylinder. This pump is worked by the up-and-down movement of a handle placed on the squared outstanding end of the turned crank shaft C S. To the centre of the crank shaft there is fixed a crank C, which gears with a slot in the force-pump plunger P, and thus the motion of the handle is communicated to the pump plunger. By comparing the right-hand section of the water reservoir, and the section on the line A B, with the vertical left-hand section of the jack, it will be seen where the inlet and delivery valves I V and D V are situated. On raising the pump plunger P, water is drawn from W R into the lower end of the pump barrel through I V, and on depressing the plunger this water is forced through the delivery valve D V, into the hydraulic cylinder, thus causing a pressure between the upper ends of the cylinder and the ram, and thereby forcing the cylinder, with its grooved head H, and footstep S, upwards, and elevating whatever load may have been placed thereon. Both the inlet and outlet valves are of the kind known as "mitre valves." They have a chamfer cut on one or more parts of their turned spindles, so as to let the water in and out along these channels. The valves are assisted in their closing action by small spiral springs S S, bearing in small cups or hollow centres, as shown more clearly in the case of D V by the enlarged section on A B.

When it is desired to lower the jack, the relief valve R V is screwed back and the water is thus allowed to be forced up again into W R.

EXAMPLE IX.—Mr. Croydon Marks, in his book on *Hydraulic Machinery*, illustrates and describes another method of lowering the jack-head (first introduced by Mr. Butters, of the Royal Arsenal, Woolwich), where, by a particular arrangement, the inlet and delivery valves are acted upon by an extra depression of the handle, and consequent movement of the pump plunger. He also gives the main dimensions, with a drawing, of the standard 4-ton pattern as used by the British Government, where the ram has a diameter $D = 2$ ins., the pump plunger a diameter $d = 1$ in.; and the ratio of the leverage of the handle to the crank is 16 to 1. Therefore, from the previous formula we find that :—

$$\textit{The Theoretical Advantage} = \frac{W}{P} = \frac{A\,F}{B\,F} \times \frac{D^2}{d^2} = \frac{16}{1} \times \frac{2^2}{1^2} = \frac{64}{1}.$$

And he instances two trials by Mr. W. Anderson, the Inspector-General of Ordnance Factories, to determine the efficiency of these jacks, where, with a pressure on the end of the working handle of

76 lbs., the theoretical load should have been 76 lbs. × theoretical advantage = 76 × 64 = 4,864 lbs., instead of which it was only 3,738 lbs. :—

$$\therefore \qquad 4{,}864 \text{ lbs.} : 3{,}738 \text{ lbs.} : 100 : x.$$

$$\text{Or,} \qquad x = \frac{3{,}738 \times 100}{4{,}864} = 77 \text{ per cent. efficiency.}$$

In a second trial, a load of 1,064 lbs. required a pressure of 22 lbs. on the handle, and consequently the efficiency at this lighter load, as might be expected, was less, or only 74 per cent.

Example X.—With a hydraulic jack of the dimensions given above, and of 77 per cent. efficiency, it is desired to lift a load of 4 tons; what force must be applied to the lever handle?

Answer.—By the previous theoretical formula :—

$$W = \frac{P \times AF}{BF} \times \frac{D^2}{d^2}.$$

$$\therefore \qquad P = \frac{W \times BF}{AF} \times \frac{d^2}{D^2}$$

$$\text{,,} \quad = \frac{4 \times 2{,}240 \times 1}{16} \times \frac{1^2}{2^2} = 140 \text{ lbs.}$$

But the efficiency of the machine is only 77 per cent., consequently 140 lbs. is 77 per cent. of the force required :—

$$\therefore \qquad 77 : 100 :: 140 \text{ lbs.} : x \text{ lbs.}$$

$$x = \frac{140 \times 100}{77} = 181{\cdot}81 \text{ lbs.}$$

Example XI.—Show, with the aid of sectional sketches, the construction of the ordinary hydraulic lifting jack. If, in such a machine, the mechanical advantage of the lever or handle is 12 to 1, and the diameter of the lifting ram is 2 inches, while the diameter of the plunger is $\frac{7}{8}$ of an inch, what weight can be lifted theoretically when a pressure of 50 lbs. is applied to the lever handle? (S. & A. Exam., 1891.)

Answer.—The hydraulic jack has been fully described and illustrated in this lecture.

Let D = Diameter of ram = 2 inches.
,, d = ,, plunger = $\frac{7}{8}$ inch.
,, n = Mechanical advantage of lever = 12 : 1.
,, P = Effort applied at end of lever = 50 lbs.
,, W = Weight raised.

If Q denotes the pressure on the plunger, caused by the effort P applied at the end of the lever, then :—

$$Q = P\,n.$$

But, $$\frac{W}{Q} = \frac{\text{area of ram}}{\text{area of plunger}} = \frac{D^2}{d^2}.$$

$$\therefore \qquad \frac{W}{P n} = \frac{D^2}{d^2}.$$

This is a formula giving the relation between W, P, n, D, and d, which is also true for the hydraulic press.

Substituting the above values in this formula, we get :—

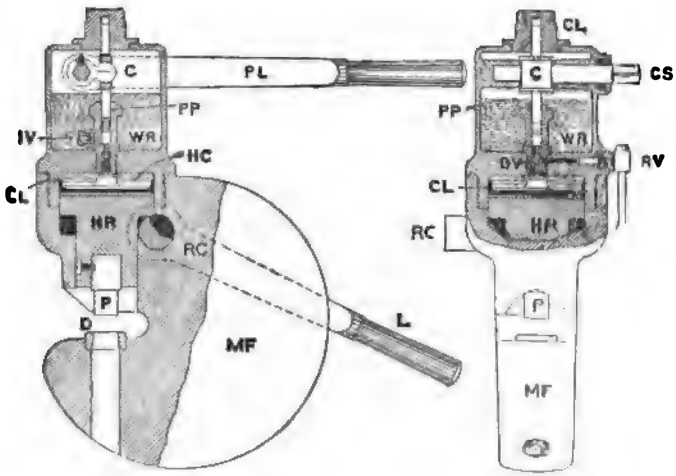
$$\frac{W}{50 \times 12} = \frac{2^2}{(\frac{7}{8})^2}.$$

$$\therefore \qquad W = 3{,}134 \text{ lbs., } or = 1{\cdot}4 \text{ tons, nearly.}$$

Hydraulic Bear.—This is another very useful application of the hydraulic press and force pump. It is used in every shipbuilding-yard and bridge-building works. By comparing the drawing with the index to parts, it may be seen that its construction and action are similar to the hydraulic jack just described in full detail, and we need say nothing more than direct the student's attention to the action of the raising cam, and to the means by which the apparatus is lifted and suspended. In order to raise the punch P, for the admittance of a plate between it and the die D, the relief valve R V, must first be turned backwards, and the lever L, depressed. This causes the corner of the raising cam R C, to force the hydraulic ram H R, upwards, and the water from the hydraulic cylinder H C, back into the water reservoir W R. The relief valve R V, may now be closed and the plate adjusted in position. Then the pump lever P L, can be worked up and down until the punch P, is forced through the plate, and the punching drops through the die hole D, in the metal frame M F, to the ground, or into a pail placed beneath to receive it.

The whole bear is suspended by a chain (worked by a crane or other form of lifting tackle) attached to a shackle, whose bolt passes through a cross hole in the back of the metal frame M F, just above, but a little to the front of, the centre of gravity of the machine. This hole and shackle are not shown in the drawing, but the student can easily understand that the hole would be bored a little above where the letters R C, appear on the side view,

and that the chain would pass clear of the pump lever, since this works well to the right-hand side of the bear.



SIDE VIEW AND SECTION. END VIEW AND SECTION.

THE HYDRAULIC BEAR, OR PORTABLE PUNCHING MACHINE.

INDEX TO PARTS.

P L	represents	Pump lever.	H C	represents	Hydraulic cylinder.
C S	,,	Crank shaft.	C L	,,	Cup leather.
C	,,	Crank.	H R	,,	Hydraulic ram.
P P	,,	Pump plunger.	R C	,,	Raising cam.
W R	,,	Water reservoir.	L	,,	Lever for R C.
I V	,,	Inlet valve.	P	,,	Punch.
D V	,,	Delivery valve.	D	,,	Die ring
R V	,,	Relief valve.	M F	,,	Metal frame.

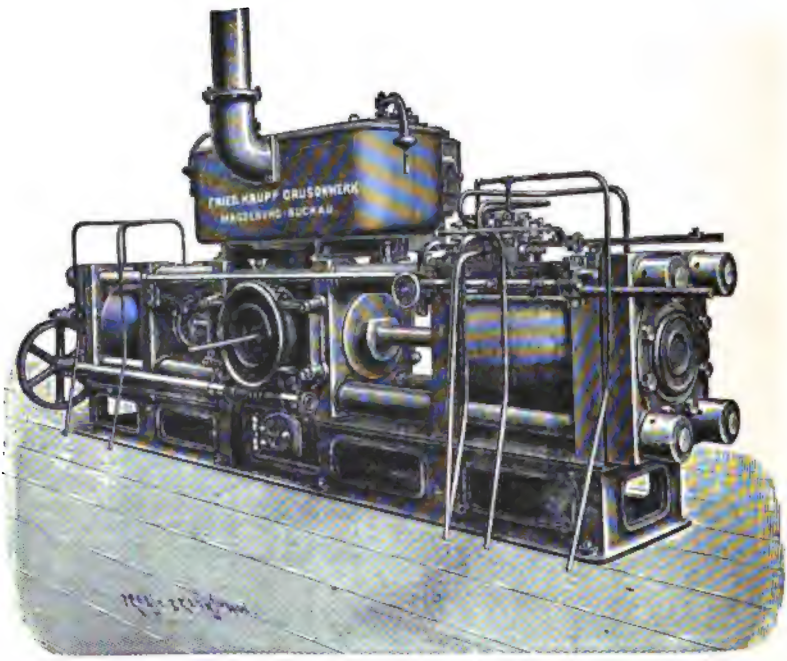
Lead-covering Cable Press.—Hydraulic presses are now employed for making lead pipes, and for covering electric light cables with a close-fitting tube of lead. For the former purpose the lead is heated until it is nearly melted in a strong chamber from which it is forced by rams to squirt through a die at the top of the machine. A mandrel projects into the centre of the die, and, consequently, the lead issues as a continuous tube.

The accompanying figure illustrates a press for covering electric cables with lead in this manner, as carried out by Messrs. Siemens Brothers, at their Woolwich works. It consists essentially of a receiver in which two rams work, and which contains a mandrel

and matrice. The lead is melted in the melting pot at the top by gas or petroleum, and is then poured into the receiver below.

The cable enters at one side of the receiver, and passes through the mandrel and leaves at the other, while as soon as the rams begin to force their way into the receiver, the lead casing is formed round the cable.

The matrice can be so nicely adjusted by means of steel cones, that a lead casing of perfectly uniform thickness of a fraction of a millimetre, can be obtained. An excellent feature of this press is,



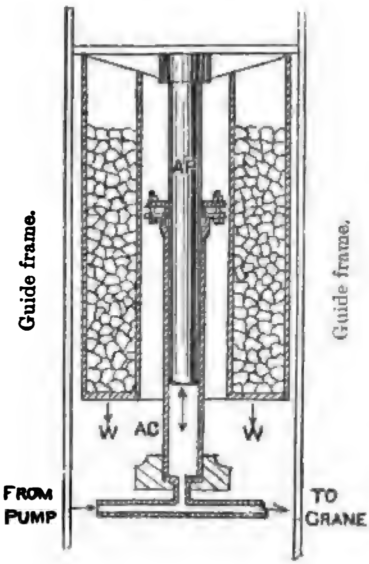
PRESS FOR COVERING CABLES WITH LEAD, BY FRIED. KRUPP GRUSONWERK, GERMANY.

that it possesses two rams which, by distributing the pressure more uniformly, ensure a more regular casing than is possible with only one, as is usually the case. The two rams are connected together by an adjusting apparatus so that they will both move forward at the same rate. The mandrels and matrices can easily be changed to suit cables whose diameters range from $\frac{1}{8}$ in. to $2\frac{3}{4}$ ins.

Hydraulic Accumulator.—The demand for hydraulic power to work elevators, cranes, swing bridges, dock gates, presses, punching and riveting machines, &c., being of an intermittent nature—at one moment requiring a full water supply at the maximum pressure, and at another a medium quantity, whilst in many cases all the machines may be idle—it is evident that if an engine with pumps were devoted to supplying this demand in a direct manner, the power thereof would have to be equal to the greatest requirements of the plant, and would have to instantly answer any and every call from the same. In the case of a low-pressure supply, as for lifts, this difficulty is best overcome by placing one tank in an elevated position at the top of the hotel or building where the lift is required, and another tank below the level of the lowest flat. Then a small gas engine working a two or three-throw pump, or a Worthington duplex steam pump, may be used to elevate the water more or less continuously from the lower to the higher tank. The "head" of water in the elevated tank will, if sufficient, work the lift at the required speed, and the discharged water from the hydraulic cylinder will enter the lower tank, to be again sent round on the same cycle of operations. Should the lift be stopped for any considerable time, then a float in the upper tank, connected by a rope or chain with the shifting fork for the belt-driven pumps (in the case of the gas engine) will force the belt over on to the loose pulley, or shut off the steam from the Worthington pump. And when the water falls in the upper tank, the float will cause a reverse movement of the rope and shift the belt to the tight pulley, or open the steam valve, and so start the pumps.

When the pressures required are great, such as for cranes, &c., where 700 lbs. on the square inch is considered a very medium pressure, an elevated tank would be out of the question, for it would have to be fully 1600 feet high in order to exert this force and to overcome friction. Under these circumstances recourse is had to a very simple and compact arrangement called an accumulator, of which we here give a lecture diagram, without any details of cocks or valves, and automatic stopping and starting gear. A steam engine, or other motor, works a continuous delivery pump, of the combined piston and plunger type, without an air vessel, as already illustrated in this lecture. The water from the pump enters the left-hand branch pipe leading into the foot of the accumulator cylinder, and forces up the accumulator ram with its crosshead or top T-piece and the attached weight or dead load, until the ram has reached nearly to the end of its stroke. Then the top of the T-piece, or a projecting bracket on the side of the wrought-iron cylinder containing the dead load, engages with and lifts a small weight attached to a chain passing over a pulley

fixed to the guide frame or to the wall of the accumulator house. This chain is connected directly to the throttle valve of the steam supply pipe, or to the belt-shifting gear if the pump is driven by belt gearing, and being provided with a counter-weight, the motor and pump are automatically stopped by the raising of the weight and the chain in the accumulator house. Should the water which has been forced into the accumulator cylinder be now used by a crane or other machine, the load on the ram causes it to follow up and keep a constant pressure on the water. The starting weight falls as the receding T-piece or bracket descends, and thus pulls the starting chain, and opens the steam engine throttle valve, or shifts the belt from the loose to the fixed pulley, and again sets the pump to work. Should the hydraulic machines be working continuously, then the pump is kept going, for the water from it passes directly on to the machines, and only the surplus water finds its way into the accumulator cylinder if the pump's supply exceeds the demand of the machines for water.



THE HYDRAULIC ACCUMULATOR.

INDEX TO PARTS.

A C for Accumulator cylinder.
A P ,, Accumulator plunger or ram.
W ,, Weight or load contained in an annular cylinder of wrought iron and suspended from the top of T-piece or crosshead.

The annular cylinder of wrought iron is generally filled with scrap iron, iron slag, sand, or other inexpensive heavy material. The accumulator cylinder A C, has a stuffing-box and gland at its upper end. A coil of hemp woven into a firm rectangular section and smeared with white lead is placed in the bottom of the stuffing-box. The gland is screwed down on the top of this packing until at the normal pressure the water in the cylinder cannot leak past it. Cup leather packing is seldom used for this simple form of accumulator; just the ordinary packing that would be used for pump rods is found to answer all requirements. This is the

simplest form of accumulator which we have here described, but other forms will be illustrated in the next lecture.

EXAMPLE XII.—Describe and sketch in section an hydraulic accumulator, showing how the ram is kept tight in the cylinder. An hydraulic press, having a ram 16 inches in diameter, is in connection with an accumulator which has a ram 8 inches in diameter and is loaded with 50 tons of ballast; what is the total pressure on the ram of the press? (S. & A. Exam., 1892.)

ANSWER.—The first part of the question is answered by the previous figure and by the text.

By Pascal's Law the pressure *per square inch* in the accumulator is equal to the *pressure per square inch in the* hydraulic press. Consequently :—

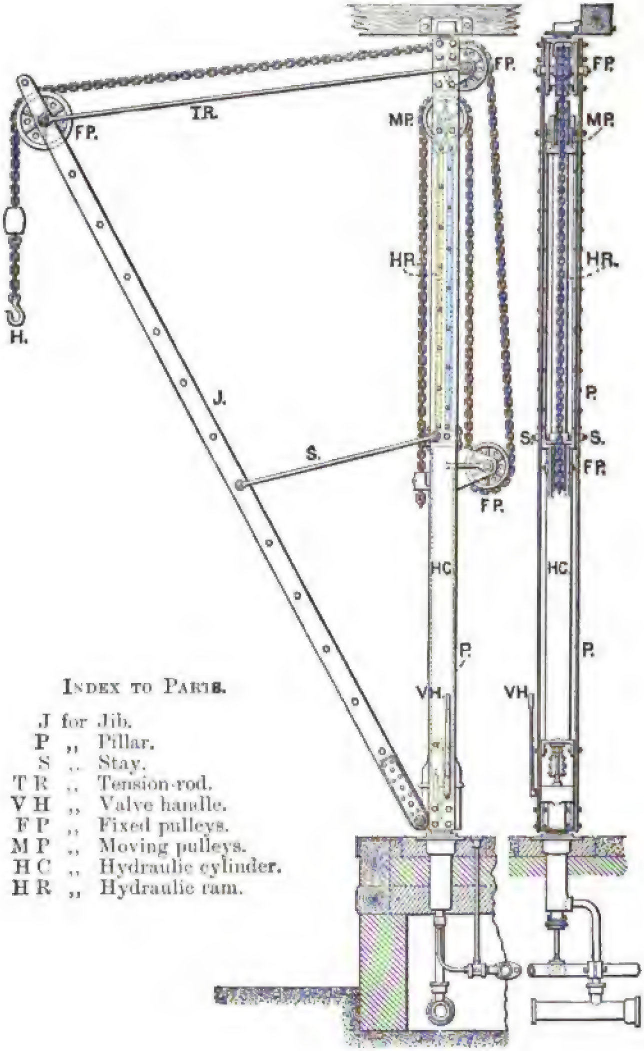
$$\frac{\textit{Total Pressure on Press}}{\textit{Total Load on Accumulator}} = \frac{\textit{Cross Area of Press Ram}}{\textit{Cross Area of Accumulator Ram}}$$

$$\frac{P}{50} = \frac{\pi}{4} \times 16^2 \div \left(\frac{\pi}{4} \times 8^2\right) = \frac{16^2}{8^2}.$$

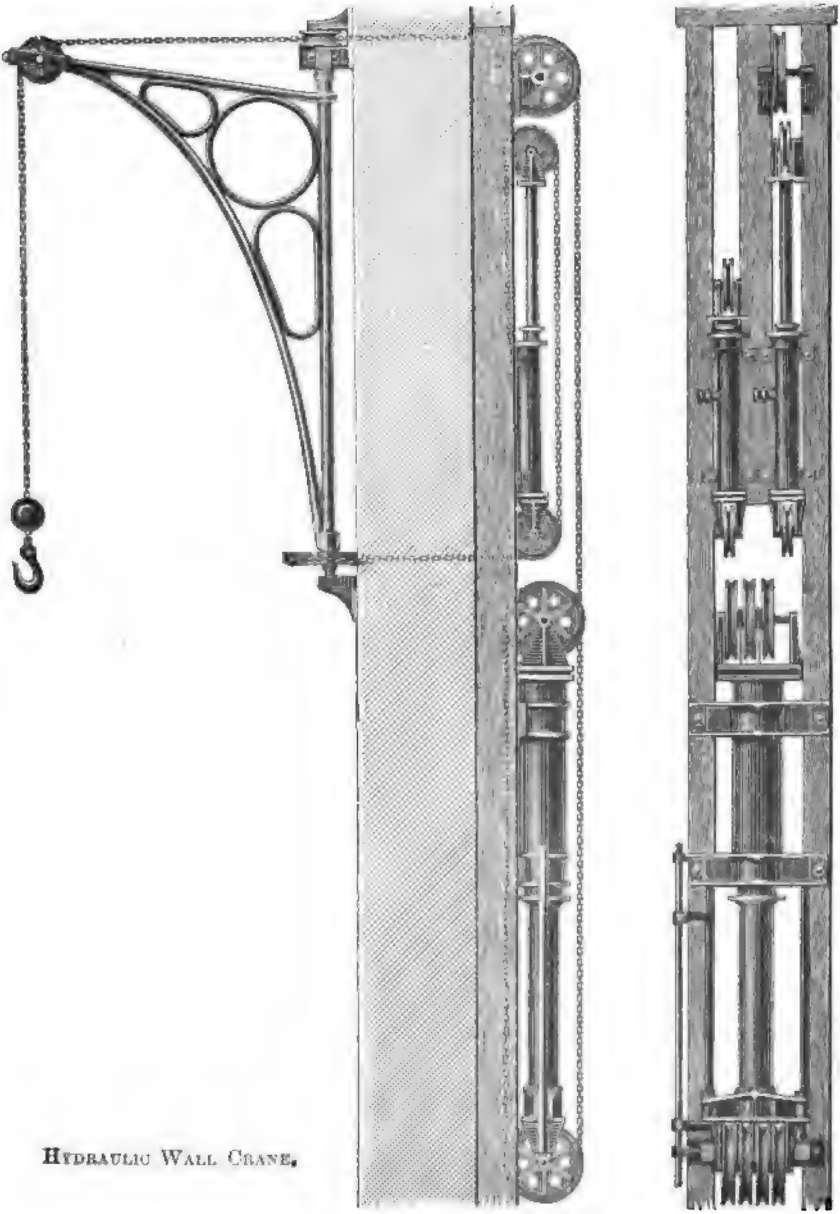
$$\therefore \qquad P = \frac{50 \times 16 \times 16}{8 \times 8} = 200 \text{ tons.}$$

Hydraulic Cranes.—Another very common application of hydraulic power is the working of cranes for handling goods at wharves, warehouses, or railway depôts, and we illustrate a simple one for the latter purpose In this crane, the lower part of the pillar forms the cylinder for a ram H R, which carries a pulley M P, at its upper end and is actuated by the water pressure. A chain, fastened at one end to the crane pillar, passes over this pulley and then round two fixed ones F P, before going to the jib pulley. Hence, when the ram is forced up, the chain will be pulled up twice as far. A small slide valve, actuated by the handle V H, controls the supply of water to the hydraulic cylinder. By means of this valve, the cylinder is put into communication with either the pressure or the exhaust pipe, or it may be cut off from both. The weight of the ram and load attached to the hook H, are utilised to drive out the water on the return stroke.

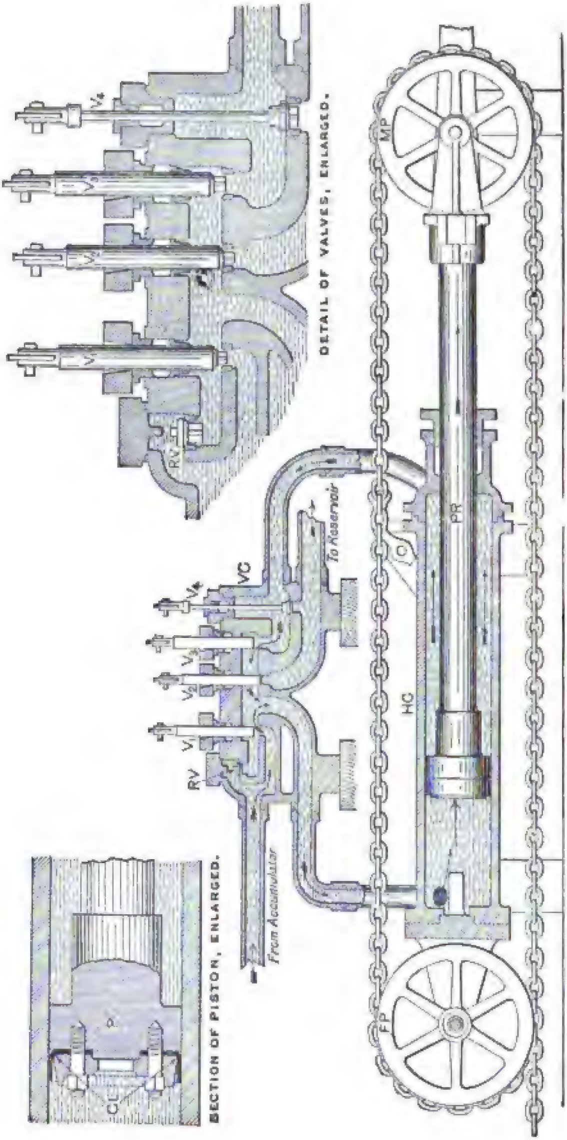
Hydraulic Wall Crane.—Our next figure shows a crane on the same principle fixed to the wall of a warehouse or shed. In this case, however, the motion of the ram is magnified eight times by placing four pulleys side by side on the end of the large ram which here moves downwards. The slewing of this crane is also accom-



HYDRAULIC CRANE.

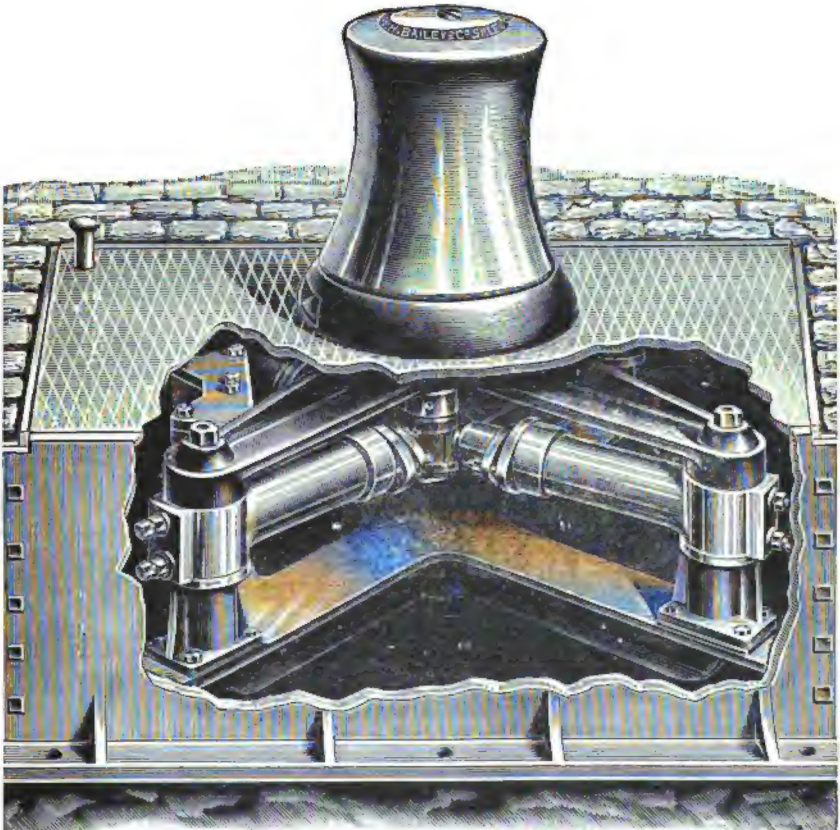


Hydraulic Wall Crane.



ARMSTRONG'S DOUBLE POWER HYDRAULIC CRANE.

H C for Hydraulic cylinder.	V_1 for Inlet valve.
P R ,, Piston-rod.	V_2 V_4 ,, Outlet valves.
P ,, Piston.	V_3 ,, Connecting valve.
C L ,, Cup leather.	R V ,, Relief valve.
V C ,, Valve chamber.	



Hydraulic Capstan by W. H. Bailey & Co., Ltd., Manchester

LECTURE XXXIII.—QUESTIONS.

1. One side of a reservoir has a slope of 12 vertical to 5 horizontal; what is the whole amount of the pressure of the water against 50 feet of its length, when the depth of the water is 12 feet? *Ans.* 243,750 lbs.

2. In an empty dock the water is level with the sill at the lowermost edge of the dock gate, the level of the water on the opposite side of the gate being 10 feet above the sill. The dock gate is 10 feet wide, find the pressure in pounds on the dock gate. If the water were at a level of 5 feet in the dock, the level outside being the same as before, how much would the pressure on the gate be relieved? *Ans.* 31,250 lbs.; 7,812·5 lbs.

3. State Archimedes' principle. A cylindrical can is 6 inches in diameter and 30 inches deep, and is required, when empty, to stand in a bath of water 30 inches deep without being lifted up. To what weight must the can be loaded, the weight of a cubic foot of water being 62½ lbs.? *Ans.* 30·679 lbs.

4. A rectangular tank, 4 feet square, is filled with water to a height of 3 feet. A rectangular block of wood, weighing 125 lbs., and having a sectional area of 4 square feet, is placed in the tank, and floats with its sides vertical and with this section horizontal. How much does the water rise in the tank, and what is now the pressure on one vertical side of the tank? *Ans.* 2 inches; 1,253 lbs.

5. The total force in the direction of the motion of a piston is the cross-sectional area of the cylinder multiplied by the pressure. Why is this so, the piston not having a plane surface? (S. & A. Adv. Exam., 1898.)

6. An escape valve, loaded partly by a weight and partly by a spring, is fitted to a main conveying water under pressure, and is required to open automatically when the water pressure rises above a certain amount. Sketch and describe the construction of such a valve when arranged on the double beat principle, and explain clearly the hydrostatic principle involved therein.

7. Prove that when a thin spherical shell is exposed to the bursting pressure of gas or liquid the stress in the material is half as great as that within the curved surface of a thin cylindrical shell exposed to the like pressure, each shell being of the same thickness and diameter.

8. Sketch a combined plunger and bucket pump with index of parts, explaining its use and action, also sketch its application to the lifting of water from deep mines. Suppose a pump raises 5,000 gallons every half minute from a depth of 600 feet with 30 per cent. loss in system, what is the horse-power required? *Ans.* 1,874 H.P.

9. Describe a force pump for supplying water to the accumulator of hydraulic cranes. Sketch a section through the plunger and valves.

10. Sketch and describe a force pump having a solid plunger, showing the construction of the valves. The diameter of the plunger is 2¼ inches, and it is driven by a crank 2 inches in length making 30 revolutions per minute. Find the cub. ins. of water pumped in 5 minutes. *Ans.* 2,945 cub. ins.

11. Describe and illustrate by a longitudinal section, and such other views as may be necessary, the construction and action of a double-acting pump

and its valves, supposing the pump cylinder to be of $3\frac{1}{4}$ inches internal diameter, and to work at a pressure of 700 lbs. per square inch. Of what materials would the several parts be constructed?

12. A vertical single-acting pump has to elevate water 50 fathoms. The bore of the pump is 6 inches; stroke, 6 feet; number of up strokes, 10 per minute. Find (*a*) the pressure per square inch on the pump backet when it is at the bottom of its stroke; (*b*) the weight of water discharged per minute; (*c*) the horse-power of the engine required to drive the pump, supposing 30 per cent. of the engine-power to be lost by friction, &c. Sketch an arrangement of the kind. *Ans.* (*a*) 130·28 lbs. per square inch; (*b*) 736·31 lbs.; (*c*) 9·56 H.P.

13. Give any method with which you are acquainted whereby the valves in a pumping engine may be relieved from the shock due to the inertia of the water in the mains.

14. A pump for exhausting air from a receiver has a solid piston and one valve in the casing. Describe, with a sketch, the construction of the pump, and explain the nature of the improved form known as Sprengel's pump, where a small portion of mercury forms the piston, and no valve is required.

15. Describe and show, with the aid of necessary sketches, the construction of the "Pulsometer." Describe how it works, and indicate the contrivances introduced to promote the steady flow of water and to prevent sudden shocks upon the apparatus. Is the pulsometer an economical arrangement for raising water? Give reasons for your answer. What, if any, are its advantages over the ordinary piston pump? (S. & A. Hons. Exam., 1896.)

16. Sketch in section the cylinder, ram, and leather collar of an hydraulic press. Explain the principle of the press and the manner in which the escape of water is prevented. Example—The sectional area of the plunger of the force pump is $\frac{1}{50}$ that of the ram, and the leverage gained by the pump handle is 12 to 1, find the pressure on the ram when a force of 60 lbs. is exerted at the end of the pump handle. *Ans.* 36,000 lbs.

17. Describe, with a sectional sketch, a hydraulic press where the ram is actuated in both directions. Show the position and forms of the cup leathers.

18. The return stroke in an hydraulic press is often accomplished by forming the ram like a piston with a very large piston-rod. Sketch in longitudinal section such a press, showing the arrangement of the leathers. What will be the relative speeds of the forward and return strokes of the ram when the larger and smaller diameters are 15 inches and 14 inches respectively, the pumps for the supply of water running at the same speed in both cases?

19. In some hydraulic presses a single valve, held down by a lever and weight, is used both to indicate and relieve the pressure. Sketch the valve in position and explain its action.

20. Describe clearly and show with sketches the construction and action of any one form of portable hydraulic riveting machine with which you are acquainted. Show clearly the valves and connections by which the pressure is applied to close the rivet, and how the pressure is released and the tool withdrawn from the rivet head when the riveting is completed. How is the water pressure conveyed to the machine? (S. & A. Hons. Exam., 1895.)

21. What are the advantages in forging large masses of steel by hydraulic pressure over the same operation performed by the steam hammer? Show clearly, with the assistance of the necessary sketches, the method em-

ployed in hydraulic forging presses for bringing the ram or pressing surface rapidly back from the work after each application of the pressure. (S. & A. Hons. Exam., 1896.)

22. Sketch a section through an hydraulic lifting jack and hydraulic bear, and describe the manner in which the pressure exerted on the handle is transmitted to the ram.

23. A 4-ton hydraulic lifting jack has a lifting ram of 2 inches in diameter, and a pump plunger of 1 inch in diameter. The jack is worked by a lever handle, the leverage being 16 to 1. What pressure must be applied at the end of the handle in order to lift a load of 25 cwts., if the efficiency of the machine is 80 per cent.? Make a vertical section of the jack, showing the valves and the mode of connecting the lever with the pump plunger. How can the weight be lowered slowly and regularly without jerks?

24. Explain, with the aid of a sectional sketch, the action and construction of the hydraulic jack. How is the pressure taken off and the load slowly lowered? If the ram is 2 inches in diameter, the pump plunger $\frac{7}{8}$ inch, and the mechanical advantage of the handle 10, what is the total mechanical advantage, neglecting friction? (S. & A. Adv. Exam., 1897.)

25. Make a sketch of a 10-ton hydraulic jib crane in which the lifting cylinder is carried in the pillar or post of the crane. What would be the diameter of ram required in the arrangement you adopt, supposing water to be supplied to the crane at a pressure of 700 lbs. per square inch, neglecting friction, &c.?

26. What is the object of a relief valve in an hydraulic crane, and where is it placed? Sketch a longitudinal section through the cylinder and ram of a crane working at two powers. Explain the mode of action.

27. Describe, with sketches, some form of hydraulic lift, and the manner in which it is worked.

28. Describe with sketches a direct-acting 80 foot hydraulic lift, to carry a load of 1 ton, the ram is 4 inches diameter, and the moving parts are balanced by a counterweight and chain. The lift is worked from a tank of water 40 feet above the highest level of the platform and an intensifier must be used. (S. & A. Hons. Exam., Part II., 1898.)

29. Make a section through the cylinder of an hydraulic crane with two powers as applied for raising weights. Give a general description of the crane, and explain the mode of working it by referring to your sketch.

30. Describe the general construction of the lifting apparatus in the hydraulic crane, making a sketch of the cylinder and ram. Show the method of obtaining *two* powers from a single cylinder. In what way is the *pulley principle* applied in such cranes?

31. In an hydraulic crane with two powers show (1) the apparatus for lifting the weight, (2) the method of slewing or rotating the jib of the crane.

32. Explain the advantage to be derived from making the length of stroke of an hydraulic engine adjustable. Describe and give sketches of a construction for this purpose.

33. Prove the formula used for finding the resultant pressure on a sloping plane interface, with any shape of boundary underneath the surface of a liquid. Also find the position of the resultant. Apply the formula to the case of a rectangle with its highest horizontal side 5 feet long at 20 feet below the surface, its other sides being 12 feet long and making 30° with the horizontal. (S. and A. H., Part I., 1899.)

34. A rectangle with its highest horizontal side 2 feet long at 10 feet below still water level, its other sides being 3 feet long, making 43° with the horizontal; find the amount and position of the resultant pressure upon the area. Prove the truth of your formulæ. *Ans.* 4,121 lbs.

(B. of E. H., Part I., 1900.)

35. The hydraulic company's pipes are to stand a maximum working pressure of 1,200 lbs. per square inch; internal diameter 4 inches. Find the outside diameter, the working tensile stress in the material being 3,200 lbs. per square inch. When this stress is reached inside, what is the stress outside? *Ans.* Outside diameter 5·93 inches, say 6 inches.

(B. of E. H., Part I., 1900.)

36. Describe, with sketches, a hydraulic lift with hydraulic method of balancing. (B. of E. H., Part I., 1901.)

37. Describe, with sketches, any machine with which you are acquainted, in which water at great pressure is employed. Point out at what places solid friction takes effect. What are the differences in character between solid and fluid friction? If 3 cubic feet of water at 700 lbs. per square inch gauge pressure are employed in a crane to lift a weight of 2 tons through a height of 50 feet, what is the efficiency of the crane?

(B. of E. Adv., 1901.)

38. Describe, with detail sketches, a good form of hydraulic crane, with arrangements for varying the water consumption with the load lifted.

(C. & G., 1900, H., Sec. C.)

39. Describe, with sketches, either (*a*) a hydraulic riveter, or (*b*) a hydraulic boiler flanging press. Obtain an expression for the maximum pressure either can exert. Assume your own factors for friction losses.

(C. & G., 1900, H., Sec. C.)

40. In a hydraulic crane the water pressure employed is 750 lbs. per square inch; to lift the full load of 10 tons three cylinders are used, each 10 inches in diameter, and the load by an arrangement of pulley blocks is lifted at six times the speed at which the rams move out of their cylinders. How many foot-pounds of work are done on the rams in lifting the 10 tons 30 feet, and how many foot-pounds are wasted?

(C. & G., 1901, O., Sec. A.)

41. Explain, with careful sketches, the construction and working of either (*a*) a hydraulic jack, *or* (*b*) a hydraulic compression or baling press. In each case the method of packing the moving parts must be fully described. How are the sizes of the rams determined? What data are needed? (C. & G., 1901, H., Sec. C.)

LECTURE XXXIII.—A.M.INST.C.E. EXAM. QUESTIONS.

1. Prove that a body of any form when immersed or floating in a liquid is acted on by a force equal to the weight of the liquid displaced, acting vertically upwards through the centre of gravity of the displaced volume. A bulb weighing 12 oz. is found to weigh 8 oz. when immersed in water, and 7 oz. when immersed in another liquid. What is the specific gravity of the liquid? (I.C.E., *Oct.*, 1897.)

2. Show how to find the resultant thrust due to the pressure of water on a rectangular plane surface, two of the sides of the rectangle being horizontal. A vertical wall, 2 feet thick and 18 feet high, weighing 124 lbs. per cubic foot, supports the pressure of water on one side. How high may the water rise without causing the resultant force on the base of the wall to pass more than 8 inches from the middle of the wall's width? (I.C.E., *Feb.*, 1898).

3. The centre of a spherical body, 6 inches diameter, is 3 feet below the surface of water, what is the resultant pressure on the surface of the body? (I.C.E., *Oct.*, 1898.)

4. A dock entrance, whose level floor is 24 feet below the water, has a width of 62 feet at the water-level and 50 feet at the floor, the side walls being built with a straight batter. The entrance is closed by a caisson, and on one side of the caisson the floor is dry. Calculate the total horizontal pressure upon the caisson, and the height of its centre of action above the floor. (I.C.E., *Oct.*, 1898.)

5. A rectangular area inclined at 20° to the horizontal is submitted to water-pressure. Its side is 11 feet and is horizontal, and is 30 feet below the water-level; its parallel side is 62 feet below the water-level; what is the total pressure on the area counting from atmospheric pressure? (I.C.E., *Oct.*, 1898.)

6. Describe a bucket-and-plunger pump. What causes limit its efficiency? (I.C.E., *Feb.*, 1899.)

7. In a common form of hydraulic crane the piston is 8 inches in diameter; the speed at which the weight is raised is 8 times that of the piston. If the water acting on the piston is at a pressure of 750 lbs. per square inch, find what weight may be lifted by the chain if the efficiency of the mechanism be 55 per cent. What is the chief cause of the low efficiency? How many gallons of water are used in a lift of 30 feet? (I.C.E., *Feb.*, 1899.)

8. A block, in the form of an isosceles wedge, 1 foot high and having a base 1 foot square, is immersed in water with its base horizontal and uppermost at a depth of 4 feet below the surface. Determine the pressure, and the vertical component of the pressure on each face. Hence show what must be the weight of the wedge so that it may just float. (I.C.E., *Feb.*, 1899.)

9. If a spherical buoy 6 feet in diameter floats in sea-water with half its volume immersed, what is the weight of the buoy? Find also what will be the load draught of a flat-bottomed pontoon of rectangular form, whose length is 175 feet and its width 20 feet, when the weight of pontoon and load amounts to 400 tons. (I.C.E., *Feb.*, 1899.)

10. Suppose that the pontoon last mentioned is to be symmetrically loaded in such a manner as to bring the centre of gravity to a height of 6 feet above the deck; and determine the question whether the pontoon would in that case float upright in calm water, or whether it would capsize. (I.C.E., *Feb.*, 1899.)

11. How high may water rise on one side of a vertical wall 9 feet high, 27 inches thick, of material weighing 143 lbs. per cubic foot, before the resultant force at the base of the wall passes outside the edge? (I.C.E., *Feb.*, 1899.)

12. A square sluice-opening, 3 feet by 3 feet, is formed in the vertical face of a dam, and the upper edge of the opening is 6 feet below water. Calculate the hydrostatic pressure upon the square area, and find the exact position of its centre of action. (I.C.E., *Feb.*, 1899.)

13. A vessel with a square base has three vertical sides and one sloping side, so that the shape of the top is a rectangle, with one pair of sides equal to those of the base and the other pair half that amount; the height of vessel is equal to the side of base. If it is filled with water, what is the amount and distribution of the pressure on the base. If the water freezes into a block, how is the distribution of pressure altered. (I.C.E., *Oct*, 1899.)

14. Sketch a lift-pump, a double-acting force-pump, and a bucket-and-plunger pump, and state generally their relative advantages. (I.C.E., *Oct.*, 1899.)

15. The sides of a circular tank 15 feet in diameter are constructed of iron plates ½ inch thick, riveted together. Omitting the effect of the joints and the connection with the flat bottom of the tank, find the depth to which it may be filled with petroleum weighing 55 lbs. per cubic foot, so that the maximum stress on the metal may not exceed 1,000 lbs. per square inch. (I.C.E., *Feb.*, 1900.)

16. Give the theory of the simple hydraulic press, and state the chief causes of loss of mechanical advantage in practice. A hydraulic punch has a ram 8 inches diameter; ⅝-inch holes are being punched, each requiring a force of 70,000 lbs. What is the water pressure? This water comes from a steam intensifier; the area on which the steam acts is 300 square inches; the area of the ram is 30 square inches. What is the pressure difference of the steam, neglecting friction? (I.C.E., *Oct.*, 1900.)

17. Describe carefully the action of the air vessel on the delivery side of a vertical single-action bucket-pump with three sets of valves, suction, bucket, and delivery. Under what circumstances would you consider an air vessel on the suction side useful? Under what circumstances would you expect that the suppression of the suction-valves would improve the delivery? (I.C.E., *Oct.*, 1900.)

18. Give an expression for the tension of the bucket-rod or piston-rod in a common suction pump. The barrel of the pump is 8 inches diameter and the stroke of the bucket or piston is 10 inches. Find the work done per stroke in foot-pounds when the spout is 16 feet above the surface of the water in the well. (I C.E., *Oct.*, 1900.)

19. Explain the limits of action of the common lifting pump and find the tension of the bucket rod at any point of the upward stroke when the pump is in full working. (I.C.E., *Feb.*, 1901.)

20. A vertical sluice gate of rectangular form closes a channel which is 6 feet in width. On one side of the gate the water stands 5 feet above the cill, and on the other side only 2 feet above the cill. Find the horizontal pressure which is exerted upon the gate as a whole and resisted by its supports. (I.C.E., *Feb.*, 1901.)

21. In the case described in the previous question, find the centre of action of the whole horizontal pressure. (I.C.E., *Feb.*, 1901.)

22. The inner face of a masonry dam is a plane vertical surface 12 feet in height. The section of the dam is a right-angled triangle 9 feet wide at the base, and the weight of the masonry is 125 lbs. per cubic foot. When the reservoir is filled to the upper edge of the dam, find the point at which the resultant line of pressure intersects the base. (I.C.E., *Feb.*, 1901.)

23. In designing a hydraulic forging press to exert a maximum pressure of 3,000 tons what water pressure would you recommend; how many hydraulic cylinders would you use; would you cast these in steel or iron; and what thickness would you give them? (I.C.E., *Feb.*, 1901.)

24. Write a short essay on the special advantages of hydraulic over other kinds of cranes for dock work. (I.C.E., *Feb.*, 1901.) (*Refer back to Lecture X., Vol. I., before answering this question.*)

25. A box in the form of a cube, of internal dimensions 1 foot, has its base horizontal and is half filled with water. One vertical side is kept in its position by four screws only, one at each angular point. Find the tension in these screws due to the water pressure. (I.C E., *Feb.*, 1901.)

26. State what is meant by the centre of pressure of a plane immersed in water. A rectangular door 10 feet long and 4 feet broad is placed in the side of a tank with its plane vertical, and one of its long edges at the surface of the water. It is hinged at this edge. Find the horizontal force at the lower edge to keep the door in its vertical position (1 cubic foot of water weighs 62½ lbs.). (I.C.E., *Oct.*, 1901.)

27. Show how the gland in a plunger-pump may be packed. Sketch suction and delivery valves suitable for such a pump delivering 50,000 gallons per hour against a head of 300 feet. (I.C.E., *Oct.*, 1901.)

28. The ram of a hydraulic accumulator is 10 inches diameter, and has a stroke of 11 feet. When fully loaded the water-pressure is 800 lbs. per square inch. If the whole energy of the accumulator water could be used in three minutes, what horse-power would it exert? (I.C.E., *Feb.*, 1902.)

29. If the area of a horizontal section of a ship at the water-line is 15,000 square feet, and the sides vertical where they cut the water, find the extra depth the ship will sink when loaded in fresh water with 7.0 tons of cargo. What depth would the ship sink if floating in salt water of specific gravity 1·026 when loading? (1 cubic foot of fresh water weighs 62·5 lbs.) (I.C.E., *Feb.*, 1902.)

30. A flap valve is 2 feet square, and stands inclined at 60° to the horizontal. Find the force needed to open this valve when the sea-water level is 6 feet above the horizontal hinge of the valve. The force to open the valve is applied at right angles to an arm 16 inches long attached at right angles to the upper edge of the valve close to the hinges.
(I.C.E., *Feb.*, 1902.)

31. Explain the construction of the common type of hydraulic jack.
(I.C.E., *Feb.*, 1902.)

32. Give a rough diagram showing the construction of the ordinary form of Worthington pump. (I.C.E., *Feb.*, 1902.)

33. A rectangular area is immersed in water with one edge in the surface; express the resultant fluid pressure upon one side of it, and state where this resultant acts. The width of a lock is 12 feet, and that of each gate 6½ feet; if the height of the water be 10 feet inside the lock and 4 feet outside, find the resultant fluid pressure on each gate, and also the pressure (assumed to be acting symmetrically) between the two gates.
(I.C.E., *Oct.*, 1902.)

34. Give sketches and describe the construction of a hydraulic crane. Estimate the volume of the ram necessary, if a weight of 5 tons is to be lifted 20 feet, the water-pressure being 700 lbs. per square inch and efficiency of machine ⅓. (I.C.E., *Oct.*, 1902.)

LECTURE XXXIV.

HYDRAULIC APPLIANCES IN GAS WORKS.

CONTENTS.—Labour-Saving Appliances in Modern Gas Works—Pumping Engines and Accumulator—Example I.—Differential Accumulator—Brown's Steam Accumulator—Small Hydraulic Accumulator Plant—Arrol-Foulis' Gas Retort Charging Machine—Foulis' Withdrawing Machine for Gas Retorts—Results of Working - Questions.

Labour-Saving Appliances in Modern Gas Works.*—In no industry have the conditions of labour undergone more radical and rapid changes than in large modern gasworks. Until recently, the coal was emptied by hand from railway waggons or from carts at the most convenient position for the retort benches. If the lumps of coal were too large, they were broken up by manual labour, and the retorts were charged, as well as discharged, by hand. The surplus coke, beyond what was required for the furnaces, was quenched, wheeled into a yard, and there screened and filled into carts or waggons, all by hand labour.

In modernised works, the coal is usually delivered direct from the railway truck into the hopper of the mechanically-driven coal breaker. After passing through this machine, it is raised by an elevator to a large hopper, which is so fixed that the coal can be automatically delivered into the hopper of any of the charging machines. The charging of the retorts is then performed by means of a hydraulic charging machine more evenly and otherwise better than by hand. After carbonisation, the coke is withdrawn from the retorts by a hydraulic drawing machine. These two operations now entail a minimum of labour and inconvenience from heat to the attendants. The surplus

* I am indebted to Mr. Andrew S. Biggart, of Sir William Arrol & Co., Ltd., Glasgow, for the drawings from which the three electros of the General Arrangement, Charging, and Drawing Machines were made. I am also indebted for the information contained in Mr. Biggart's paper which he read before the 1895 Glasgow meeting of the Institution of Mechanical Engineers, as well as to Mr. Foulis, the General Manager of the Glasgow Corporation Gas Trust for showing me the whole of the plant in operation.

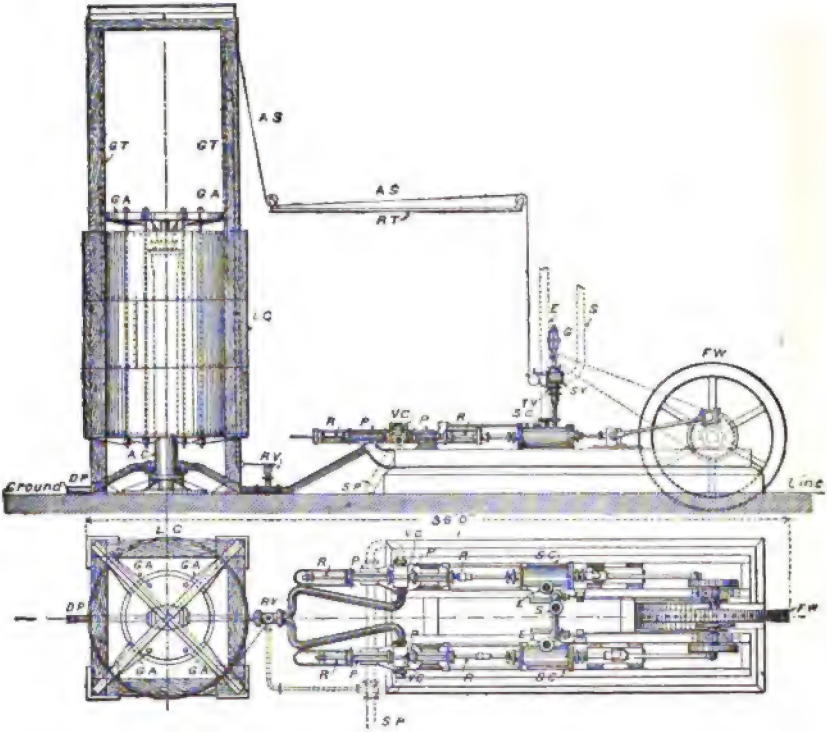
coke, guided in its descent by shoots, falls into a bogie underneath the floor, and is run out to the yard by small locomotives. In some instances conveyors, which are placed underneath the stage floor, carry the coke to circular revolving screens, whence it is delivered into large storage hoppers, railway waggons, or bags for small consumers.

The old methods involved continuous and repeated operations performed by hand; while the new are such that no hand labour is employed in dealing with the coal from the time it leaves the railway truck until it arrives there again in the form of coke. Hand labour is thus entirely superseded by mechanical power, to the great advantage of both the labourer and the employer. The number of men required in the retort house under the new system is less than half that required under the old method. The saving which this represents, after allowing for maintenance of plant and interest on additional capital, will average about one shilling per ton.

Pumping Engines and Accumulator.—The general arrangement of this portion of the plant is illustrated by the following side elevation and plan. The steam engine and pumps are situated either in the main engine room of the gasworks or in a convenient house not far removed from the steam boilers; while the accumulator may be placed in an annex, where there is plenty of head room for the guide timbers. From what has already been said about the construction and action of the Armstrong accumulator in the preceding Lecture, and by carefully comparing the present figures with the index to parts (which has been tabulated in nearly the exact sequence of the operations) the student will have no difficulty in understanding the generating plant which serves to supply hydraulic power to the charging and drawing machines in the retort houses.

The steam engine drives the rams R of the pumps P direct from a back extension of the piston-rods. From the pumps the water is forced into the accumulator cylinder A C at such a pressure that, acting on the accumulator ram, it is capable of lifting the heavy dead-weight bolted to the upper end. This weight is provided with guide arms G A, bearing upon planed iron rails fixed to the vertical guide timbers G T. When the guide arms have reached a certain height, the one at the right hand begins to lift a weight attached to a chain on the weighted lever of the throttle valve T V. During the remainder of the ascent of the accumulator the automatic starting chain A S is slackened until the throttle valve is closed, and the engine is stopped before the guide arms reach the cross beam, which connects the upper ends of the guide timbers. Should

the chain stick, or anything fail about this automatic system of stopping the engine, then a second chain, which connects the weight at the end of the relief valve R V to the counterpoise ball



General Arrangement of Pumping Engines and Accumulator for Working Gas Retort Charging and Withdrawing Machines.

Index to Parts.

S	for	Steam Pipe.	V C	for	Valve Chest for Pumps.
S V	,,	Stop Valve.	R	,,	Rams.
T V	,,	Throttle Valve.	P	,,	Pumps.
A S	,,	Automatic Starting and Stopping Apparatus.	R V	,,	Relief Valve.
S C	,,	Steam Cylinder.	A C	,,	Accumulator Cylinder.
E	,,	Exhaust Pipe.	D P	,,	Delivery Pipe.
G	,,	Governor.	G A	,,	Guide Arms.
F W	,,	Flywheel.	G T	,,	Guide Timbers.
S P	,,	Suction Pipe.	L C	,,	Load Casing.
			R T	,,	Roof Tie-Rods.

attached to the chain A S, is tightened up just before the guide arms reach the uppermost limit of their travel.* This action releases the downward pressure on the relief valve, and permits sufficient water to escape to prevent any further elevation of the accumulator ram. The governor G now prevents the engine from racing. When water is used by any of the hydraulic machines it flows to them from the accumulator cylinder through the delivery pipe D P. This allows the accumulator ram and weight to sink until the ball attached to A S re-asserts its pull on the throttle valve, and, automatically opening it, starts the engine.

It is evident that if a set of engines and pumps were devoted to the direct supply of high-pressure water to several machines, their output would have to be equal to the greatest requirements of the plant at any instant; but by the introduction of the accumulator and its automatic starting and stopping gear we have a simple means of ensuring a constant supply of water at the desired pressure with a smaller engine.

An accumulator, therefore, performs several very important functions in a most efficient manner:—

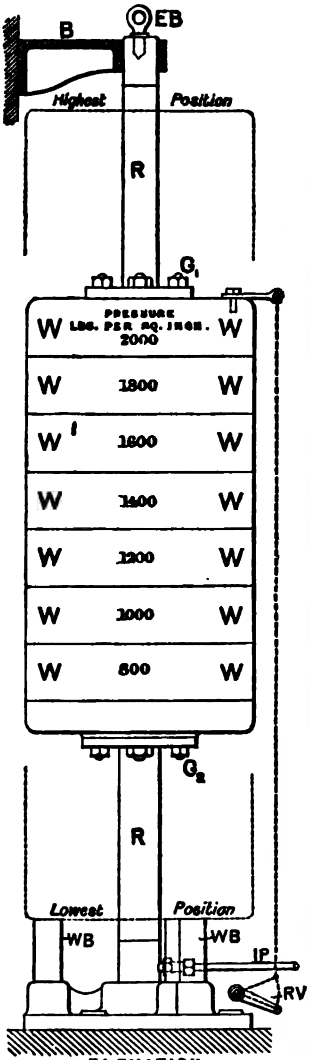
(1) It acts as a reservoir for the storage of energy.
(2) It acts as a regulator of pressure.
(3) It acts like a flywheel in taking up and giving out power in direct sympathy with the immediate wants of supply and demand.
(4) It acts as an elastic buffer, and prevents the breakage of joints, &c
(5) It automatically controls the motive power.

The efficiency of an accumulator has been proved to be as high as 98 per cent., only 1 per cent. being lost through friction in charging, and 1 per cent. in discharging it, as tested by pressure gauges on the supply and discharge pipes. Its total store of energy is, however, comparatively small, since it is only equal to the potential energy of the weight raised. Hence, if W lbs. be the total weight raised in the accumulator, and H feet the difference of height between its highest and lowest positions, we have:—

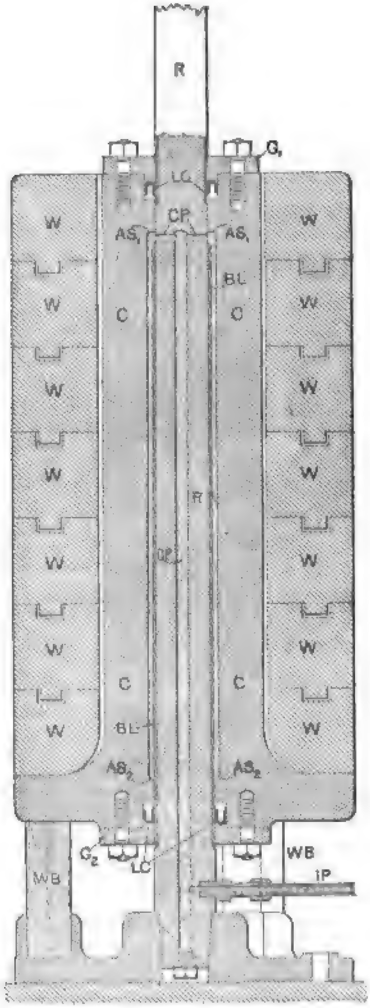
$$\left.\begin{array}{l}\textbf{Energy Stored in}\\ \textbf{Accumulator}\end{array}\right\} = \mathbf{W\,H}\ \textbf{ft.-lbs.} = \frac{\mathbf{W\,H}}{33{,}000 \times 60}\ \textbf{H.P.-hours.}\dagger \qquad \text{(I)}$$

* Another arrangement is to pass this second vertical chain through a hole in a projecting plate fixed to the right-hand guide arm G A, and then attach its upper end to the top cross beam. If the accumulator should rise too high, then the plate catches an enlarged portion of the chain and opens the relief valves.

† One horse-power = 33,000 ft.-lbs. of work per minute, hence, 1 horse-power hour is 33,000 × 60 or 1,980,000 ft.-lbs.



ELEVATION.



VERTICAL SECTION.

TWEDDELL'S DIFFERENTIAL ACCUMULATOR.

EXAMPLE I.—The accumulators used in connection with the hydraulic power supply in Glasgow are 18 inches in diameter, and have a free lift of 23 feet. The total load on each is 127 tons. Find the pressure of the water and the maximum energy stored in each accumulator, neglecting friction.

ANSWER.—

Pressure of water $= p = \dfrac{W}{A}$.

" " $= \dfrac{127 \times 2{,}240}{\cdot 7854 \times 18 \times 18} =$ **1,120 lbs. per sq. in.**

Energy stored $= W H = 127 \times 2{,}240 \times 23$.

" $= 6{,}543{,}000$ ft.-lbs., or **3·3 H.P.-hours.**

INDEX TO PARTS.

I P	for	Inlet Pipe from pump.
R	"	Ram of accumulator.
B L	"	Brass Liner on lower part of R.
C P	"	Central and Cross Passages for water.
C	"	Cylinder.
A S_1, A S_2	"	Annular Spaces from top and bottom of cylinder.
G_1, G_2	"	Glands (top and bottom).
L C	"	Leather Cup Packings.
W	"	Weights.
W B	"	Wooden Blocks.
B	"	Bracket (at top).
E B	"	End Bearing.

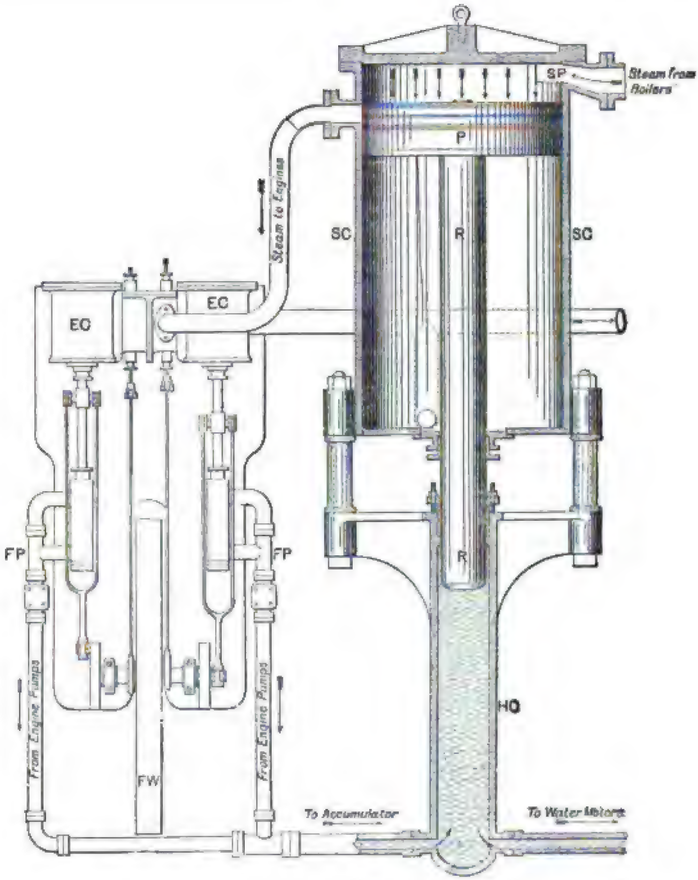
It will be seen from this example how small the total store of energy is, and that accumulators would be quite useless to maintain the supply for any length of time. One of the real advantages of the accumulator arises from the fact that we may use its energy for a very short time at a high rate. For instance, although the accumulator in the above example is only capable of maintaining 3·3 H.P. for a whole hour, it could exert 19·8 H.P. for ten minutes, or 198 H.P. for one minute.

Differential Accumulator.—Although it has only been found necessary to employ a water pressure of 400 lbs. per square inch in the charging and drawing machines for gas retorts, which we will describe later on, and, further, since it is advisable with them to have a considerable volume of water in the accumulators when many machines have to be worked simultaneously, yet it may not be out of place to describe here the differential accumulator designed by Mr. Tweddell for his smaller kinds of hydraulic

tools, since there may be cases in which space and compactness are of considerable importance. From the foregoing elevation and enlarged vertical section it will be seen that the ram R consists of a vertical fixed shaft secured at the top by a bracket B, and at the bottom by a footstep. The lower half of this shaft is of larger diameter than its upper half, a brass liner B L being shrunk on the former part. Moreover, this lower portion of the ram has a central passage C P drilled axially along it, with a cross passage just above the upper end of the brass liner. Through these passages water is admitted from the inlet pipe I P, which is connected directly to the force pumps. This water finds its way into an annular space $A S_1$, $A S_2$, which is the clearance between the outside of the brass liner and the inner bore of the heavy press or cylinder C. Surrounding the outside of the cylinder are placed a number of cast-iron or lead weights W which fit into each other, and form the dead load along with the weight of the cylinder. At the top and the bottom of the cylinder there are suitable glands G_1 and G_2 containing the usual leather cup packings L C. When the machine is idle the bottom flange of the cylinder rests upon wooden beams W B. It will now be readily understood that the effective area of the ram is *only* the difference between the cross areas of the brass liner B L and the upper part of the ram R, instead of the whole area of the ram as in the previous case. Hence, a very great pressure may be obtained from a small weight. For example, should the annular area representing the difference in size between the brass liner and the upper part of the iron ram be 5 square inches, and the total weight of the cylinder and its surrounding cast-iron blocks be 2,000 lbs., then, neglecting the friction at the glands, the pressure would be $2,000 \div 5$, or 400 lbs. per square inch. This accumulator will store up an equal amount of energy, as in the previous case, if the dead weight and height of the lift are the same since their stored energy depends directly upon $W \times H$. The volume of water contained in the accumulator will, however, be comparatively small, and hence it will fall more quickly for a certain amount of water used by the hydraulic machines which it drives. As will be seen from the left-hand figure, a relief valve R V is worked by a chain connecting an outstanding arm on the uppermost weight to the end of the lever; also any desired pressure up to 2,000 lbs. per square inch, or more, may easily be obtained from this accumulator with a comparatively small dead load and space.

Brown's Steam Accumulator.—Another very simple form of accumulator, which has proved very effective both for land purposes and on board ship, is that designed and made by Mr. A.

Betts Brown, of the Rosebank Iron Works, Edinburgh. It consists of a steam cylinder S C fitted with a piston P and a piston-rod or ram R. Steam is supplied direct to this cylinder from the boiler and presses on the piston P in opposition to the



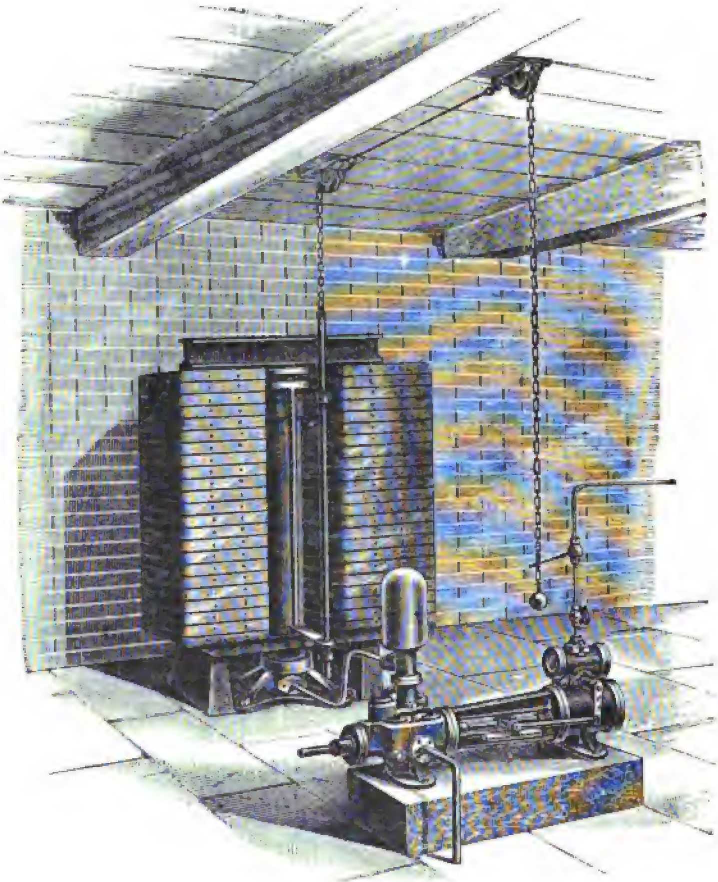
BROWN'S STEAM ACCUMULATOR OR COMPENSATED STEAM PUMP.

INDEX TO PARTS.

S P for Steam Pipe from Boilers.
S C ,, Steam Cylinder.
P ,, Piston working in S C.
R ,, Ram attached to P.

H C for Hydraulic Cylinder.
E C ,, Engine Cylinders.
F P ,, Force Pumps.
E ,, Exhaust Pipe.

water forced into the hydraulic cylinder H C by the force pumps F P, which are worked by a pair of engines. An exhaust pipe E carries away the exhaust steam from the engine cylinders E C and the bottom of the large steam accumulator cylinder S C.

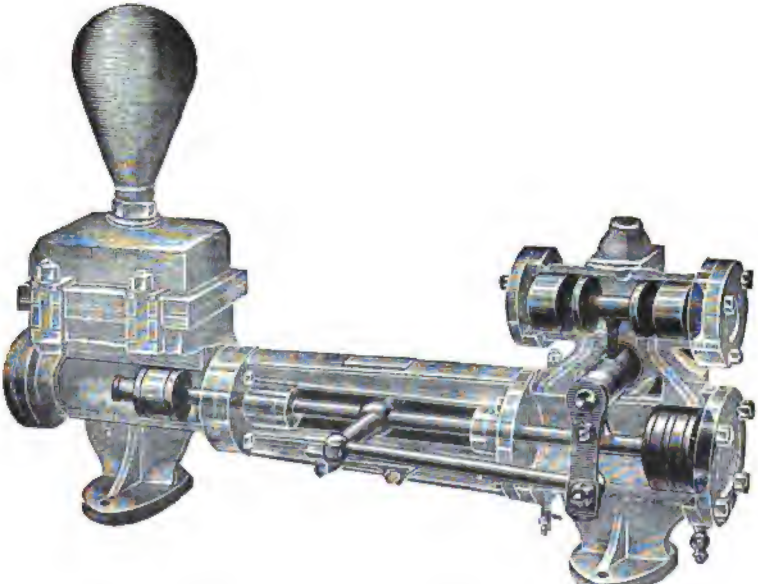


SMALL HYDRAULIC ACCUMULATOR PLANT.

Suppose that the piston P is at the bottom of its cylinder, then the boiler steam not only fills the portion above the piston but passes on to the engine cylinders and therefore works the pumps

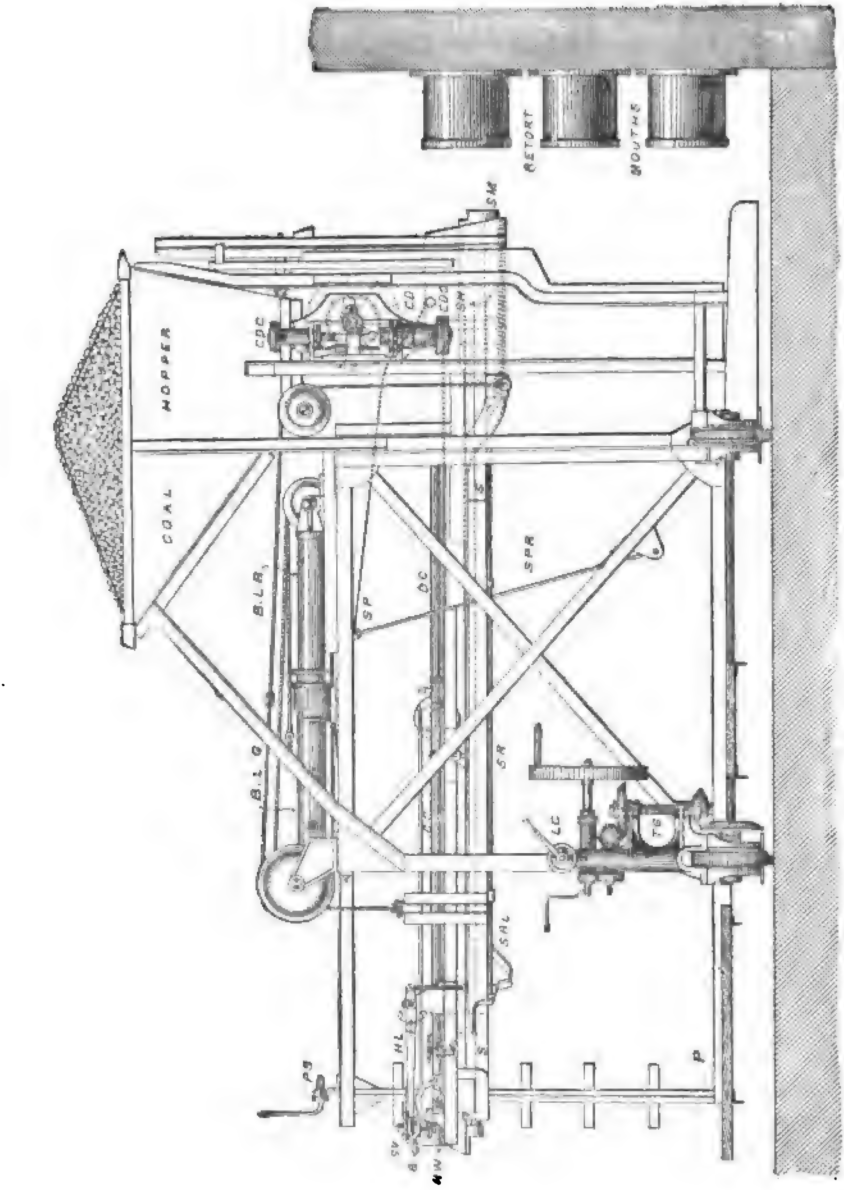
and forces up the ram and piston to the top of their stroke, when the piston gradually closes the steam passage leading to the engines until they stop. Should any of the hydraulic machines be now put into action the water flows to them from the hydraulic cylinder, the ram and piston descend and the engines are again set into motion to keep up the demand and again close the engine steam pipe. With a steam cylinder of 36 inches diameter and a ram of $9\frac{1}{4}$ inches (or, ratio of areas 15 to 1) Mr. Brown is able with about 50 lbs. steam pressure per square inch to maintain a pressure of 750 lbs. per square inch in the hydraulic mains leading to water motors, steering, stopping, and starting gear, or, in the case of a gas works, to the charging and drawing machines.

Small Hydraulic Accumulator Plant.—Should the gas works be a small one, then a much less expensive accumulator plant may be employed, such as that illustrated. This figure is sufficiently self-descriptive after what has been said about the larger plant.



DIRECT ACTING STEAM PUMP.

The small pumping engine, or donkey pump, which supplies water of the desired pressure to the accumulator is also shown in a perspective view. Steam is admitted to the steam



Arrol-Foulis Gas Retort Charging Machine.

cylinder by piston valves, actuated by a connecting-rod and lever, while the double-acting pump is fitted with a solid piston plunger and an air vessel.

Arrol-Foulis Gas Retort Charging Machine.—In replacing hand labour for the charging gas retorts by a hydraulic machine several important requirements have to be effected:—

(1) The machine should be easily moved parallel to the retort bench at the proper distance therefrom.

(2) It must be capable of being connected with the hydraulic pressure pipe at every position.

(3) The shoot mouth should be easily raised or lowered to suit the highest or lowest retort.

(4) The shoot mouth should be easily entered into the mouth of the retort before filling it with coal.

(5) The coal hopper should contain at least a sufficient quantity of coal to charge a complete set of retorts without requiring to be refilled.

Index to Parts.

P	for Platform (for attendant).	C D	for Coal Drum.
T G	,, Travelling Gear.	S V	,, Slide Valve.
P S	,, Pressure Swivel.	C C	,, Charging Cylinder.
L C	,, Lifting Cock.	S	,, Spear.
B L C	,, Beam Lifting Cylinder.	S H	,, Spear Head.
B L R	,, Beam Lifting Ram.	D C	,, Drawing Cylinder.
S R L	,, Shoot Rod Lever.	S P	,, Swing Plate.
S R	,, Shoot Rod.	S P R	,, Swing Plate Rods.
S M	,, Shoot Mouth.	H W	,, Hand Wheel.
H L	,, Hand Lever.	B	,, Bell.
C D C	,, Coal Drum Cylinders.		

(6) The exact quantity of coal required for the charge of each retort should be easily regulated, and the regulator should be capable of dealing with all the sizes of coal in use.

(7) The charge should be laid evenly and of the desired depth from back to front without any special effort on the part of the attendant, and without loss of time.

(8) The platform of the attendant should be placed in the most convenient position for actuating all the motions, and at the same time protecting him from the heat emanating from the retorts.

(9) The whole machine should require a minimum of attention and repair. It should also be able to withstand the rough usage of retort house workmen, as well as the hydraulic pressure, without undue leakage or giving way in any of its vital parts.

These several requirements have been very ingeniously worked out and applied by Sir William Arrol and Mr. William Foulis in their patent hydraulic charging machine, shown by the accompanying figure, as follows:—

(1) The whole machine with its load of coal is supported on a pair of rails, placed parallel to the retort bench, and moved by travelling gear T G. In the illustration ordinary hand gear is shown, but in the latest machines a hydraulic motor of special design has been applied with excellent results.

(2) A pipe is fixed the whole length of the retort bench at a convenient height and distance therefrom, and it is always kept charged with water from the accumulator at the full working pressure of 400 lbs. to the square inch. At suitable distances along this pipe there are attached ordinary armoured flexible hose pipes, which may be connected to the pressure swivel P S. From this swivel copper pipes lead to the lifting cock L C and slide valve S V.

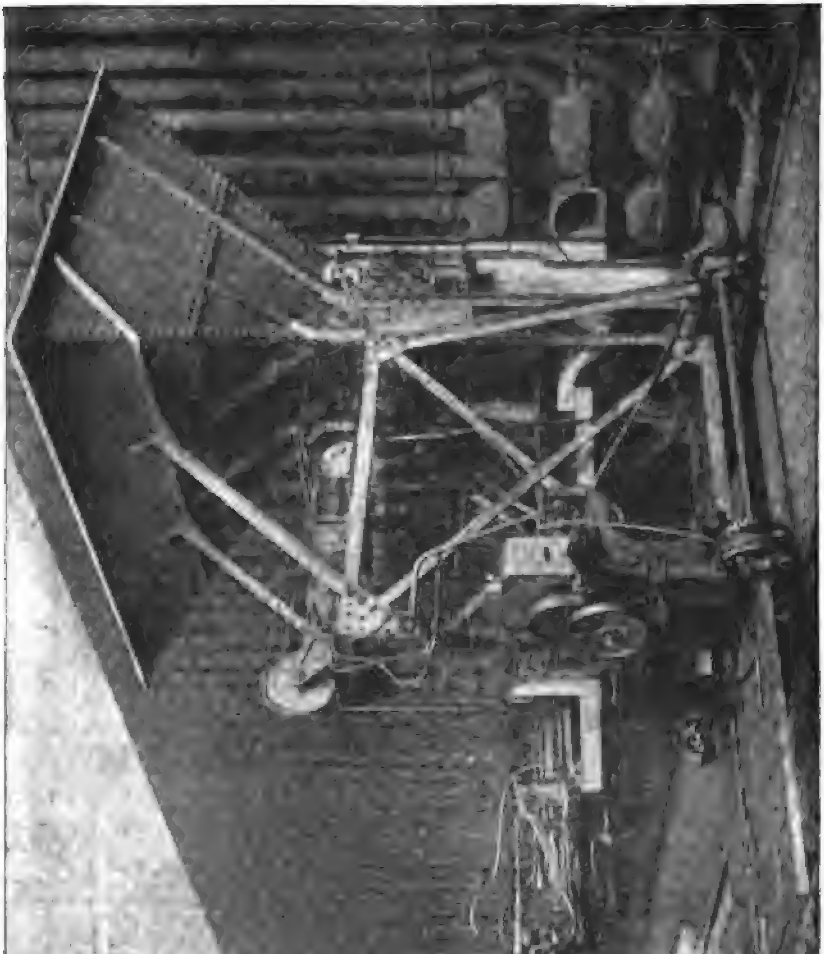
(3) The beam carrying the shoot mouth, shoot rod S R, spear S, charging cylinder C C, and drawing cylinder D C, is raised and lowered by turning the lifting cock L C to the right or left, and thereby admitting water to the beam lifting cylinder B L C.

(4) When the shoot mouth S M is fairly opposite one of the retort mouths it is pushed forward by the shoot-rod lever S R L, and the hand lever H L is turned to admit water to the slide valve S V and work the charging cylinder C C, so as to push the spear S and spear head S H into the retort, having a quantity of coal in front.

(5) The coal hoppers are made to contain from 2 to 5 tons.

(6) The feeding gear consists of an open coal drum C D, which is divided into segmental compartments, each of which can contain a certain quantity of coal. It is turned through one or two divisions at regular intervals by a hydraulic ram with rack and ratchet gear, so as to permit the desired quantity of coal to fall into the shoot. A plate, which acts as a flap valve, is so fixed in front of the drum by a lever and weight, that it presses against the face of the drum, and prevents the small coal from falling down past the face of the drum.

(7) The coal falls from the hopper in front of the pusher plate or spear head S H, and is delivered into the retort by a series of six or seven successive strokes, each stroke being shorter than the previous one. This is accomplished by means of a shaft carrying a set of stops placed on the beam alongside the spear S. This shaft is automatically turned a certain amount during each return stroke, so as to bring the stops into position in rotation. The forward and return strokes of the spear are caused by two



General View of Arrol-Foulis Charging Machine.

hydraulic rams working in the cylinders C C and D C, which are regulated by the hand lever H L. This lever also serves to lower the spear head when entering the retort for pushing in the coal, and to raise it clear from the bottom of the retort during the return stroke. It also causes the revolution of the stopper shaft. The inclined form of the spear head and these successive strokes ensure the charge being evenly distributed along the retort, and the depth is regulated by putting in more or less coal at each stroke, as previously explained.

(8) The platform P is placed at the back of the machine, so that the attendant has the levers within easy reach, and he is placed as far as possible from the heat of the retorts.

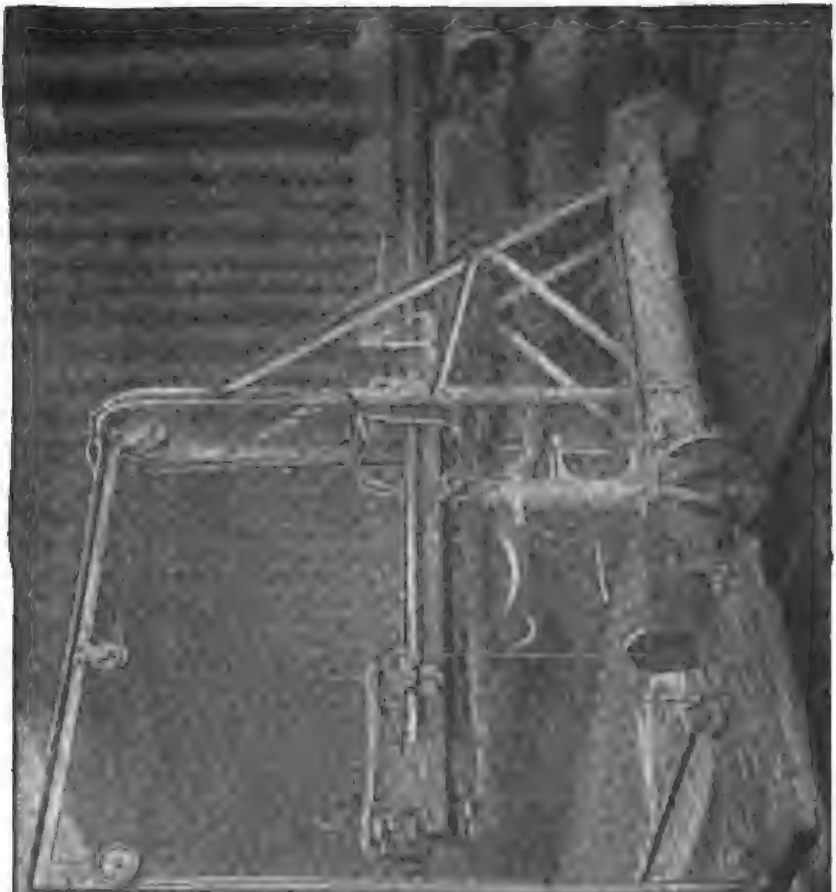
(9) Great attention has been given to the several details of the machine, so as to render it as durable as possible. The reason for adopting this comparatively low pressure is, that the power required for the different motions is so small that if a higher pressure were employed the rams would be inconveniently small. In fact, with 400 lbs. pressure it had been found in some instances that rams of only $1\frac{1}{4}$ inch diameter gave ample power.

Foulis' Withdrawing Machine for Gas Retorts.—After the coal has been carbonised and converted into coke, it has to be withdrawn from the retorts before putting in a new charge. This is done by a machine which is similar to that used for charging, and travels along the same rails. The withdrawing machine consists of a frame supported on a truck and carrying a rake for pulling out the coke. The head of this rake can swing backwards into a horizontal position so as to clear the coke when being moved inwards, but goes back to the vertical and dips into the coke on its return stroke.

The beam B carrying the rake can be raised or lowered, and the rake moved out and in, in the same manner as the corresponding parts of the charging machine. As, however, the withdrawing machine is comparatively light, it is found quite sufficient to use hand travelling gear T G to make it move along the rails.

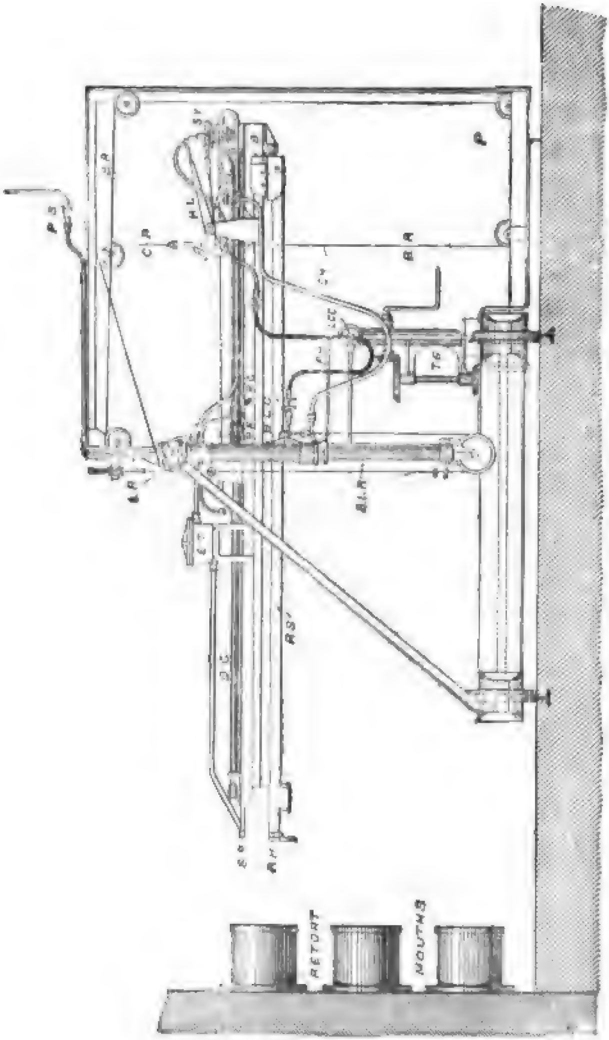
The exhaust water is led to a tank E T, from which it flows to a spray pipe S P and plays on the rake head to keep it cool and help to quench the coke.

Results of Working.—Mr. Biggart says that the number of retorts charged or drawn per hour by these machines varies to a considerable extent in actual work. In some cases, owing to special circumstances, not more than twenty-four per hour are available for each machine; while in other more favourable instances as many as forty-eight per hour are allotted to each, and even with this larger number a reasonable time remains for



FOULIS' WITHDRAWING MACHINE FOR GAS RETORTS.

rest for the attendants at the end of each hour. The labour of charging the retorts and withdrawing the coke is much lightened



FOULIS' WITHDRAWING MACHINE FOR GAS RETORTS.

by these mechanical means, and the number of retorts charged and drawn for each man employed is largely increased. It

might at first be imagined that coal placed in retorts in only six to eight large charges by the machine would not be so evenly laid as a much larger number of smaller charges put in by hand. The machine, however, lays the coal by far the most evenly, owing partly to the shape of the pusher head, which is bevelled so as to allow the small ridge of coal raised in pushing forwards to fall back when its support is removed on the withdrawal of the pusher head. Another advantage possessed by machine work over hand labour is that the charging is done more quickly, and thus there is a diminished loss of gas before the retort doors are closed.

Apart from any other consideration, the mechanical charger could not fail to prove beneficial in view of the greatly improved conditons under which it enables work of a most trying nature to be carried on. The old method of hand charging was a severe ordeal for the stokers, requiring great exertion to get through

INDEX TO PARTS.

T G	for Travelling Gear.	P H	for Pressure Hose.
P S	,, Pressure Swivel.	P C	,, Pushing Cylinder.
L C C	,, Lifting Cylinder Cock.	P R	,, Pushing Ram.
B	,, Beam.	R S	,, Rake Spear.
B L C	,, ,, Lifting Cylinder.	R H	,, ,, Head.
B L R	,, ,, ,, Ram.	E C	,, Exhaust Cock.
L R	,, Lifting Rope.	E H	,, ,, Hose.
B R	,, Balancing Ropes.	E T	,, ,, Tank.
C R	,, Check Rope.	S P	,, ,, Spray Pipes.
H L	,, Hand Lever.	D C	,, Drawing Cylinder.
S V	,, Slide Valve.	P	,, Platform (for Attendant).

the work in the shortest time possible, while exposed throughout to a high heat. Such adverse conditions are now entirely done away with where mechanical stoking obtains. The single lever by which the whole of the operations are controlled is worked from such a position that the attendant is quite removed from the discomfort of close proximity to a high heat, while at the same time the former severe bodily exertion is replaced by light and easy work. Even greater improvements in the conditions of labour arise from the introduction of the drawer, which accomplishes, under all the better conditions attending the use of the charger, work of a still more trying nature. The withdrawing of the hot coke from the retorts was work for which even the stokers themselves, accustomed as they were to it, admitted that mechanical appliances were required. Here again all is worked by a single lever, in such a position as to remove the attendant from the former discomforts of withdrawing the coke at a white heat at the mouth of the open retort.

Lecture XXXIV.—Questions.

1. Sketch and describe the general arrangement of pumping engines and accumulator as used for supplying hydraulic power to the mechanical stoking appliances in a gas works.

2. Make a vertical section of an accumulator, and explain the manner in which this apparatus enables us to store up and give out energy. If the ram of the accumulator be 17 inches in diameter, what should be the load in order to obtain a water pressure of 700 lbs on the square inch? *Ans.* 70·9 tons.

3. Sketch and describe some arrangement of an hydraulic accumulator by which a pressure of 10 tons to the square inch can be obtained for testing purposes, with pumps working at a pressure not exceeding 3 tons to the square inch.

4. Describe, without going into detail, the engines, pumps, accumulator, and one or two of the appliances likely to be used by customers of an hydraulic company. (S. and A. Adv. Exam., 1897.)

5. If you desire to obtain great pressure with a small dead load, what form of accumulator would you employ? Give a vertical section and a description of the construction and action of the accumulator you have selected. Why is it inadvisable to use one with a very small ram?

6. If you desire to obtain a pressure of 1,200 lbs. per square inch, and you are limited to a dead weight of 1 ton, what effective area would you require, and what would be the diameter of the larger part of the ram if the smaller be 6 inches in diameter, in the case of a differential accumulator having a total efficiency of 90 per cent.? What would be the diameter of the ram if you used a solid one?

7. Sketch and describe Brown's Steam Accumulator. Mention any advantages and disadvantages which you consider it possesses with respect to other forms of accumulator.

8. Suppose the effective steam pressure in the steam cylinder of a Brown's Accumulator to be 60 lbs. on the square inch, and that you require a water pressure of 1,000 lbs. per square inch, with a ram of 20 square inches in cross section, what will be the diameters of the ram and steam piston if 2 per cent. be lost in friction?

9. Sketch the construction and describe the action of the Arrol-Foulis hydraulic apparatus for charging gas retorts. Mention the several advantages of employing this machine as against hand labour.

10. Sketch the construction and describe the action of the Foulis hydraulic apparatus for withdrawing the coke from gas retorts. Mention the several advantages of employing this machine as against hand labour.

11. What is the use of an intensifier or intensifying accumulator in the working of hydraulic machinery? Sketch such an apparatus, and explain fully its principle and construction: give also one example of the application of the intensifier to hydraulic machinery. (S. and A. Adv. Exam., 1896.)

12. A differential accumulator, loaded to 1,400 lbs. per square inch, has a 5-inch spindle and a ½-inch bush, and supplies a riveter ram of 9 inches diameter through 30 feet of 1-inch piping. Find the "equivalent mass" at the riveter ram on the assumption that no water is pumped into the accumulator during the working stroke of the riveter ram.

(C. & G., 1902, H., Sec. C.)

13. Sketch an automatic governor suitable for use on a main pumping engine which, by operating on the steam valve, reduces the speed of the pump when the pressure in the delivery pipe exceeds a certain amount.

(C. & G., 1902, H., Sec. C.)

Lecture XXXIV.—A.M.Inst.C.E. Exam. Questions.

1. Give a short account of the arrangements for distributing power by pressure water. Sketch an accumulator. Find the energy stored in foot-pounds per cubic foot of water, in an accumulator loaded to 700 lbs per square inch. (I.C.E., *Feb.*, 1898.)

2. Sketch, dimension, and describe a proper packing gland for an accumulator 12 inches in diameter and 4,000 lbs. per square inch water pressure. (I.C.E., *Feb.*, 1901.)

3. What amount of energy is stored in p lbs. of water at q lbs. per square inch pressure? A differential hydraulic press has two stuffing boxes in line and a vertical rod passes through both, the diameters of the pistons at its lower and upper ends being 5 inches and 8 inches respectively. What load does the press take when supplied with water at a pressure of 1,400 lbs. per square inch? What store of energy can it take in, in foot-pounds, if it can rise through 10 feet, and how many gallons of water are required for the operation? (1 gallon of water = 10 lbs.) (I.C.E., *Feb.*, 1901.)

4. Sketch any form of direct-acting compensated steam pump, explaining the action of the compensator by reference to combined diagrams. (I.C.E., *Oct.*, 1902.) (*See Brown's Steam Accumulator.*)

LECTURE XXXV.

HYDROKINETICS.

CONTENTS.—Energy of Flowing Water—Bernouilli's Theorem—Jet Pumps, Injectors and Ejectors—Hydraulic Ram—Example I.—Velocity of Efflux and Flow of Water from a Tank—Measurement of a Flowing Stream—Rectangular Gauge Notch—Thomson's Triangular Notch—Measurement of Head—Measurement of Large Streams—Horse-Power of a Stream—Vortex Motion—Free Vortex—Forced Vortex—Pressure due to Centrifugal Force—Reaction of a Jet—Reduction of Pressure Round an Orifice—Impact—Loss of Energy—Resistance of a Pipe—Hydraulic Mean Depth—Loss of Head due to Friction in Water Pipes—Examples II. and III.—Notes on Measurement of Streams, &c.—Questions.

Energy of Flowing Water—Bernouilli's Theorem.—When a liquid is flowing in a pipe or channel, it possesses kinetic energy in virtue of its motion in addition to the potential energy due to its position and pressure; and the total energy is the sum of these three.

Let v = Velocity of the liquid.
„ h_1 = Length of its pressure column.
„ h_2 = Height above the datum level.
„ H = Height of the free surface of the still water above the datum level.
„ m = Mass of the portion of the liquid under consideration.
„ g = Acceleration due to gravity.
„ f = Frictional loss of head.

Then,

The energy of pressure $= m\,g\,h_1$

„ of position $= m\,g\,h_2$

And, „ of motion $= \frac{1}{2}\,m\,v^2$.

Hence, The total energy $$= m\,g\left(h_1 + h_2 + \frac{v^2}{2g}\right). \quad \text{(I)}$$

Or, Energy per unit mass $$= g\left(h_1 + h_2 + \frac{v^2}{2g}\right) \quad . \text{(I}_a\text{)}$$

It follows from the principle of the *Conservation of Energy* that so long as no work is spent on friction, this total remains constant whilst the water flows along the pipe or channel. Therefore, for a frictionless liquid in which there are no eddies :—

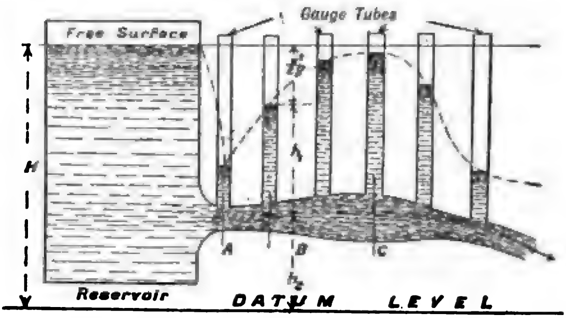
$$h_1 + h_2 + \frac{v^2}{2g} = \text{a constant} = \mathbf{H}. \quad . \quad . \quad . \quad \text{(II)}$$

If, p = Pressure per unit area at any point in the liquid,
And, w = Weight of unit volume of the liquid,

Then, $h_1 = \frac{p}{w}$;

Hence, $\frac{p}{w} + h_2 + \frac{v^2}{2g}$ = a constant = H. . . . . . . (III)

This equation is known as *Bernouilli's Theorem.*



PRESSURES IN A FRICTIONLESS PIPE.

If a certain amount of energy mgf be absorbed by friction between some vertical datum section such as A and the section under consideration which may be at B or C.

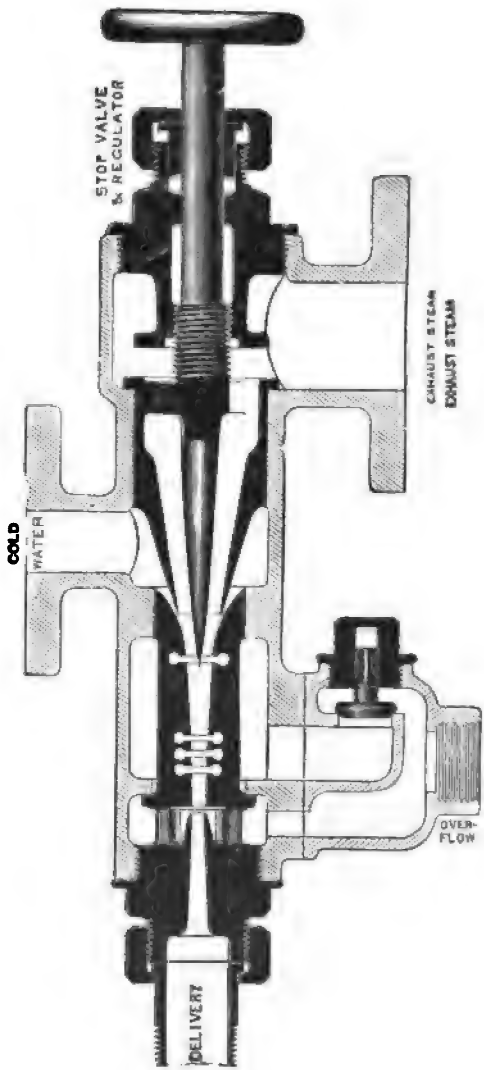
Then, $\frac{p}{w} + h_2 + \frac{v^2}{2g} + f$ = a constant = H. . . (IV)

Let a = The cross area of the pipe at any section.
,, v = ,, velocity of the water at the same section.
,, Q = ,, quantity or volume of water passing every section in unit time. This volume is constant for all sections during a steady flow of the liquid.

Then, $a v = Q$; or, $v = \frac{Q}{a}$. . . . . . . (V)

This shows, that the velocity is great when the cross section of the pipe is small, and *vice versâ.*

In the figure, we have shown small vertical gauge tubes placed at intervals A, B, C, &c., along the pipe, in order to indicate the pressure of water at these points by the height to which it rises in them. It will be observed that where the pipe is level or nearly so, the pressure is greatest where its cross section is largest and consequently the velocity is least. This follows directly from



EXHAUST STEAM EJECTOR.

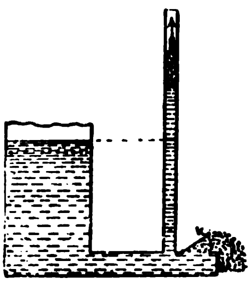
Bernouilli's theorem, since the height h_2 above the datum level is practically constant.

Jet Pumps, Injectors, and Ejectors. — From the previous illustration and explanation of the alteration in the pressure of water as it flows through a pipe of varying size, it will be readily understood that if a stream of water be rapidly forced through a tapered passage, its pressure may be so lowered below the surrounding atmosphere, that it can draw in more water from another source connected to the smaller end of the taper. If the whole of the water be then conducted along a gradually en'arged passage, its pressure will increase and the outflow can take place at a higher level than the intake of the induced stream, but lower than the free surface of the driving water. The late

Prof. James Thomson, of Glasgow University, designed his jet pump upon this principle, and Prof. Bunsen used a jet of water to produce a vacuum in his air pump. Steam jets, compressed air, and water under pressure, have frequently been used to create a blast of air, to feed petroleum into furnaces, to produce a sand blast for engraving or cleaning purposes, and to transfer granular materials from one position to another.

Injectors for feeding steam boilers with water also work upon this principle; but since they are greatly assisted by the condensation of the steam as it comes into intimate contact with the suction water, the latter can be forced into the same boiler or another vessel having the same pressure as, or even a higher pressure than, that which supplies the injector with steam.* Exhaust steam ejectors also depend upon the above action, and are sometimes used to replace both the ordinary jet condenser and air pump in connection with condensing engines. As will be seen from the accompanying figure, when the regulating stop valve is screwed upwards by turning the hand-wheel, the exhaust steam from the engine cylinder enters by the right-hand upper pipe and mixes with cold water coming through the left-hand one, thereby becoming condensed and producing the desired vacuum. The combined condensed steam and condensing water then flow from the delivery pipe into the hot well, whilst any throttled discharge, or some of the live steam that may have been used for blowing through and starting the ejector, can escape into the same place by the overflow pipe. As we shall prove further on, apparatus of this kind cannot have a very high efficiency since the mixing up of quick and slow moving streams results in a considerable loss of mechanical energy.

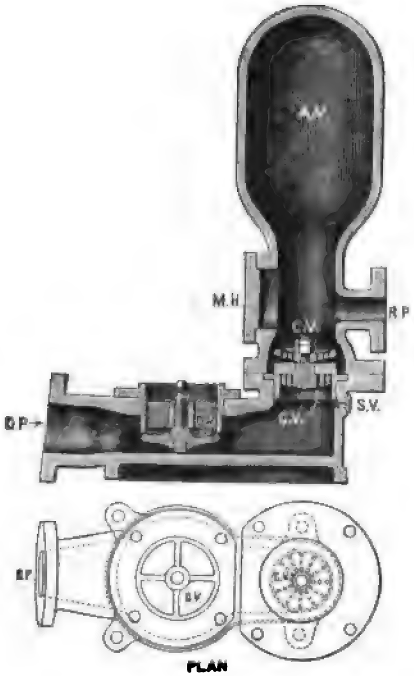
Hydraulic Ram.—This apparatus was invented about 100 years ago by a Frenchman named Montgolfier. It is one of the simplest, most durable, and efficient machines for raising water to a greater height than the source of supply. The energy stored up in water descending from a comparatively low elevation is utilised to raise part of the same water to a much higher level, of from three to thirty times the vertical height of the original fall. The principle upon which the apparatus works will be understood from



PRINCIPLE OF THE HYDRAULIC RAM.

* For a description of Giffard's and other steam injectors see the author's *Text-Book on Steam and Steam Engines.*

a consideration of the foregoing figure. If the valve V be held down firmly on its seat, and the left-hand vessel be filled with water to a certain height, it will rise to the same level in the right-hand open pipe. If the valve be now released for a short time, water will flow under the action of gravity along the horizontal passage and escape at the open valve with a velocity proportional to the square root of the "head" or vertical height of the free surface above the valve. On suddenly closing the valve, the kinetic energy of the moving water will be partly spent in raising the right-hand column to a greater height than the free surface of the water in the left-hand vessel. Now, if we introduce a check valve at the foot of the long column, so as to prevent this water from falling down again, and an air vessel to act as a cushion, we can repeat the operation continually, so as to produce a flow of water up the pipe.



HYDRAULIC RAM, BY THE GLENFIELD COMPANY, KILMARNOCK.

The machine, as made and supplied by the Glenfield Company of Kilmarnock, is illustrated by a vertical section and plan in the accompanying figure. Water flows from a cistern, tank, pond, or dam, through a cast- or wrought-iron pipe, technically called the drive pipe D P, to a hollow casting containing two valves. The first of these is named the escape or dash valve D V, which opens downwards, and the other the check valve C V, which opens upwards. Over the latter is fixed an air vessel A V, having a manhole door M H to the left and a delivery pipe, which is technically termed a rising pipe R P, to the right. If the

apparatus and all the pipes are duly connected to the supply and delivery tanks, and the dash valve D V be held up, until the water from the source of supply has filled not only the drive pipe D P, but also risen through the check valve C V and rising pipe R P to nearly the same level as the free surface in the supply tank, the whole will remain motionless or in a static condition. If we now depress the dash valve D V, and then let it go, the machine will immediately begin to work, and continue to work automatically without any attention or even oiling for years, until stopped by some accident or by the wearing out of one or both of the valves. Of course, the supply of water must be maintained, so that the drive pipe is always kept full. This pipe should not be throttled in any part, and the weight on the dash valve must be so carefully adjusted, that it will just overcome the internal pressure—*i.e.*, drop from its seat—and permit water to escape thereat. Then, the acceleration produced by gravity on the water coming down the drive pipe very soon produces a greater pressure on the dash valve than that due to the mere static pressure. This increased force suddenly raises it again to its seat, when the kinetic energy which has been imparted to the water lifts the check valve and forces some water into the air vessel. Whenever this kinetic energy has been spent, the compressed air in the air vessel, together with the weight of the check valve, causes it to close, and immediately thereafter the dash valve automatically opens. The same cycle of operations takes place over and over again, the air in the air vessel gets more and more compressed, and water rises higher and higher in the rising or delivery pipe, until it issues as a continuous stream from its mouth into the cistern or receiving tank. From this tank it may be drawn off at pleasure for all the various uses of a mansion-house or farm steading, &c.

The air vessel plays two important parts in each cycle of the operations of this interesting and useful apparatus. (1) The air contained therein acts as a cushion by minimising the water hammer action, which would otherwise stress the various parts, and tend to break the joints. (2) The air acts as a store of energy by taking up, during its compression, a part of the kinetic energy of the water, and then giving out the same gently, thus producing a constant flow of water through the delivery pipe. If the vertical height of the column of water in the rising pipe be about 34 feet above the check valve, the pressure per square inch on the upper surface of this valve will be one atmosphere, or, say, 15 lbs. on the square inch, and the air in the air vessel will be compressed to nearly that pressure,

and therefore occupy about half its original volume. If the column be 68 feet high in the delivery pipe, the pressure on the valve will be about 30 lbs. on the square inch, and the air in the air vessel will occupy one-third of its original volume, and so on. Hence, it is necessary to proportion the size of the air vessel to the vertical height through which the water has to be forced in the delivery pipe. Besides this, air becomes absorbed by water, and in a short time the air vessel, if small, would become entirely filled with water. The air vessel may, however, be kept charged with air in a very simple manner by the introduction of a snifter valve S V, screwed into the ram casing, immediately below the check valve. In its simplest, and probably its most efficient form, it consists of a brass plug with a very small hole drilled through its axis. Every time that water is forced through the check valve a very small quantity also passes through this tiny opening in the snifter valve; and each time that the check valve is forced down upon its seat a rebound or reaction of the water takes place, and produces a partial vacuum immediately underneath the check valve. Consequently, a little air is forced into this vacuum by the atmospheric pressure, and this air is carried up into the air vessel at the next stroke or pulsation, thus keeping up the necessary supply for effecting a continuous flow of water into the receiving tank. If everything about this machine is thoroughly tight and in good working order, and the valves are made of the best proportions and weights, an efficiency of from 80 to 90 per cent. can be obtained therefrom, and it has been found possible to work it with a minimum driving head of only three feet. The several causes for loss of efficiency are:—

(1) Eddies caused by the sudden stoppage of the water's motion.

(2) The friction of the water passing along the drive pipe D P, and the casing of the apparatus.

(3) The weight and friction of the dash valve D V, which has to be lifted at each stroke or pulsation.

(4) The weight and friction of the check valve C V, which has also to be lifted at each stroke.

(5) The slip of the check valve C V—*i.e.*, a slight quantity of water may slip back past this valve when in the act of closing.

(6) The friction of the water passing along the rising pipe R P.

(7) Any defects of tightness in the faces of the dash and check valves.

The sudden closing of the dash valve is only necessary to prevent the water spending a large portion of its energy in friction

at the restricted orifice while it is closing. Large rams are now made with special valves to stop the water gradually without throttling it, and so avoid the shocks caused by sudden closing without losing anything in efficiency.*

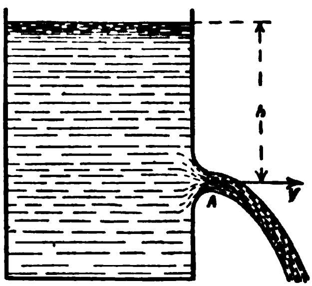
EXAMPLE I.—If 1,000 lbs. of water pass per minute through the drive pipe under a head of 6 feet, and 60 lbs. of water are delivered into the receiving tank, which is 87 feet above the check valve, what is the efficiency?

$$\text{Efficiency} = \frac{\text{work got out}}{\text{work put in}} \text{ (in the same time).}$$

$$\therefore \qquad \text{Efficiency} = \frac{87 \times 60}{6 \times 1000} = \cdot 87$$

Or, **Efficiency = 87 per cent.**

If the length of the drive and rising pipes be considerable, and if there be many bends and much throttling of the passages, then the efficiency will thereby be reduced to a considerable extent. By a simple modification of the ram shown in the illustration, river or impure water may be made to raise spring or pure water; the two waters are separated by a diaphragm, and the pumping action actuates two valves, the one being a suction and the other a delivery valve.



EFFLUX OF WATER FROM A CIRCULAR ORIFICE.

Velocity of Efflux and Flow of Water from a Tank.—Consider a jet of water issuing from a small circular orifice A at a depth h below the free surface of the water in the tank. If we take the datum line at the orifice, then inside the vessel, where the water is still, the energy is entirely potential and equal to $m g h$; whereas, it is all kinetic just outside the opening and amounts to $\frac{1}{2} m v^2$.

$$\text{Hence,} \qquad \tfrac{1}{2} m v^2 = m g h\text{; or, } v^2 = 2 g h\text{;}$$

$$i.e., \qquad v = \sqrt{2 g h}. \qquad \text{(VI)}$$

It will be observed from this equation (VI) that the velocity

* See § 30 of Gordon Blaine's *Hydraulic Machinery* for an illustrated description of Mr. H. D. Pearsall's improved hydraulic ram.

v is the same as that attained by a body in falling freely from a height h; and further, that it would be the same even if the water had no free surface, so long as the pressure at the level of the orifice was equal to that due to a head h.

If there be a loss of head f due to friction and eddies formed by the water in passing through the orifice,

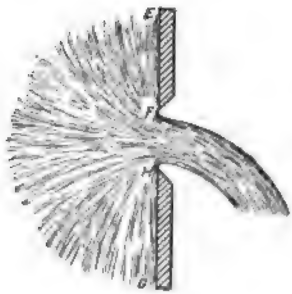
Then, $\frac{1}{2} m v^2 = m g (h - f)$; or, $v = \sqrt{2 g (h - f)}$. (VII)

If, a = The cross area of the orifice in square feet.
r = " radius of the orifice in feet.
v = " average velocity of the water in feet per second.
h = " head of the water in feet.
g = " acceleration due to gravity or 32·2 feet per second per second.
And Q = " quantity of water flowing out from the orifice in cubic feet per second.

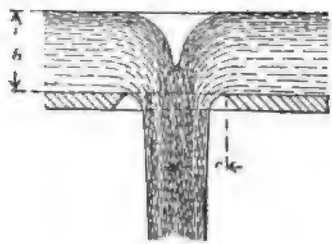
We find from equations (V) and (VI) that :—

$$Q = a v = a \sqrt{2 g h} \text{ cubic feet per second.}$$

This formula is very nearly true for a small tapered opening, but with a flat one, as shown in the next figure, the cross area of the



Efflux of Water from a Vertical Flat Orifice.



Efflux of Water from a Horizontal Flat Orifice.*

stream where the stream lines are parallel, at a short distance outside the opening, is less than that of the orifice. The ratio of these two areas is called the *coefficient of contraction*, and it is found experimentally for a flat opening to be 0·64. The contraction is caused by the water flowing along the inner flat surfaces E F and G H and then leaving them at a tangent. It has also been found by experiment that for a sharp-edged orifice the velocity v is only

* By mistake the figure has been drawn with a vortex, but when measuring water we must prevent the formation of a vortex by putting in radial blades.

$0 \cdot 97 \sqrt{2 g h}$. Hence, the actual flow for *a small circular orifice* will be :—

$$Q = 0 \cdot 64\, a \times 0 \cdot 97 \sqrt{2 g h} = 0 \cdot 62\, a \sqrt{2 g h}$$

$$\therefore \qquad Q = 0 \cdot 62\, \pi r^2 \sqrt{2 g h} \text{ cubic feet per second.} \qquad \text{(VIII)}$$

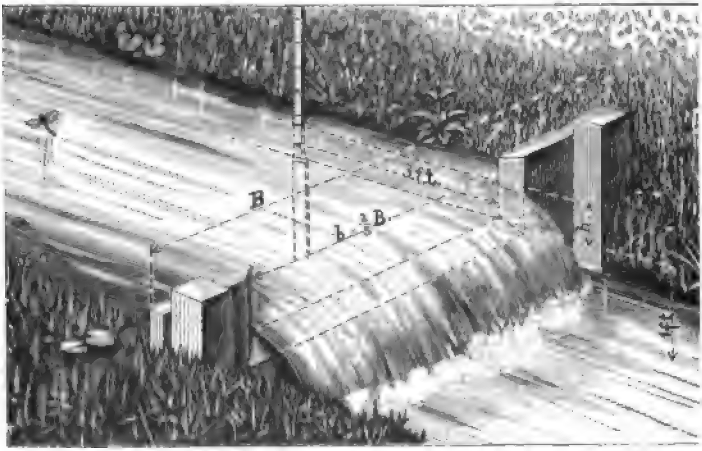
In measuring a small flow of water by this method, it is run into a tank having a carefully made clean cut orifice of known area, until the surface level is just sufficiently above the outflow to cause the water to run out as fast as it runs into the tank. This difference of level, or head h, is measured and the above formula applied.

Accurate results cannot be obtained from a large circular opening placed in the side of a tank, because the parts of it would have different depths from the free surface, and consequently the water would have different velocities at these parts. If we, however, put the orifice in the bottom of the tank, as shown in the right-hand figure, then the velocity will be approximately the same at all parts of the opening, and we can enlarge it so as to measure a much greater flow of water.

Measurement of a Flowing Stream.—In order to ascertain the available power from a stream or river, as well as to test the efficiency of a hydraulic installation, it is of the first importance to determine the rate of flow—*i.e.*, the number of cubic feet or gallons of water passing a given point per unit of time. At first sight, this would appear to be a very simple matter; but, as will be shown, it is not so easy to do so with accuracy, for several special precautions have to be observed and constants obtained.

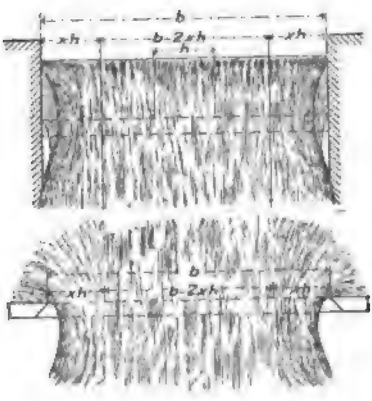
Rectangular Gauge Notch.—When we have to measure large quantities of moving water, then such orifices as we have previously dealt with are quite unsuitable. In such cases, it is usual to pass the whole of the water over a special form of weir or gauge notch. This consists of a board placed across the stream between stakes and carefully puddled, so that all the water must flow over it. The top bevelled edge of this board must be well above the surface of the water on the delivery side. The length of the notch is usually less than the width of the stream, and the board should be continued from each end in a straight line (as shown in the perspective view) so as to give definite conditions; or parallel guides may be placed in the stream before the gauge board. In the former case, the inflow of water at the ends of the notch board causes end contraction, while in the latter, this effect is avoided. In each case, contraction of the stream is caused by the water flowing upwards from E to F, as shown by the sectional figure a few pages further on. Further, the surface directly over the notch board is

lower than that of the still water in the pond above the board. If, however, we neglect these effects we see that with different lengths



PERSPECTIVE VIEW OF A RECTANGULAR GAUGE NOTCH.

of notch board the total flow will be proportional to its length. Now, consider any horizontal strip of the cross section of the stream over the notch (say, one-thousandth of its total depth); then, if we increase the depth of the stream over the notch, the vertical width of this strip and consequently its area will be proportionally increased, as well as its depth below the free surface of the water. But the velocity of the water passing through this strip varies as the square root of its depth, and the quantity as the area multiplied by the velocity. This result will hold good for each elementary strip, and will, therefore, apply to the whole stream.



FRONT VIEW AND PLAN OF A RECTANGULAR GAUGE NOTCH.

Hence, $Q \propto a\,v \propto h \times \sqrt{h} \propto h^{\frac{3}{2}}$ for different depths;

And, $Q \propto b$ for different breadths of stream at the notch;

$\therefore$ $Q = k\,b\,h^{\frac{3}{2}}$ for a rectangular notch. . . . . . (IX)

Here, k is a *coefficient of discharge* which must be found by experiment. This equation, however, would not give us accurate results if the proportions of the stream passing the notch were much different from that used to determine the constant k. With a very long shallow notch a considerable error will arise from the fact, that the water may adhere to the horizontal bevelled edge, and with a very deep narrow notch a similar effect would be produced by the bevelled sides. The above formula is often used and is quite correct for similar streams; but if the flow is variable, the depth will change with the flow of the water while the breadth remains constant, so that the proportions of the stream obtained with different flows in a gauge notch of this kind are not the same. (*See Note 1 at the end of this Lecture for Mr. Ritchie's Rule.*)

The late Professor James Thomson showed how we may obtain a formula which will apply to all ordinary proportions of rectangular notches. In the central part of the stream the lines of flow are practically parallel and unaffected by the sides of the notch. Consequently, the water passing through this part will be proportional to its breadth. Suppose the influence of each end to extend perceptibly to a distance $x\,h$ from it. Then the breadth of the central part will be $b - 2\,x\,h$. Consider a portion of this central part whose breadth is equal to h. This will be a square and therefore similar for different sizes of stream. Hence, since the area is proportional to h^2 and the velocity to $\sqrt{h}$, the flow through this square will be:—

$$k_1 \times h^2 \sqrt{h} \text{ or } k_1\,h^{\frac{5}{2}}$$

where k_1 is a constant.

Consequently, the flow through the whole of the central part $(b - 2\,x\,h)$ will be:—

$$\frac{b - 2\,x\,h}{h}\,k_1\,h^{\frac{5}{2}} = k_1\,(b - 2\,x\,h)\,h^{\frac{3}{2}}.$$

If we now imagine the two side portions to be placed together, we will get another stream which will be of similar form whatever its actual size may be; for, it will always be a rectangle of depth h and length $2\,x\,h$. Consequently the flow through this will be:—

$$k_2 \times 2\,x\,h \times h^{\frac{3}{2}} = 2\,k_2\,x\,h^{\frac{5}{2}}$$

where k_2 is another constant.

Hence, the flow of the whole stream will be :—

$$Q = k_1 (b - 2 x h) h^{\frac{3}{2}} + 2 k_2 x h^{\frac{5}{2}}$$

$$\therefore \qquad Q = k_1 \left\{ b - \frac{2 x (k_1 - k_2)}{k_1} h \right\} h^{\frac{3}{2}}.$$

If we write c for $\frac{x (k_1 - k_2)}{k_1}$ which is a constant, we get:—

$$Q = k_1 \{b - 2 c h\} h^{\frac{3}{2}}. \quad . \quad . \quad . \quad . \quad . \quad (X)$$

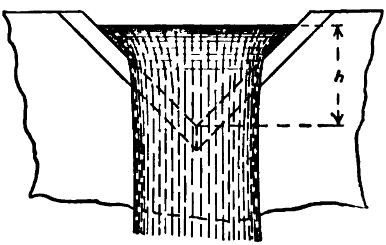
If one of the sides of the stream had a guide board we should have had x throughout instead of $2 x$, and therefore $c h$ in our final results instead of $2 c h$. If both sides of the stream be guided, this term would disappear and we would get the former result.

Mr. Francis, an American engineer, deduced the following empirical formula from a large number of experiments which he made :—

$$Q = 3{\cdot}33 \left(b - \frac{1}{10} n h\right) h^{\frac{3}{2}} \text{ cubic feet per second.} \qquad (X_a)$$

Here n is the number of end contractions (viz., 2, 1, or 0, as explained above), and the units employed are feet and seconds. It should be noted that this equation is of the same form as the previous one, and that neither is applicable to a notch whose length is less than $2 x h$.

Thomson's Triangular Notch.—Professor James Thomson proposed and used a gauge notch in the form of a right-angled isosceles triangle with its sides equally inclined to the vertical. It has the advantage of giving a similar form of stream whatever may be the size of the notch or the height of the water passing through it, and is, therefore, more accurate for measuring variable streams. As, however, less water is passed for a given height than with the rectangular notch, it is not so convenient for large flows; but by cutting a number of such notches, side by side like the teeth of a saw, considerable quantities of water may be dealt with.



THOMSON'S TRIANGULAR GAUGE NOTCH.

If we consider corresponding elements of two such notches, we see that their areas are proportional to the square of their depths,

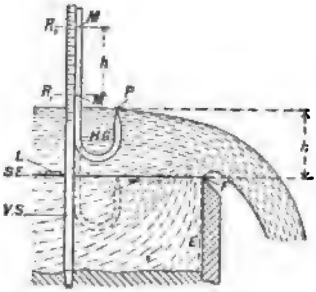
whilst, as before, the velocities of the water are as the square roots of the depths. Hence, the flow through a V notch will be :—

$$Q = k \times h^2 \times \sqrt{h} = k\,h^{\frac{5}{2}} \text{ cubic feet per second.} \qquad \text{(XI)}$$

In this case, the *coefficient of discharge k*, has been found by careful experiment to be 2·64.

Measurement of Head.—When using either of the previously mentioned notches for determining the flow of a stream or river we must ascertain the head h, with great accuracy. This may be done by aid of a level, straight edge, graduated staff, and a bent wire or hook-gauge in the following manner :—

Drive the vertical stake V S, into the bed of the stream at a position above the notch where the surface has no appreciable velocity. To obtain such a position the pond above the weir should not be too small. Level a straight edge S E, by the level L, with its lower edge resting on the inner edge F, of the bevelled board and on the point of the hook-gauge. Note the position R_1 on V S, opposite a mark M, on the longer limb H G. This gives us once for all the zero from which to reckon h.



APPARATUS FOR MEASURING HEAD OF WATER.

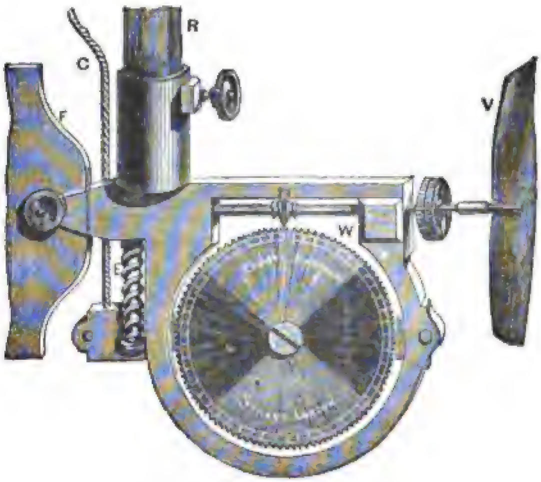
When the water is flowing in the normal condition for making the test, raise H G until the point P just touches the surface of the water and note the reading R_2 on V S opposite M, as shown by the full outlined hook in the figure. The difference between this reading and the former one gives the head h.

As may be seen from the previous formulæ any mistake made in determining h will produce a larger percentage error in the result with the V and rectangular notches than with an orifice in the bottom of a tank. The latter is, therefore, preferred where great accuracy is desired and the quantity of water is not too large, such as when measuring the circulating water used by a steam engine.

Measurement of Large Streams.—When it is inconvenient or impracticable to place a weir gauge in a river, then the flow may be estimated by measuring the cross section a, of the river and finding its mean velocity v, at that section :—

Then, as before, $\qquad Q = a\,v.$

To do this, a number of equidistant points are marked along a rope which is then stretched across the river at right angles to the direction of the current. By means of a graduated pole the depth at each of these points is ascertained from a boat. The results are then plotted to scale and give us a cross section of the river and an estimate of its sectional area. The surface velocity at midstream may be roughly found by noting the time taken for a float to move a given distance down stream, and the *mean* velocity may be taken as 0·65 of this. It is, however, much more accurate to ascertain the velocity at a number of points of the section by means of a current meter and then calculate the mean value. The following illustration shows a current meter



Elliott's Current Meter.

made for this purpose by Messrs. Elliott Brothers, London. It has a screw-shaped vane V, which is rotated by the water flowing past it. The revolutions of this vane are counted by the wheel W, which is driven by a worm on the same spindle as the vane. When the apparatus is immersed by means of the rod R to the required depth, with the vane pointing up stream, the cord C is pulled up and kept tight for a definite interval of time. This cord is attached to the end of a lever which carries the bearings of the counting wheel and is pushed down by a spring E. The wheel only gears with the worm when the cord is pulled, and the reading gives the number of revolutions of the vane from which the velo-

city of the water may be deduced. F is a rudder to keep the appaıatus pointing directly upstream.

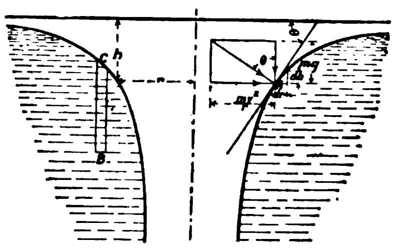
Horse-Power of a Stream.—After having obtained the quantity of water flowing in a stream, we have only to measure the available head in order to find its horse-power H.P. The head may be measured in feet by aid of a surveyor's level and staff. Then, if w be the weight of a cubic foot of water, and W the weight of the total flow of water per second, we get:—

$$Q = a\,v. \quad \text{And } W = Q\,w.$$

Hence, $$\text{H.P.} = \frac{\text{ft.-lbs. per second}}{550} = \frac{h \times Q\,w}{550}.$$

$$\therefore \quad \text{H.P.} = \frac{62 \cdot 5}{550} \times h\,Q = 0 \cdot 114\,h\,Q = 0 \cdot 114\,h\,a\,v. \qquad \text{(XII)}$$

Vortex Motion.—A whirling mass of fluid is termed a *vortex* and may be either *forced* or *free*. A *free vortex* is one which can be formed naturally, as when water flows through a hole at the bottom of a basin, and is such, that the energy of the fluid per unit mass is the same at all points in it. A *forced vortex* can only be produced artificially, and in it the energy of the fluid is different at different places. Such a vortex may be obtained by rotating a cup containing water.



FREE VORTEX.

Free Vortex.—From the above condition of constant energy, the velocity at a point A, on the surface of the vortex and at a depth h below the level of the still water, must be the same as that due to a body falling freely from a height h.

$$\therefore \qquad v = \sqrt{2\,g\,h}.$$

A particle on the surface of the vortex is acted upon vertically by its own weight $m\,g$, and horizontally by a centrifugal force $\frac{m\,v^2}{r}$. The resultant of these two forces must make an angle θ, with the vertical, so that:—

$$\tan \theta = \frac{\frac{m v^2}{r}}{m g} = \frac{v^2}{g r} = \frac{2 h}{r}.$$

Here m is the mass of the particle and r the radius of the circle in which it revolves.

This resultant must also be perpendicular to the surface of the fluid at A, since the fluid cannot resist any shearing component. Consequently, if the slope of the surface at A be $\tan \theta$,

$$-\frac{d h}{d r} = \tan \theta = \frac{2 h}{r}.$$

Or,
$$\frac{d h}{2 h} = -\frac{d r}{r},$$

i.e.,
$$\tfrac{1}{2} \log h = -\log r + \log c,$$

$\therefore$
$$\log \sqrt{h} = \log \frac{c}{r},$$

$\therefore$
$$h = \frac{c^2}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XIII)}$$

Here, c is a constant of integration which will depend on the size of the vortex. It is the radius of the vortex at unit depth.

Since the total energy is the same at all places in the fluid and the pressure at B due to the column B C, just makes up for the lower level of B, the velocity in any thin vertical tube of fluid such as B C must be constant and equal to that at the surface of the tube.

A vortex of this kind was applied to centrifugal pumps with radial blades by Professor James Thomson in order to convert, as far as possible, the kinetic energy of the water into potential energy. The vortex commences at the circumference of the wheel and the radial component of the water's motion is outward.

Forced Vortex.—In a vortex whose angular velocity ω is constant throughout, we have at any point A on the surface :—

$$\text{Centrifugal force} = m r \omega^2,$$

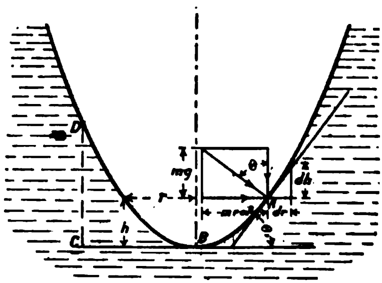
$\therefore$
$$\tan \theta = \frac{m r \omega^2}{m g} = \frac{r \omega^2}{g}.$$

$\therefore$
$$\frac{d h}{d r} = \frac{\omega^2}{g} r.$$

Or,
$$d h = \frac{\omega^2}{g} r \, d r.$$

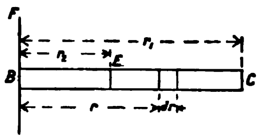
$\therefore$
$$h = \frac{\omega^2 r^2}{2 g} = \frac{v^2}{2 g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XIV)}$$

This is the equation to a parabola with its axis vertical, and, therefore, the surface of the vortex will be the paraboloid formed by rotating this parabola about its axis. The velocity of any particle D, on the surface of the vortex is that which would be attained by a body falling a distance equal to DC. The kinetic and potential energies are both least at the centre B, and become greater as we move upwards or outwards. We have a vortex of this kind in the wheel of a centrifugal pump with radial blades.



FORCED. VORTEX.

Pressure due to Centrifugal Force.—When a liquid is rotating, its pressure is not the same for all points on one level. Thus, in the previous figure the pressure at B is atmospheric, while at the point C on the same level, the pressure is, in addition, that due to the head C D.



WHIRLING COLUMN.

Consider a uniform column of liquid B C, rotating round the axis B F, with an angular velocity ω and cross section a. Then, if ρ be the density of the liquid, the centrifugal force due to a small element of length dr, at a distance r from the axis is :—

$$\rho\, a\, d r \times \omega^2 r.$$

$\therefore$ The total centrifugal force of the column B C $= \rho\, a\, \omega^2 \int r\, d r.$

This force is spread over an area a, at the end C, and we must divide it by this area to get p, the pressure per unit area. If the whole column from B to C is full of liquid :—

$$\text{Then,} \qquad p = \rho\, \omega^2 \int_0^{r_1} r\, d r = \tfrac{1}{2} \rho\, \omega^2 r_1^2 = \tfrac{1}{2} \rho\, v_1^2 . \quad . \quad . \quad \text{(XV)}$$

But, if the part of the column from B to E is empty :—

$$\text{Then,} \qquad p = \rho\, \omega^2 \int_{r_2}^{r_1} r\, d r = \tfrac{1}{2} \rho\, \omega^2 (r_1^2 - r_2^2). \quad . \quad . \quad (\text{XV}_a)$$

Students who are not acquainted with the integral calculus will understand the following proof of these equations.

The centre of gravity of B C is at a distance of $\frac{1}{2}$ B C from B.

$$\therefore \left.\begin{array}{l}\text{Centrifugal force due to}\\ \text{the whole column BC}\end{array}\right\} = \rho\, a\, r_1 \times \omega^2 \frac{r_1}{2} = \tfrac{1}{2} \rho\, a\, \omega^2 r_1^2.$$

The centre of gravity of the part E C is distant $\frac{r_1 + r_2}{2}$ from B.

$$\therefore \text{ Centrifugal force due to EC} = \rho\, a\, (r_1 - r_2)\, \omega^2 \frac{r_1 + r_2}{2}$$

$$\text{,, \quad ,,} \quad = \tfrac{1}{2} \rho\, a\, \omega^2 (r_1^2 - r_2^2).$$

This formula may be applied to finding the equation for the forced vortex. For, in the figure of the *forced vortex*, if B C = r, and C D = h, then, $p = h\,w = h\,\rho\,g$.

Therefore, from equation (XV), we get :—

$$h\,\rho\,g = \tfrac{1}{2} \rho\, v^2,$$

Or, $$h = \frac{v^2}{2\,g} \text{ (as before).}$$

Reaction of a Jet.—In general, when a fluid issues from an orifice it exerts a force on the vessel which contained it. This force is the reaction of the jet, and is due to the momentum given to the escaping fluid.

Let v = Velocity of efflux.
,, w = Weight of unit volume of the fluid.
,, W = Weight of fluid issuing per second.
,, m = Mass issuing per second.
,, Q = Quantity or volume issuing per second.
,, F = Force exerted on the vessel containing the fluid.

Then, the momentum given to the water per second = $m\,v$. And, since force is the rate of change of momentum :—

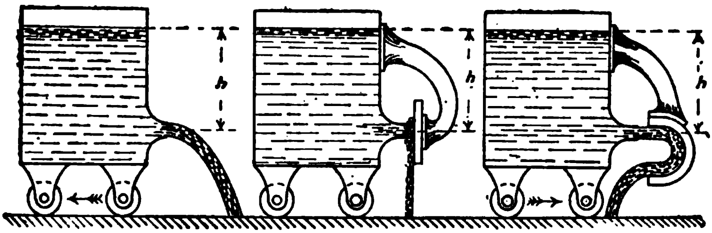
$$F = m\,v = \frac{W}{g} v.$$

But, $\quad W = w\,Q = w\,a\,v.$

$$\therefore \quad F = \frac{W}{g} v = \frac{w\,a\,v^2}{g} = \frac{w\,a\,.\,2\,g\,h}{g} = 2\,w\,a\,h. \quad \text{(XVI)}$$

This formula also gives the force acting on a vane of a turbine or waterwheel which alters the direction of flow of the water, and

in that case, v is the change of velocity produced by the vane. In applying the formula to any particular case, v is the total change of velocity produced until the water is quite clear of the vessel, and the direction of the force acting on the vessel is exactly opposite to that of v. The following figures will explain the results in three cases :—



BACKWARD. NO MOTION. FORWARD.

MOTION PRODUCED BY A JET.

In the left-hand figure the jet of liquid issues towards the right and urges the containing vessel to the left. The jet in the central figure strikes a plate fixed to the tank close to it, and then drops vertically downwards; consequently, the water receives no horizontal momentum and the tank no motion from it. The water will, however, exert a pressure on the plate, and this pressure balances the force produced by issuing from the orifice. In the remaining figure the jet is turned backwards by a curved guide attached to the tank and the whole momentum imparted to the water is backwards, consequently the tank is pressed forwards. If there be no loss from friction or eddies, the backward velocity of the stream, when the tank is at rest, must be the same as that with which it left the orifice. The blade has not only stopped the water, but has given it an equal backward momentum, and must, therefore, be pressed with a force twice as great as the flat one in the previous case. The force on it, is therefore, 2 F or 4 $w\,a\,h$, which is four times the pressure on a plug stopping the orifice. When the tank is in motion, the momentum given to the water is modified thereby, as will be explained in connection with the Pelton wheel in the next Lecture. Steam life-boats are now frequently propelled by jets of water instead of by screw propellers.

As this reaction astonishes everyone who hears of it for the first time, we will consider it from another point of view. Water under a pressure of 100 lbs. per square inch issues out of a nozzle of 1 square inch in area, and impinges on a fixed vane of

such a shape as to gradually deflect the water and turn it back in the opposite direction, without loss by friction, and consequently without loss of velocity. Thus, the vane may be a double U, as in the Pelton wheel; or a semicircle; or it may deflect the water in more than one plane. Then, the pressure on the vane is 400 lbs., although the statical pressure on a plug or valve stopping the jet is only 100 lbs. Although it requires very little elementary dynamics to prove this fact, the statement is generally received with incredulity.

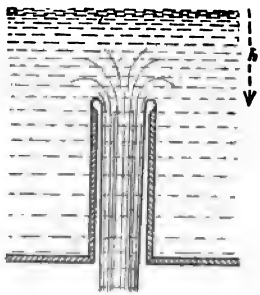
In the first place, it will be seen that the vane not only arrests the water—that is, imparts a negative acceleration to it, equal and opposite to what it received from the nozzle—but accelerates it equally in the opposite direction. Therefore, the total pressure on the vane is double the reaction on the nozzle.

The nozzle reaction at first sight appears to be 100 lbs.; that is, area of jet multiplied by pressure. But wait! Imagine a square vertical column in which water is maintained at a constant head. Near the bottom is a suitably formed horizontal nozzle closed by a plug. The system is then in equilibrium, since every square inch of wall has another square inch opposite to it which is acted on by an equal and opposite pressure. Opposite the nozzle, on the other side of the vessel, is a small circle equal to the area of the plug and equally pressed by the water. Now, remove the plug and let the water issue. It really does look as if the force pushing the vessel back was the pressure due to the head acting on this small circle on the back wall—that is, equal to the pressure multiplied by area of jet. But by Newton's second law, whenever we find a body moving at any velocity, we know the product of the force which has urged it and the time during which it has acted. Let us take a simple case and suppose the height of the water surface above the nozzle to be 16 feet. Then we know that the water issues as fast as if it had fallen 16 feet, and that its velocity is 32 feet per second. Consequently, every second a cylinder of water equal in section to the area of the jet and 32 feet long has a velocity of 32 feet per second impressed on it. Therefore, the urging force must be equal to its own weight; because when falling from rest by gravity—that is, when urged by its own weight—it acquires that velocity in one second. But the statical pressure is only half of this, being the weight of a cylinder of water equal in section to the jet and 16 feet long; and, since action and reaction are equal and opposite, the recoil of the vessel is equal to the force urging the jet, or twice the statical pressure.

Of course, if the jet strikes a flat vane at right angles the motion is stopped in its own direction, but not returned, and the pressure on the vane is twice the statical pressure. But, if the vane

returns the motion undiminished the pressure is four times the statical pressure.

Reduction of Pressure Round an Orifice.—Let h be the depth of a bell-shaped orifice and a its cross sectional area; then, if this orifice be completely closed by a flat plate, the force required to keep the plate in position will be $w\,a\,h$. It might therefore be supposed that, when the plate is removed and the water allowed to flow, this would be the force exerted on the vessel, but on looking at our former result (see equation XVI) we find that the actual force is twice as great, or $2\,w\,a\,h$. This difference is caused by a diminution of pressure on the surface round about the orifice. At these places the water has a certain velocity, and Bernouilli's theorem shows us that the pressure there must be less than when the liquid was at rest.



RE-ENTRANT ORIFICE.

There can be no reduction of pressure on the flat surface round the bottom of a re-entrant orifice, because there the water is practically at rest. The amount by which the pressure is reduced on opening the orifice must, therefore, be that on the area of the orifice itself, or $w\,a\,h$. Every second, this will set in motion a mass m as given by the equation :—

$$w\,a\,h = m\,v.$$

But, $$m = \frac{W}{g} = \frac{w\,Q}{g} = \frac{w\,a'\,v}{g}.$$

Where a' is the cross area of the jet, and $v = \sqrt{2\,g\,h}$.

$$\therefore \qquad w\,a\,h = \frac{w\,a'\,v^2}{g} = \frac{w\,a' \times 2\,g\,h}{g}.$$

Hence, $$a = 2\,a'. \qquad \text{(XVII)}$$

That is, the area of the jet will only be half that of the orifice. This result has been found experimentally to be correct. With a sharp-edged orifice, such as we considered in the early part of this lecture, there is a certain reduction of pressure round the orifice, and, therefore, the contraction must be less than in this case.

Impact—Loss of Energy.—When a stream of fluid meets an obstruction which causes a sudden change in its motion, its kinetic energy is partially, or perhaps wholly, spent in forming eddies or

little whirls of water, and is thus lost so far as useful effects are concerned. The eddies are, however, soon stilled by the viscosity of the liquid and their energy is converted into heat.

Let us suppose a body of mass m_1 moving with a velocity v_1, to overtake and strike another body of mass m_2, and velocity v_2. After collision let the bodies move on together with a common velocity v.

Then, from the laws of momentum,

$$(m_1 + m_2)v = m_1 v_1 + m_2 v_2.$$

Or,
$$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}.$$

But, before impact, the total energy was :—

$$E_1 = m_1 v_1^2 + m_2 v_2^2$$

And, after impact :—

$$E_2 = (m_1 + m_2) v^2.$$

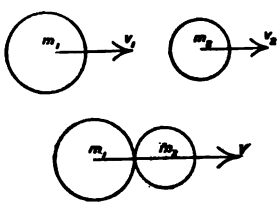
$$\therefore \quad E_2 = (m_1 + m_2)\left\{\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}\right\}^2 = \frac{(m_1 v_1 + m_2 v_2)^2}{m_1 + m_2}.$$

The energy lost is :—

$$E_1 - E_2 = \frac{(m_1 v_1^2 + m_2 v_2^2)(m_1 + m_2) - (m_1 v_1 + m_2 v_2)^2}{m_1 + m_2}$$

$$\therefore \; E_1 - E_2 = \frac{m_1 m_2 (v_1^2 + v_2^2 - 2 v_1 v_2)}{m_1 + m_2} = \frac{m_1 m_2}{m_1 + m_2}(v_1 - v_2)^2. \text{(XVIII)}$$

This lost energy is converted into heat. If one or both of the bodies be fluids the lost energy at first shows itself in eddies; but, as already explained, it is ultimately converted into heat. We thus see that when two streams moving at different velocities mix together, energy is lost and this loss is greater the greater the difference of the velocities. A similar effect takes place when water flows through a pipe with sudden changes of area, and even to a slight extent when the area is gradually varied, and also when water is flowing in a pipe or channel above a certain speed. In all these cases, different parts of the water move with



COLLISION.

different velocities, and these parts get mixed with one another. It also shows, why a turbine or water-wheel in which the water collides with the vanes, must have a low efficiency.*

Resistance of a Pipe.—When a fluid is passing through a pipe it rubs against the sides and experiences a certain resistance to its motion. This resistance limits the flow in a long pipe and causes a loss of head or pressure. Had the water no viscosity its flow would not be affected by the friction of the inner surface; because, this friction could only act on the thin layer of water actually in contact with the pipe.

Professor Reynolds found that the manner in which water flows depends upon its velocity. When this is below a certain critical point the flow depends chiefly on the viscosity and is along smooth stream lines. On passing the critical velocity, the water no longer moves steadily, but breaks up into numberless little whirls or eddies which move along with it, and absorb energy from the main stream. The former condition may be called *Steady Flow* and the latter *Eddy Flow*. This was beautifully demonstrated by Professor Reynolds by passing water from a large tank through a glass tube and then injecting a fine stream of coloured liquid with the same velocity into the centre of the stream. The water in the tank was quite steady and entered the tube through a bell mouth. At low velocities the coloured liquid formed a thin line along the centre of the tube, but at a certain velocity it was seen to suddenly spread out through a considerable part of the water, and on photographing it by means of an electric spark it was found to be all twisted into little whirls.

RESISTANCE OF SMOOTH LEAD PIPE AT DIFFERENT SPEEDS OF A FLUID.

He also found, that the law connecting the resistance R, with the velocity v, was different for these two conditions. For the lower speeds he ascertained that the resistance was proportional to the velocity, but at greater speeds it varied as some higher power, ranging from 1·7 to 2 depending on the roughness of the pipe. Near the critical velocity, which depends on the diameter of the pipe and on the temperature, the law is uncertain. The temperature affects the viscosity on which the critical velocity also depends.

On plotting the logarithms of his results as obtained with a smooth lead pipe $\frac{1}{2}$ inch in diameter, we get the lines shown in the accompanying figure. This consists of two straight lines joined by a

* See Note 2 at end of this Lecture.

curve of indefinite shape. The two straight parts have slopes of 1 and 1·72, showing that in one case the increase of log R is equal to the corresponding increment of log v, whereas beyond a certain point it is 1·72 times the corresponding increment of log v.

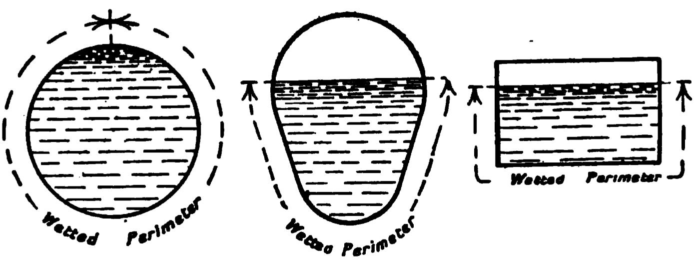
This shows us, that if c_1, c_2, and c_3 be constants, then :—

$$\text{For steady flow, } R = c_1 v.$$

$$\text{For eddy flow, } R = c_2 v^{1\cdot72} + c_3.$$

The resistance of the pipe for any velocity is most conveniently expressed in terms of the difference of pressure per unit length required to force the necessary quantity of liquid per second through it when level. This is called the *Slope of Pressure.* Whether the pipe is level or not, if it be of uniform bore, this slope is given by the difference of level between the free surfaces of the pressure columns at any two points, divided by the length of pipe between them. This has been termed the *Slope of Free Level,* and is shown graphically by a line passing through the surfaces of a series of little pressure columns.

Hydraulic Mean Depth.--The resistance of a pipe or channel is directly proportional to the extent of the wetted surface in a given



WETTED PERIMETER OF DIFFERENT SECTIONS.

length, and inversely to the cross area of the stream. The transverse length of the whole surface wetted by the stream is called the *Wetted Perimeter*, and the ratio of the cross area of the stream to its wetted perimeter is termed the *Hydraulic Mean Depth*, which we shall denote by D.

The friction of the pipe or channel will consequently vary inversely as the hydraulic mean depth.

Combining this with the former results we have for water pipes or channels :—

$$R = c\frac{v^n}{D}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (XIX)$$

Where c is a constant depending on the nature of the surface and the temperature of the water, and n is a number varying from 1·7 for smooth lead pipes to 2 for very rough pipes. The critical velocity for water in ordinary pipes is so low, that in all practical cases in which we wish to know the resistance, the actual velocity is always above it.

For a circular pipe, whose diameter is d, and which is full of running water, the wetted perimeter is the circumference of the circle:—

$$\therefore \qquad D = \frac{\frac{\pi}{4} d^2}{\pi d} = \frac{d}{4} \text{ for a circular pipe.} \quad . \quad . \quad \text{(XX)}$$

For a rectangular open stream of depth h and breadth b, the wetted perimeter $= 2h + b$ and the area is $b\,h$:—

$$\therefore \qquad D = \frac{b\,h}{2h + b} \text{ for an open rectangle.} \quad . \quad . \quad \text{(XXI)}$$

Loss of Head due to Friction in Water Pipes.*—Having dealt with the loss of "head" where water enters a pipe due to the shape of its mouth, and mentioned Prof. Reynold's experiments on steady and eddy flows of the liquid through a smooth pipe, as well as explained what is meant by the "wetted perimeter," we are in a position to state the generally accepted rules for the loss of head in ordinary clean cast-iron water pipes.

From many tests with piezometers or pressure columns, gauges or vertical tubes, inserted at known distances apart into the upper sides of straight pipes of different sizes and internal roughness, but of uniform bore, without bends, it has been found that *the loss of head due to friction* for water flowing through these pipes at different velocities, is approximately proportional to—

1. The length of the pipe, l.
2. Inversely to the diameter of the pipe, d.
3. The square of the velocity, v.
4. The roughness.
5. But it is independent of the water pressure.

Let h_T = Total head in feet or pressure per sq. in.
h_L = Lost head due to friction " " "
h_E = Effective head at point of delivery " "

Then, $$h_L = h_T - h_E$$

* This "loss of head due to friction" is termed the "friction head."

Now, if f = coefficient of friction found by experiment, which will vary between 0·012 for a $\frac{1}{2}$-inch pipe when v = 1 foot per second, and 0·003 for a 3-foot pipe when v = 15 feet per second.

Then, loss of head,

$$h_L = f\left(\frac{\text{wetted surface}}{\text{cross area}} \times \text{loss of head due to velocity}\right).$$

$$\text{Or,}\quad h_L = f\left(\frac{\pi\, d\, l}{\frac{\pi}{4} d^2} \times \frac{v^2}{2\, g}\right).$$

$$\therefore\quad h_L = f\left(\frac{4\, l\, v^2}{d\; 2\, g}\right). \qquad \text{(XXII)}$$

Example II.—What will be the loss in head for every 100 feet of a 3-inch pipe when water flows through it with a velocity of 3 feet per second, if the coefficient of friction be ·0065?

Here, l = 100 feet; d = 3 inches = ·25 feet; v = 3 feet per second; and f = ·0065.

By formula XXII., Answer—

We see that, $$h_L = f\left(\frac{4\, l\, v^2}{d\; 2\, g}\right).$$

$$\therefore\quad h_L = \cdot 0065\left(\frac{4 \times 100 \times 3 \times 3}{\cdot 25 \times 2 \times 32\cdot 2}\right).$$

Or, $$h_L = 1\cdot 46 \text{ feet.}$$

Now, looking at the following table by Prof. Merriman, we see that opposite to a pipe of 0·25 foot in diameter, and directly under the velocity 3 feet per second, the loss of head is printed as 1·46 feet for every 100 feet of length. In that table, the "friction factor," f_F, is reckoned as four times the previously mentioned value for the coefficient of friction, f.

Or, $$f_F = 4\, f;$$

because the constant 4 appears in the numerator of the natural equation XXII.

Hence, when using the table, the "friction factors," f_F, will vary from 0·05 for a $\frac{1}{2}$-inch pipe when v = 1 foot per second, to 0·012 for a 3-foot pipe when v = 15 feet per second.

The formula for the "friction head" therefore becomes, using the following table:—

$$h_L = 4\, f\left(\frac{l\, v^2}{2\, d\, g}\right) = f_F\left(\frac{l\, v^2}{2\, d\, g}\right). \qquad \text{(XXIII)}$$

TABLE OF FRICTION HEAD FOR 100 FEET OF CLEAN IRON PIPE.*

Diameter of Pipe.	Velocity in Feet per Second.						
	1.	2.	3.	4.	6.	10.	15.
Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
0·05	1·46	5·10	10·30	16·90	34·70	...	...
0·10	0·59	1·99	4·20	6·97	14·50	37·30	...
0·25	0·20	0·70	1·46	2·40	5·37	13·70	29·40
0·50	0·09	0·32	0·70	1·14	2·46	6·22	13·30
0·75	0·05	0·21	0·45	0·73	1·57	3·94	8·40
1·00	0·04	0·15	0·32	0·55	1·12	2·80	5·95
1·25	0·03	0·11	0·25	0·42	0·85	2·11	4·48
1·50	0·02	0·09	0·20	0·33	0·67	1·66	3·50
1·75	0·02	0·07	0·16	0·26	0·54	1·33	2·80
2·00	0·02	0·06	0·13	0·21	0·45	1·09	2·27
2·50	0·01	0·05	0·10	0·16	0·34	0·81	1·68
3·00	0·01	0·04	0·07	0·12	0·26	0·67	1·40
3·50	0·01	0·03	0·06	0·10	0·21	0·53	...
4·00	...	0·02	0·05	0·08	0·17	0·42	...
5·00	...	0·02	0·04	0·06	0·13	...	...
6·00	...	0·01	0·03	0·05	0·10	...	...

EXAMPLE III.—Find the friction in a 4-inch wrought-iron pipe delivering 200 gallons per minute. What is the loss in head and H.P. per 100 feet? (See *The Practical Engineer*, Sept. 11, 1903, p. 264.)

ANSWER.—Gallons per second $= \dfrac{200}{60}$.

Cubic feet per second $= \dfrac{200 \times 10}{60 \times 62 \cdot 3} = \cdot 535.$

Area in sq. ft. of a 4″ pipe $= \cdot 087.$

Velocity (v) of water $= \dfrac{\cdot 535}{\cdot 087} = 6 \cdot 1$ ft. per sec.

Let the friction factor $= \cdot 023.$

Then, by formula XXIII. and the above table—

$$\text{We get,} \quad h_L = f_F \left(\frac{l\,v^2}{2\,u\,g} \right) = \cdot 023 \left(\frac{100 \times 6 \cdot 1^2}{2 \times \cdot 3 \times 32 \cdot 2} \right) = 4 \text{ ft.}$$

$$\text{H.P. absorbed} = \frac{4 \times 200 \times 10}{33{,}000} = \cdot 24 \text{ (approximately).}$$

* From *Treatise on Hydraulics*, 8th Edition, by Prof. Merriman, Lehigh University.

NOTES ON MEASUREMENT OF A FLOWING STREAM (see p. 469).*

Note 1. **Ritchie's Rule for the Flow of a Stream with a Rectangular Notch.**—"Cube the depth of the water in inches, flowing through the notch, extract square root, multiply by 5 (or more exactly by 4·83)—this gives the number of cubic feet per minute flowing over each foot in width."

Or, $$Q = k\,b\,h^{\frac{3}{2}} = \left(5 \times 1 \times h^{\frac{3}{2}}\right).$$

"I have found the best proportion of width of notch to depth flowing over the notch, to be about 10 to 1."

Note 2, *re* p. 481, at top.—Mr. J. Ritchie says: "I am not quite clear about this. As a matter of fact, the turbine buckets at the point of entry are nearly always at right angles, or square on to the jet; especially, in all the mixed flow turbines, which are rapidly superseding the older forms of Jonval and other Continental makes. Most of the Continental turbine makers are now taking up the manufacture of mixed flow turbines. In Girard turbines, it is usual to have the outer edge of the bucket slightly curved in the direction of the flow, but the circumferential speed of a Girard turbine is generally lower in proportion to the velocity of the water, than a pressure turbine of the Jonval, inward, or mixed flow type. Some of these Girard turbines give the best results at ·45 of the theoretical velocity due to the head, whereas the usual proportion of a pressure turbine is ·65 of the theoretical velocity due to the head. Some of the buckets of American wheels are actually rounded on the face, instead of hollow, such as the Risdon turbine, and these give very high efficiencies. In some of the very latest American turbines the vanes are perfectly radial in plan, the turning action on the water being entirely done on the lower part of the bucket. In these cases, the water issues from the port almost at right angles to the bucket, so that even allowing for the forward motion of the wheel there must be collision to some extent."

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* I am indebted to Mr. J. Ritchie, of Carrick & Ritchie, Engineers, Waverley Engineering Works, Edinburgh, for these *Notes*.

Lecture XXXV.—Questions.

1. Prove the law for changing p, v, and h along a stream line in a frictionless fluid. Apply the law to find the funnel shape of the surface of water in a basin from which the water is flowing by a central hole. (S. & A. Hons. Exam., Part I., 1898.)

2. Prove the law for changing p, v, and h along the stream lines in a frictionless fluid. Apply, neglecting change of level, to the case of adiabatic flow of air from one vessel to another through a small orifice, and deduce the rule for maximum quantity flowing. (S. & A. Hons. Exam., Part. II., 1898)

3. Describe the action of a jet pump, or of a good form of injector.

4. Water flows through a round sharp edged orifice 3 inches in diameter in a flat plate, at about 12 inches below still-water level. Show by a sketch your notion of the shapes of the stream lines. If we wish to know the pressure at any point, why is it not sufficient to know only the depth? (S. & A. Adv. Exam., 1898.)

5. Deduce a formula giving the velocity with which water issues from an orifice, and show how to apply it to water under pressure.

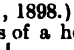
6. Discuss briefly the relative advantages under various circumstances of the different methods of measuring a stream of water.

7. Find the horse-power of a waterfall 70 feet high, when the stream is such that it passes over a gauge notch 6 feet long with a head of 15 inches. If it is employed to drive a turbine of 80 per cent. efficiency, what B.H.P. would you expect to obtain?

8. Investigate the form of the surface of water which flows out of a hole in the bottom of a basin with a vortex motion.

9. What is meant by a constrained vortex, and why will such a vortex rapidly disappear when left to itself? Find the form taken by the surface and show how to apply this to finding the pressure in a centrifugal pump.

10. The wheel of a centrifugal pump 2 feet outside diameter has a very large case and rotates at 100 revolutions per minute about a vertical axis, and almost no water is being delivered. Calculate and show in a curve the pressure at points in a horizontal plane, at various distances from the axis. The vanes are bent backwards at an angle of 30° with the radius at the outer part; if the radial flow becomes 2 feet per second and the circumferential openings are 200 sq. inches in area, what is the kinetic energy of the water leaving the wheel? What is the pressure in excess of that at the inner rim of the wheel where the water enters without shock? If there is no frictional loss to what height will the water be lifted above the well? In an actual pump with small wheel case what is the probable lift? (S. & A. Hons. Exam., Part II., 1898.)

11. "Barker's mill" consists of a horizontal pipe with a nozzle at right angles to it at each end thus  Water enters it by a vertical pipe at the centre, and the whole is so mounted that it can rotate about the axis of this pipe. Find the torque when the water is issuing under a head h, and the mill is revolving n times per minute. Show that the power is greatest when the velocity of the nozzles is half that due to the head, and that the efficiency then cannot be over 50 per cent.

12. What are the chief conclusions to be drawn from Reynold's experiments on the flow of water through pipes?

13. What is meant by the *Hydraulic Mean Depth* of a pipe or channel, and show how we use it in calculating the resistance to the flow of water?

14. If a hydraulic company supplies a 1,000 gallons of water at 700 lbs. pressure per square inch for 17 pence, how much is this per horse-power per hour? Another company supplies water at a "head" of 260 feet: what price ought it to charge per 1,000 gallons if the consumer is to have his energy at the same price as in the other case? *Ans.* 2s. 8d.; 2¾d.

(B. of E. Adv., 1900.)

15. What is the usually accepted rule for loss of head or energy per pound of water passing along a straight pipe? What is the law when the water flows very slowly? And why is there a difference between the two laws? (B. of E. Adv., 1900.)

16. 60 horse-power is being delivered hydraulically into a mile of straight horizontal pipe at a pressure of 700 lbs. per square inch. It is found that 52 horse-power is available at the far end. What power will be found available at the end of 2 miles of a pipe three-fourths of the diameter of the first-named, if the entering pressure is 800 lbs. per square inch, and the entering horse-power is 40? *Ans* 26·61 H.P.

(B. of E. H., Part I., 1900.)

17. A horizontal pipe of 12 inches diameter gradually becomes 3 inches diameter, and then becomes of 12 inches diameter again. There is a flow of 5 cubic feet per second. Neglecting friction, state how the pressure alters along the axis of the pipe. (B. of E. Adv., 1901.)

18. Take the loss of energy through friction of every pound of water flowing along a straight circular pipe of length l feet and diameter d feet, the velocity being v feet per second, to be $0{\cdot}0007\ l v^2/d$ foot-pounds. If water at 700 lbs. per square inch conveying 300 horse-power enters a pipe of 6 inches diameter, find the number of cubic feet entering per second and the velocity in the pipe. What is its loss of energy per pound in a length of pipe of 3,000 feet? What is the total loss of power? Obtain an expression for the horse-power lost in transmission in terms of the total horse-power entering the pipe, the entering pressure, p, and l and d.

(B. of E. Adv. & H., Part I., 1902.)

19. Eight gallons of water per second flow through a 6-inch pipe in which there is a right-angled bend. State the speed. What is the change in the velocity of the water (that is, the *vector* change, as there is no change in mere speed)? What is the change in the momentum of the water per second? What is the resultant force exerted by the water on the pipe at the bend, neglecting friction? (B. of E. Adv. & H., Part I., 1902.)

20. A stream is gauged over a rectangular weir or notch, the width of the notch is 3 feet 6 inches, and the height of the still water over the edge of the notch is found to be 14½ inches. Find how many gallons pass over the weir per 24 hours (the velocity of approach may be neglected), and what H.P. could be obtained from turbines supplied by this stream if the available head is 35 feet and the mechanical efficiency of the turbines is 81 per cent. (C. & G., 1900, H., Sec. C.)

21. Compare the loss of head by skin friction in a 3-inch pipe with that in a 6-inch pipe, when the velocity of flow is the same in both, and also when the velocity in the small pipe is so increased that it discharges as much water as the big pipe. (C. & G., 1900, H., Sec. C.)

22. Water is flowing steadily at a velocity of 3 feet a second through a perfectly horizontal pipe 6 inches in internal diameter; if on a length of 1½ miles there is a drop of pressure owing to friction of 25 lbs. per square inch, what H.P. is being wasted in friction? (C. & G., 1900, O., Sec. A.)

23. A large tank, in which a constant head of water is maintained, has a cylindrical hole in the bottom 3 inches in diameter, with sharp edges. How many gallons of water will escape from the tank per hour if the constant head is 9 feet? (C. & G., 1901, H., Sec. C.)

24. Obtain an expression for the loss of head, owing to surface friction, in a long straight pipe through which water is flowing. Hence obtain an expression for the probable discharge in gallons per hour from a main of given diameter and length, when the total head of the water in the reservoir above the point of discharge is known.

(C. & G., 1901, H., Sec. C.)

25. The quantity of condensing water used by a stationary engine is gauged by first allowing the water to flow into a tank supplied with baffle plates, in order to get rid of any agitation in the water, and then allowing it to flow out of the tank over a rectangular notch 6 inches wide. Estimate the discharge in pounds per minute when the head over the sill is 4 inches. Explain why the flow may be more accurately determined by a V-notch than by a rectangular notch, the head over the notch being supposed subjected to variation. (C. & G., 1902, H., Sec. C.)

26. A horizontal water pipe, 3 feet in diameter, gradually contracts to a diameter of 18 inches. The difference of head, as shown by water gauges, between two sections of these diameters is 28 inches. Neglecting all losses, estimate the flow along the pipe in gallons per hour.

(C. & G., 1902, H., Sec. C.)

Lecture XXXV.—A.M.Inst.C.E. Exam. Questions.

1. Explain what is meant by coefficients of contraction, velocity, and discharge for orifices. What are the ordinary values of these coefficients for a sharp-edged orifice in a plane surface? (I.C.E., *Oct.*, 1897.)

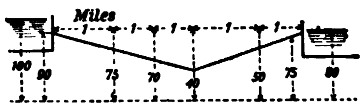
2. Deduce an expression for the discharge from a rectangular notch, and explain why the theoretical expression requires to be modified by a coefficient varying with the proportions of the notch. (I.C.E., *Oct.*, 1897.)

3. A weir is 30 feet long, and has 18 inches of head above the crest. Taking the coefficient at 0·6, find the discharge in cubic feet per second. State the relative advantages of notches with and without end contraction in the practical gauging of water. (I.C.E., *Oct.*, 1897.)

4. A pipe of 12 inches diameter connects two reservoirs 5 miles apart, and having a difference of level of 20 feet. Find the velocity of flow and discharge of the pipe, taking the coefficient of friction at any value known to you, or at 0·01. Explain how the coefficient of friction for a pipe varies with roughness and size. (I.C.E., *Oct.*, 1897.)

5. Describe carefully any one method of determining the discharge of a river by float or current-meter observations. Show roughly by a sketch how the velocities are distributed in the cross-section of a rectangular channel Describe any instruments used in determining the velocity, in the method adopted in your description. (I.C.E., *Oct.*, 1897.)

6. Explain the meaning of the term hydraulic gradient of a pipe. Suppose the levels of the reservoirs and pipe at mile distances are 100, 90, 75,



70, 40, 50, 75 and 80 feet above datum, write down the corresponding pressures in the pipe in feet of head. (I.C.E., *Oct.*, 1897.)

7. On what principle is the presence of a jet striking a plane determined? Find the pressure of a 3-inch jet having a velocity of 80 feet per second against a wall struck normally. (I.C.E., *Oct.*, 1897.)

8. A jet of water 1 inch in diameter coming from a reservoir at a height of 200 feet strikes a fixed hemispherical cup so that the direction of its motion is reversed. Find the force it exerts upon the cup, assuming that the jet has 90 per cent. of the full velocity due to the head.
(I.C.E., *Feb.*, 1898.)

9. The miner's inch is defined as the flow through an orifice in a vertical plane, of 1 square inch in area, under an average head of 6½ inches. Find the water-supply per hour which this represents, and give any reasons you can for the choice of coefficients which you assume. (I.C.E., *Feb.*, 1898.)

10. A canal lock with vertical sides is emptied through a sluice in the tail-gates. Putting Ω for area of lock-basin, ω for area of sluice, and H for the lift, find an expression for the time of emptying the lock.
(I.C.E., *Feb.*, 1898.)

11. A re-entrant cylindrical mouthpiece is fixed to the vertical side of a vessel. Show that for such a mouthpiece the limiting value of the coefficient of contraction is 0·5. (I.C.E., *Feb.*, 1898.)

12. A rectangular weir, for discharging daily 10 million gallons of compensation water, is arranged for a normal head over the crest of 15 inches. Find the length of the weir. Assume your own coefficient, stating the kind of weir for which it is applicable, or take a coefficient of 0·7.

(I.C.E., *Feb.*, 1898.)

13. A water-main is 42 inches in diameter, and the velocity through it is to be 3 feet per second. Find the head lost in friction in feet per mile. Coefficient of friction 0·01. (I.C.E., *Feb.*, 1898.)

14. A horizontal main consists of 5,000 feet of 6-inch, 5,000 feet of 9-inch, and 5,000 feet of 12-inch diameter. The velocity in the 6-inch is 12 feet per second, and the pressure head 50 feet at the inlet. Sketch (not to scale) the hydraulic gradient (*a*) supposing that only the skin friction is considered; (*b*) supposing that both skin friction and the losses of head due to abrupt changes of section are considered. Write down the expressions for the several losses of head, but without calculating them numerically.

(I.C.E., *Feb.*, 1898.)

15. Deduce a general expression for the velocity of flow in an open channel. An irrigation channel, with side slopes at 1 to 1, has a bottom width of 100 feet and a depth of 10 feet. It discharges 3,000 cubic feet per second. Find the slope in feet per mile. (I.C.E., *Feb.*, 1898.)

16. Write down a formula for the discharge of pipes, and explain its derivation. What coefficients would you use to calculate the discharge for (1) pipes coated with Dr. A. Smith's composition, (2) riveted steel pipes, (3) wooden-stave pipes. (I.C.E., *Oct.*, 1898.)

17. A pipe 2 feet diameter draws water from a reservoir at a level of 550 feet above the datum; it falls for a certain distance and again rises to a level of 500 feet, 5 miles from its starting point; it then falls to a reservoir at a level of 400 feet 1 mile away. Calculate the rate of delivery into the lower reservoir. (I.C.E., *Oct.*, 1898.)

18. Describe one form of current meter, and state how you would proceed to gauge a river 100 feet wide and 20 feet deep with it. (I.C.E., *Oct.*, 1898.)

19. Give an expression for the loss of head in a pipe owing to a sudden enlargement of section. (I.C.E., *Oct.*, 1898.)

20. Describe the hydraulic ram, and give a formula for its delivery at a height h feet above it, the supply being Q cubic feet per second coming from a height of H feet. (I.C.E., *Oct.*, 1898.)

21. State Bernoulli's theorem, and explain what form of motors could be used to utilise each of the forms in which energy exists in flowing water.

(I.C.E., *Oct.*, 1898.)

22. Give a formula for the discharge of drowned orifices. What advantages do they offer for gauging purposes? (I.C.E., *Oct.*, 1898.)

23. A pipe tapers to one-tenth its original area, and then widens out again to its former size. Calculate the reduction of pressure, at the neck, of the water flowing through it, in terms of the area of the pipe and the velocity of the water. Why is this reduction of pressure a gauge of the discharge? (I.C.E., *Oct.*, 1898.)

24. Sketch a hook gauge for measuring a varying water-level, and state how you would fix it for use. (I.C.E., *Feb.*, 1899.)

25. Water flows from a pond over a weir 10 feet long, to a depth of 10 inches; it then flows along a level rectangular channel 8 feet broad, and over a second weir the width of the channel, its crest being 1 foot above the bottom. Find the depth of the water over the 8-foot weir.

(I.C.E., *Feb.*, 1899.)

26. A rectangular chamber 120 feet square contains 15 feet depth of water, which is allowed to flow out through a vertical rectangular orifice

2 feet by 1 foot, the top of which is level with the floor of the reservoir and the tail-water. Calculate the time it will take to empty.

(I.C.E., *Feb.*, 1899.)

27. How would you proceed to gauge accurately, by means of floats, the flow of a stream of uniform section 20 feet wide and 5 feet deep? Describe the floats you would use, and state how you would deduce the result from the experimental data. (I.C.E., *Feb.*, 1899.)

28. Calculate what depth of water should flow in a pipe of circular bore to give a maximum discharge, the surface slope being constant.

(I.C.E., *Feb.*, 1899.)

29. (*a*) Explain by sketches the phenomenon of contraction in the case of a weir with vertical face and ends and with a sharp edge such as is commonly employed for measuring the discharge of water, and illustrate in reference to the same sketches the meaning of the terms "coefficient of contraction" and "coefficient of discharge." (*b*) What is approximately the coefficient of discharge where the weir is 10 feet long and the water 1 foot deep? Would this coefficient be affected by reducing the length of the weir to 1 foot while maintaining the same depth of water, and if so, in what manner? (I.C.E.. *Oct.*, 1899.)

30. Explain and illustrate by reference to a stream of rectangular section, 12 feet wide and 3 feet deep, the term "hydraulic mean depth," and state in what manner and approximately to what extent the mean velocity of the stream would be affected by widening it to 24 feet, assuming the depth and inclination to remain the same. (I.C.E., *Oct.*, 1899.)

31. In the same stream as above (12 feet wide and 3 feet deep), in what manner and to what extent would the mean velocity be affected by changing the inclination from 1 per 1,000 to 4 per 1,000, assuming the depth to remain the same? (I.C.E., *Oct.*, 1899.)

32. Sketch a practical form of hydraulic ram for raising automatically to a considerable height a portion of any volume of water derived from a lesser height, and explain the points upon which satisfactory working depends. (I.C.E., *Oct.*, 1899.)

33. A certain tank A overflows with the discharge from a 12-inch pipe fed by a tank B 10 miles distant and 90 feet above A. The discharge is 900,000 gallons a day. Another 12-inch pipe, 5 miles long, is subsequently joined to the middle distance of the 10-mile length and discharges into a third tank C having its overflow at the same level as A. State approximately the discharge per day into A and C under the new conditions.

(I.C.E., *Oct.*, 1899.)

34. A stream of water of uniform rectangular section, 12 feet wide and 3 feet deep, has a surface inclination of 1 foot per 5,280 feet. Assuming the channel to be of rough masonry making the coefficient of friction 0·007, what would be the mean velocity in feet per second and the discharge in cubic feet per second? (I.C.E., *Oct.*, 1899.)

35. A large cistern is divided vertically by a partition into two compartments A and B. In the partition is a square orifice. Water is maintained at a constant level, in compartment A, of $1\frac{1}{2}$ feet, and in compartment B of 1 foot, above the centre of the orifice, and the discharge from A to B through the orifice proves to be at the rate of 0·2 cubic foot per second. (*a*) Assuming the same orifice to be 6 inches lower and the water-levels and all other things to be unchanged, what now will be the discharge through the orifice from A to B? (*b*) State approximately in decimals of a foot the depth and width of the orifice, assuming it to be cut square in a thin plate. (*c*) Would the discharge of the orifice be affected by making its width three times as great and its depth one-third as great? If so, how and why? (I.C.E., *Feb.*, 1900.)

36. Assuming that in the following pairs, A and B, of straight streams the roughness of the sides is moderate and uniform, state in the simplest way possible whether No. 1 or No. 2 would discharge more water, and by how much per cent.

A.

	No. 1.	No. 2.
Length,	1 mile.	1 mile.
Fall,	4 feet.	6 feet.
Top width,	70 ,,	50 ,,
Bottom width,	70 ,,	50 ,,
Depth,	10 ,,	10 ,,

B.

Length,	1 mile.	1 mile.
Fall,	4 feet.	4 feet.
Top width,	70 ,,	60 ,,
Bottom width,	50 ,,	60 ,,
Depth,	11 ,,	10 ,,

If in B the coefficient of friction were 0·007, what would be the mean velocity? (I.C.E., *Feb.*, 1900.)

37. Draw approximately to a scale of 1 inch to the foot and foot-second a diagram of depths and corresponding velocities parallel to the axis, and extending from the bottom to the free surface, in the centre line of a river 10 feet deep, when the mean of such velocities is 5 feet per second; and explain the causes of variation of velocity from the bottom upwards, with special reference to the velocity at the free surface. (I.C.E., *Feb.*, 1900.)

38. A straight thin cylindrical tube, 0·25 foot internal diameter, is fixed horizontally through the vertical side of a cistern, its outer end being flush with the outside thereof. The inner end is cut square and projects into the cistern as far as is consistent with allowing a jet of water issuing through it from the cistern to contract freely in air without striking the interior of the tube. (*a*) Assuming the level of the water in the cistern to be 2 feet above the centre of the tube, sketch such an arrangement in outline approximately to a scale of 3 inches to 1 foot, showing the figure of the jet, and figuring the principal dimensions. (*b*) What is the distance from the cistern to an ideal vertical plane which the centre line of the jet would cross at a level of 1 foot below the level of water in the cistern?

(I.C.E., *Feb.*, 1900.)

39. Describe shortly, with sketches approximately to scale, the different classes of instruments used for measuring the velocity of open streams of water, explaining the circumstances under which each is to be preferred.

(I.C.E., *Feb.*, 1900.)

40. A jet of water having a sectional area of 12 square inches and a velocity of 16 feet per second impinges normally on a plane surface. Find the amount of work per second necessary to maintain the jet and determine the pressure on the plane, (*a*) when it is fixed, and (*b*) when it is moving in the direction of the jet at 6 feet per second. (I.C.E., *Oct.*, 1900.)

41. What is the connection between the velocity and pressure at different parts of a smooth horizontal pipe of varying diameter and which is running full of water, all resistance being neglected? If the pressure where the diameter is 2 inches is 500 lbs. per square inch and velocity 4 feet per second, what will be the pressure where the diameter is 1 inch?

(I.C.E., *Oct.*, 1900.)

42. In a pipe 2 inches inside diameter and tarred inside, water, starting at 700 lbs. per square inch, flows at the rate of $2\frac{1}{2}$ feet per second. What is the horse-power transmitted at the beginning of the pipe, and what percentage of this is lost on friction and viscosity per 100 feet length of pipe? (I.C.E., *Oct.*, 1900.)

43. If the same horse-power with the same initial pressure as in (5) were transmitted through a $1\frac{1}{2}$-inch pipe, how much would this percentage loss on friction be increased, and what would be the saving in weight of metal pipe per 100 feet length? Make a rough allowance for weight of joint flanges in this comparison. (I.C.E., *Oct.*, 1900.)

44. Explain, without mathematical symbolism, the difference between the water-resistances to the progress of a vessel through open deep water and through a canal. (I.C.E., *Oct.*, 1900.)

45. Calculate the percentage loss of H.P. per 1,000 yards in hydraulic transmission through clean cast-iron pipes $4\frac{1}{2}$ inches in diameter, with water velocities 2 feet, 3 feet, and 4 feet per second, and with pressures 200 lbs., 700 lbs., and 1,200 lbs. per square inch, that is, in all 9 percentage losses, and represent your calculated results by curves drawn on squared paper. (I.C.E., *Feb.*, 1901.)

46 A measuring weir is constructed with a 90° angular notch, the edges being bevelled to 45° on the outside to a nearly sharp edge. Give the formula you think best for the discharge over such a weir, and apply it to calculate the discharge in gallons per minute when the water depth above the apex of the angular notch is 9·36 inches and the water level 5 feet back from the weir is found to be 0·93 inch above that of the weir. (I.C.E., *Feb.*, 1901.)

47. Define "hydraulic mean depth" and calculate its amount for a pipe 4 feet in diameter flowing full, the same pipe half-full, and for a channel 4 feet bottom width, side slopes 2 to 1, and a depth of water 4 feet. Also, find the rate of flow in the pipe when full, and in the channel. The hydraulic gradient in the former case, and the slope in the latter case being 1 in 3,000, assuming an approximate value for the coefficient. (I.C.E., *Oct.*, 1901.)

48. Develop a formula for the flow over a rectangular notch; if the velocity of approach is appreciable, how may it be allowed for in the calculation? (I.C.E., *Oct.*, 1901.)

49. Explain how the velocity at different points in the cross section of a river varies, and show how this variation is inconsistent with stream-line motion. (I.C.E., *Oct*, 1901.)

50. A circular orifice in the bottom of a tank is 0·385 inch diameter, and the mean head of water over the orifice is 22 inches. Calculate the theoretical flow of the water through the orifice, assuming the velocity to be due to the "head" only. What would be the energy of one pound of this water? (I.C.E., *Feb.*, 1902.)

51. Water flows through a straight pipe 4 feet in diameter and 100 feet long. The upper end is 10 feet higher than the lower end. At the middle of its length there is a conical contraction in the pipe, after which it is again enlarged to its original diameter. The velocity of the water through the full-sized section is 5 feet per second. Find what the diameter of the contracted section must be for the pressure there to be 10 lbs. per square inch less than at the upper end, neglecting friction. (I.C.E, *Feb.*, 1902.)

52. Give a formula for the resistance to the flow of water (1) in a smooth tube where the water moves with low velocities; (2) in a pipe where the water flows at high velocities. Find the loss of head due to friction in water flowing through a pipe 2 feet in diameter, 1 mile long, at 5 feet per second. (I.C.E., *Feb.*, 1902.)

53. Find the quantity of water which will flow through a notch 9 feet long, the head of water over the sill being 10 inches, and the area of the approach channel being 30 square feet. (I.C.E., *Feb*, 1902.)

54. Describe one form of current meter, and explain how it could be used to ascertain the discharge of a river 50 feet wide. (I.C.E., *Feb.*, 1902.)

55. Find an expression for the pressure on a plate due to the impact of a jet of water. What difference would result if the plate were replaced by a hemispherical bowl? Water is flowing through a pipe 80 feet long at the rate of 50 feet per second, and is stopped by a valve being closed in $\frac{1}{10}$th of a second. What is the increase in pressure near the valve, the pipe being supposed rigid and with no elasticity? (I.C.E., *Oct.*, 1902.)

56. State and prove "Bernoulli's Theorem" relating to the steady motion of incompressible fluids, and apply it to find the discharge through a Venturi water-meter. (I.C.E., *Oct.*, 1902.)

57. Estimate the discharge of a clean cast-iron pipe 15 inches in internal diameter, 2 miles long, under a total head of 100 feet. Give reasons for the selection of the formula you use, and state what you neglect in your calculations. (I.C.E., *Oct.*, 1902.)

58. Obtain an expression for the loss of head due to a sudden enlargement in a pipe, and calculate the loss in the case of a pipe 2 inches in internal diameter suddenly enlarged to 4 inches, the discharge being 5000 gallons per hour. (I.C.E., *Oct.*, 1902.)

59. Show that, for a rectangular notch, a formula of the form $Q = a(L - bh)h^{\frac{3}{2}}$ can be made to represent the discharge at varying heights more correctly than a formula of the form $Q = cLh^{\frac{3}{2}}$.

(I.C.E., *Oct.*, 1902.)

60. A jet of water, velocity 24 feet per second, diameter 3 inches, strikes a plane normally. Find, approximately, the total pressure on the plane and the maximum pressure, and show by a curve the approximate distribution of pressure. (I.C.E., *Oct.*, 1902.)

61. Show, by sketches relating to any case you may select, how the velocity varies at different points in the cross-section of a stream.

(I.C.E., *Oct.*, 1902.)

62. Describe and discuss (*a*) a rough method, (*b*) a more accurate method, of estimating the discharge of a stream. (I.C.E., *Oct.*, 1902.)

63. Water issues from the nozzle of a fire-engine, 1½ inch diameter, in a jet which rises to a height of 100 feet. Neglecting the contraction of the jet, find the reaction on the machine due to the jet. (I.C.E., *Feb.*, 1903.)

64. A horizontal tube is tapered slowly from a diameter of 15 inches to a diameter of 6 inches. Neglecting friction, calculate the difference in the pressures in pounds per square inch at the two sections when the discharge is 60,000 gallons per hour. (I.C.E., *Feb.*, 1903.)

65. In the formula $v = c\sqrt{mi}$ for the mean velocity in streams and channels, explain how the value of c is affected by the roughness of the sides and bottom, the slope, and the hydraulic mean depth. Selecting a suitable value of c, find the discharge in cubic feet per second of a rectangular rough masonry channel 5 feet wide, when the depth of the water is 2 feet and the slope is 1 in 250. (I.C.E., *Feb.*, 1903.)

66. What are the advantages and disadvantages attending the use of the V-gauge notch, and for what purpose is it specially suitable? The still-water surface-level is at a height of 15·5 inches above the bottom of a right-angled V-gauge notch. Calculate the discharge in cubic feet per second, taking 0·60 as the coefficient of discharge.

(I.C.E., *Feb.*, 1903.)

67. Sketch carefully a form of current meter. Explain exactly how you would employ it, and from its indications obtain the mean velocity of a river. (I.C.E., *Feb.*, 1903.)

68. At a point in a clean hydraulic supply main, 3 inches in diameter, the velocity is 10 feet per second, and the pressure is 750 lbs. per square inch; find the percentage loss of pressure per 1000 yards, and calculate the power conveyed by the main at the point (I.C.E., *Feb.*, 1903.)

69. Give a brief account of Reynolds' experiments on the flow of water through pipes and the results obtained from them. (I.C.E., *Feb.*, 1903.)

70. Describe and explain the action of a hydraulic ram, noting the chief points which must be attended to in its design and erection.

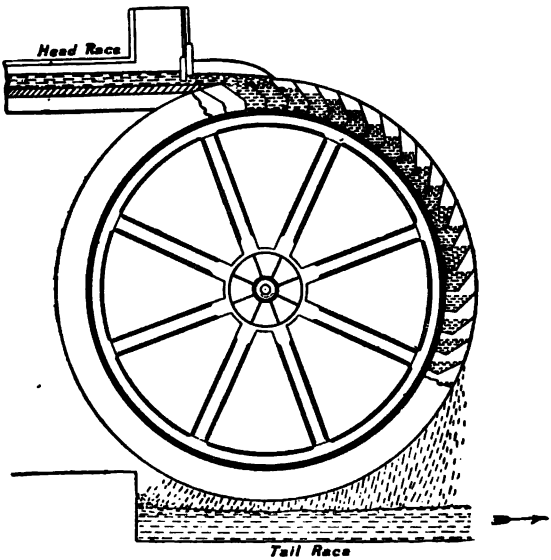
(I.C.E., *Feb.*, 1903.)

LECTURE XXXVI.

WATER-WHEELS AND TURBINES.

CONTENTS.—Hydraulic Motors—Overshot Water-wheel—Breast-wheels—Undershot Water-wheel—Fairbairn's Improvements—Clack Mill—Pelton Wheel—Turbines—Girard Turbine—Jonval Turbine—Günther's Governor—Thomson's Vortex Turbine—Little Giant Turbine—Hercules Mixed-Flow Turbine—Centrifugal Pumps and Fans—Notes on this Lecture—Questions.

Hydraulic Motors.*—In connection with hydrostatics we have already described some machines for obtaining motion by hydraulic



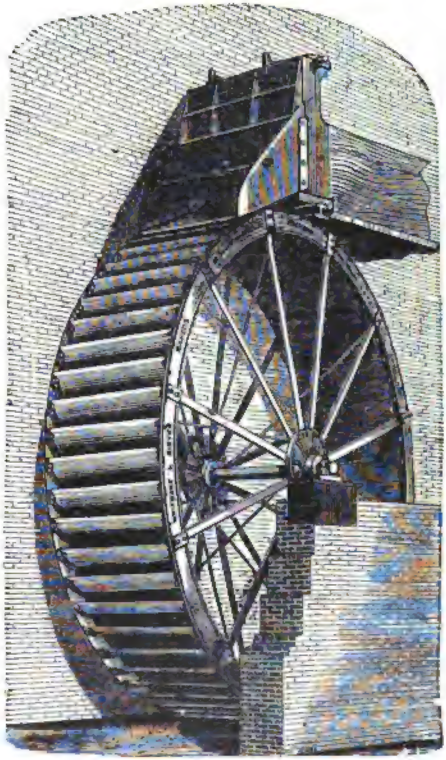
OVERSHOT WATER-WHEEL.

means, but in all these cases the water acts solely by its pressure,

* Students should refer to *Hydraulic Motors* by G. R. Bodmer (London, Whittaker & Co.), and to *Hydraulic Machinery* by R. Gordon Blaine (London, E & F. N Spon, Limited) for further information on the design of water-wheels and turbines.

We now come to the consideration of other water motors in which the weight and momentum of the water are also employed. These may be divided roughly into two classes—Water-wheels, in which the water acts for the most part directly by its weight; and turbines, in which it acts by its momentum. We cannot, however, draw a very sharp line between them, as they gradually merge into one another, and the power of both ultimately depends on gravity.

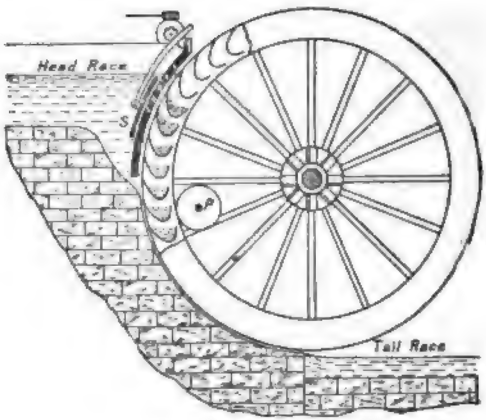
Overshot Water-wheel.—This consists of a wooden or iron frame to which is fixed a number of blades, so as to form with the inner circumference a series of buckets for holding water. The water is led along an aqueduct termed the *head race* to the top of the wheel, and there enters the buckets. Its weight forces them downwards and thus makes the wheel revolve. As each bucket in turn approaches its lowest position the water gradually drops out of it into the tail race. The motion of the water as it enters the wheel also assists to some extent in producing rotation. The buckets are so shaped and fixed to the wheel, that as little as possible of the available power shall be lost by the water spilling from them before they reach the tail race. The sectional view shows one form of bucket for this purpose whilst the outside view shows another with curved blades. One disadvantage of this wheel is, that the water leaves



OVERSHOT WATER-WHEEL BY MESSRS. WHITMORE & BINYON.

it with a velocity opposite in direction to that of the tail race, if this flows the same way as the head race. The water therefore does not get away so freely from the tail race, and more clearance is necessary at the bottom of the wheel which thereby involves a loss of head.

Breast-wheels.—This last consideration, coupled with the difficulty of supporting the head race for large overshot wheels, has brought about the introduction of breast-wheels, in which the water is introduced between the top and middle of the wheel as shown by the next illustration. The breast-wheel is also frequently made with curved blades into which the water drops almost vertically, and then acts chiefly by its weight. The motion of the

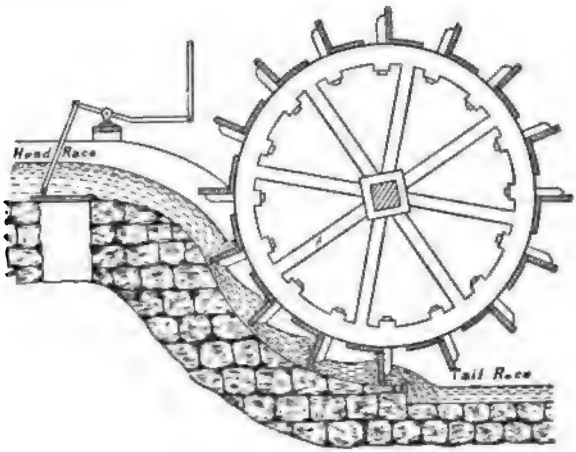


FAIRBAIRN'S BREAST-WHEEL.

wheel in this case assists the escape of the tail water instead of hindering it as in the previous one. The curved ventilated form of bucket with closed breast, as shown in the above figure, was first introduced by Sir William Fairbairn and greatly increased the efficiency of the motor. But, for small wheels, such as are used for farms, where first cost is more important than efficiency, hey are usually made radial as shown by the following figure.

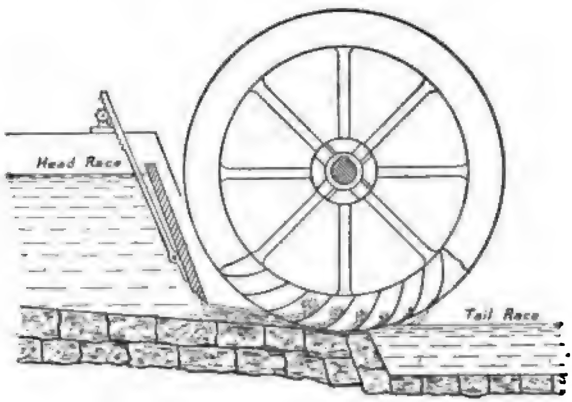
Here, the water is allowed to attain a certain amount of motion before reaching the wheel, and therefore acts partly by momentum and partly by its weight. The buckets have no ends, but the wooden breast serves to keep the water from escaping by the sides and circumference of the wheel before it reaches the bottom.

Breast-wheels into which the water enters near the top are called high breast-wheels.



BREAST-WHEEL WITH RADIAL BLADES AS USED IN COUNTRY FARMS.

Undershot Water-wheel.—The breast-wheel just described forms a connecting link between water-wheels proper and turbines, to

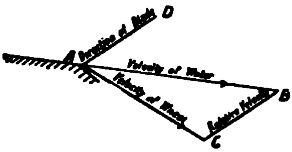


UNDERSHOT WATER-WHEEL.

which latter class undershot wheels really belong. With an undershot wheel the whole energy of the water is allowed to become kinetic and then it acts on the blades solely by its momentum. Here,

again, radial blades are common, but they are very inefficient on account of the large loss due to the eddies which are caused by the impact of the water upon them, and from the kinetic energy which the water still possesses on leaving the wheel.

In order that the water may be applied without shock and to avoid the formation of eddies, the tips of the blades should be so inclined as to be parallel to the motion of the water relatively to the wheel at its point of entering. To find this direction, draw A B to represent the velocity of the water as it leaves the head race, and A C the circumferential velocity of the wheel at the point where the water enters it. Then, by completing the triangle A B C we find C B the direction of the tip of the blade at A. The blade is curved upwards, and the position where the water leaves it is at the same level as where it enters, in order that the water may drop out with as little kinetic energy as possible.



ANGLE OF BLADE.

Fairbairn's Improvements.—In the illustration of Fairbairn's breast-wheel, we have shown the regulating sluice S connected by a curved rack and pinion to the worm gear which is turned by a hand-wheel, until the sluice admits the desired quantity of water to develop the necessary power. In the case of large wheels which have to drive textile or other machinery requiring great uniformity of speed, this worm gear is connected to a ball governor which automatically adjusts the position of the regulating sluice, to suit the different demands of the works.

In addition to other improvements effected by Fairbairn, we may mention, that instead of driving from a spur-wheel keyed to the water-wheel shaft, he bolted a segmental annular toothed wheel directly to one of its outer sides or flanges and geared it with a pinion as shown at P. He thus diminished the distance between the plummer block bearings of the water-wheel shaft, relieved its radial arms from conveying the driving stresses, and at once obtained the necessary speed without intermediate gearing. The importance of this improvement was so self-evident to mill-owners, that many large wheels which had previously given considerable trouble and shown signs of distress, were fitted with Fairbairn's drive, and are working to the present day at full power with perfect regularity and freedom from breakdowns.

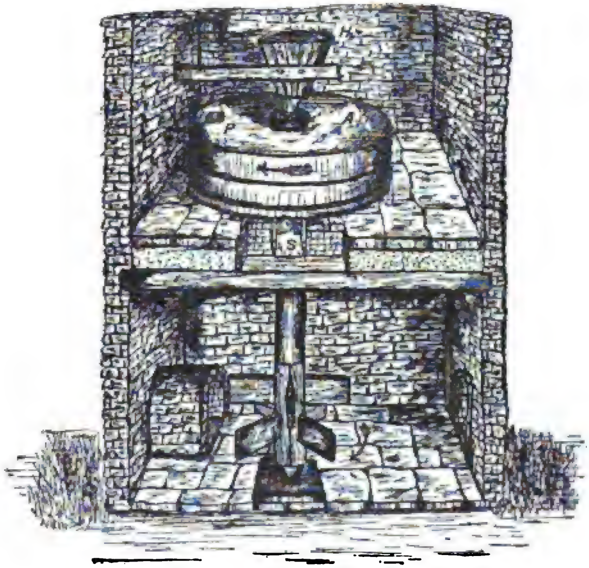
We may here summarise Fairbairn's improvements in water-wheels, viz. :—(1) The use of iron instead of wood in their construction, thus lightening and at the same time strengthening the various parts.

(2) The adoption of curved ventilated iron buckets instead of straight wooden ones, to prevent eddies and thus obtain a greater efficiency from the head and body of water.

(3) The introduction of a closed breast to prevent the escape of the water during its turning of the wheel.

(4) Driving directly from the sides and periphery of the water-wheel in order to minimise the stresses in the arms, reduce the distance between the bearings, and at once obtain the desired speed.

The Clack Mill.—One of the oldest forms of water-wheels, and one which belongs to the same class as the undershot wheel, is

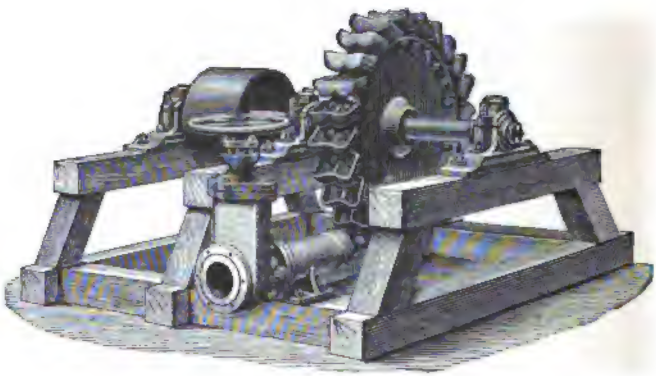


THE OLD CLACK MILL.

that known as the "clack mill." It is supposed to be of Norwegian origin, and was certainly introduced therefrom into the Orkney and Shetland Islands several hundred years ago. Our illustration is reduced from a sketch made by Mr. Sellar in 1898 of the only remaining working clack mill in Orkney, at the farm of Millbrig, in the parish of Birsay. The owner, Mr. Folster, says that it has been handed down to him from generation to generation for more than two hundred years.

The water under a head of 10 feet enters by the left-hand aperture and leaves by the right-hand one. During its passage the water impinges upon the further side of the wooden radial arms which are fixed to the vertical iron shaft. The lower end of this shaft rests in a conical footstep, whilst the upper end carries the revolving millstone, and is steadied by a bearing immediately beneath the lower or fixed stone. The corn to be ground is fed into a "head" or "harp" H, and as the upper stone revolves the projecting wooden pin P, strikes the radial arm A connected to the lower end of H. This shoggles a portion of the grain into the central opening at each revolution. In doing so, a clacking noise is made by this pin striking the outstanding arm, which has given rise to the local term of "clack mill." The grain is carried down by gravity and finds its way into groves between the two stones where it is ground into rough meal. This compound of meal and husk dribbles from the shoot S into a wooden bin, from which it is removed, and separated by shaking and blowing, or by another machine, for future use in the shape of porridge or oat-cake.

Pelton Wheel.*—The previous examples naturally lead us to the consideration of the Pelton wheel, which is very often used for



PELTON WATER-WHEEL BY MESSRS. W. GÜNTHER & SONS, OLDHAM.

great pressures and high falls. As will readily be understood from a consideration of the accompanying figure, this form of water-wheel consists of a plain disc mounted upon a central shaft, and carrying a number of curved buckets fixed at equal distances around its periphery. A conical nozzle attached to the supply pipe is so fixed as to direct a jet of water upon each of these buckets in turn and thus drive the wheel at a high speed. The

* See Notes on this water motor at end of this Lecture.

buckets have a central division, and as they curve outward towards each side, the jet is thereby deflected in two portions and then backwards. With properly designed buckets, and when the circumferential velocity of the wheel is half that of the jet, the water will simply leave the buckets with little or no remaining kinetic energy, and hence the efficiency of such a wheel may be very great. Pelton wheels are used for falls having a head of from 30 to 2,000 feet, or for corresponding pressures derived from water-pumps and city hydraulic power mains. They are sometimes made with several nozzles, each being fitted with a stop-valve, so that the power can be varied by shutting off the water from one or more of them. When there is only one jet, the power can be varied without a change of efficiency by simply unscrewing the nozzle and putting on another of a different size. If the power is varied by partially closing the stop valve, we lose a large amount of energy in friction at the valve, and the efficiency is thereby considerably reduced.

We may find the efficiency of a Pelton wheel, on the assumption that there are no eddies or friction, as follows :—

Let V = The velocity of the water as it issues from the nozzle.
„ v = „ „ vane.

Then, $V - v$ is the relative velocity of water on vane, which is not changed as it moves around the vane. Therefore, the final velocity of the water is $v - (V - v)$ or $2v - V$, which may be either positive or negative. Now, by the principle of conservation of energy, the initial energy of the water is equal to that spent on the vane + the final energy. Therefore, the efficiency of the wheel is :—

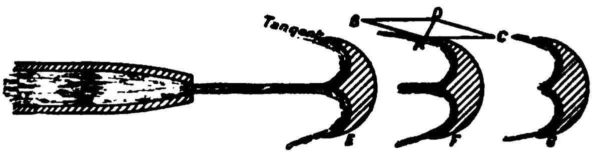
$$\eta = \frac{\textit{Power got out}}{\textit{Power put in}} = \frac{\textit{Initial kinetic energy} - \textit{final kinetic energy}}{\textit{Initial kinetic energy}}$$

$$= \frac{V^2 - (2v - V)^2}{V^2} = \frac{4v(V - v)}{V^2}.$$

This is evidently greatest, and equal to unity, when $2v - V$ is zero, or $V = 2v$, since $(2v - V)^2$ can never be negative. That is, for maximum efficiency the speed of the wheel should be half that of the water, and when it runs either quicker or slower than this the efficiency will be less. For example, if the speed of the jet is 100 feet per second and the vanes travel 20 feet per second, the theoretical maximum efficiency is :—

$$\eta = \frac{4 \times 20(100 - 20)}{100 \times 100} = 0{\cdot}64 \text{ or } \mathbf{64 \text{ per cent.}}$$

The above are ideal cases in which the water is directed backward by the vane parallel to its original direction. In practice, the Pelton vanes do not quite do this, but are slightly divergent, as shown, in order that the water may clear the following vane.



VANES FOR PELTON WHEEL.*

The final velocity of the water is easily found by the parallelogram of velocities, thus—Draw A B the velocity of the water and A C that of the vane. Then, A D is the final velocity of the water, and the kinetic energy carried away is proportional to A D^2.

We have seen, in the preceding Lecture, that when the ideal vane is stationary, the pressure on it is four times the statical pressure on the area of the jet. At maximum efficiency, when the relative velocity of the water and vane is half what it is when the vane is stationary, the force on the vane is equal to this statical pressure, because the impact varies as the square of the relative velocity. Or,

Water striking vane per second $= \dfrac{w}{g} a\,(V - v)$ *lbs.*

Change of velocity produced by vane $= 2\,(V - v)$.

$\therefore$ Momentum imparted per second $= \dfrac{w}{g} a\,(V - v) \times 2\,(V - v)$.

Or, Force of jet on vane . . . . $= \dfrac{2\,w\,a}{g}(V - v)^2$

And, when, $v = \frac{1}{2} V = \frac{1}{2}\sqrt{2\,g\,h}$,

Then, $F = \dfrac{2\,w\,a}{g}\left(\frac{1}{2}\sqrt{2\,g\,h}\right)^2 = w\,a\,h.$

Hence, in this case, the power expended on the vane is $F \times \frac{1}{2} v$. But, the total power of the jet is $F \times v$, or twice as much. This would make the efficiency to be one half, but we have proved it to be unity. Where is the discrepancy?

Suppose a single vane to move away indefinitely in a straight line in the direction of the jet, it will be seen that there is an ever-increasing quantity of water flying through the air after the

* See Notes at end of this Lecture.

vane. This represents an ever-increasing debt of uncollected kinetic energy, so that only half the water issuing per second strikes the vane in the same time.

But now, let E be the initial position of the vane (see previous figure), and E F the distance it travels in one second. When it has arrived at F, let another vane be suddenly interposed at E. There is now a rod of water between E and F running twice as fast as the vanes, and the last particle of this water will only overtake F when it arrives at G and the second vane at F. A third vane is now interposed at E and the process repeated.

This is what happens in the Pelton and other impact wheels, such as the Laval steam turbine. The fluid strikes two vanes at once, one behind the other; a statement hard to believe unless approached by the above argument.

Turbines.—These form a type of water motor which occupy much less space, are more efficient, more easily governed, suit a greater range of fall, and generally run at a greater speed than ordinary water-wheels. They are classified in several different ways according to the manner in which their special properties are considered. We shall first of all divide them into four classes, viz.—(1) inward flow; (2) outward flow; (3) parallel (or axial) flow; and (4) mixed flow turbines. In the first two kinds, the water either flows inwards from the outer circumference as in the Thomson turbine, or outwards from the inner circumference as in one type of the Girard. In the third class, it flows parallel to the axis, entering at one side of the wheel and leaving at the other as in the Jonval. The fourth type has both radial and axial flow, and the water usually enters at the circumference and leaves at one or both sides parallel to the axis.

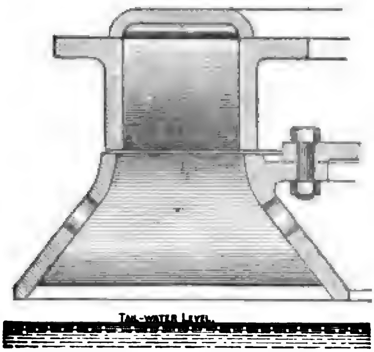
In the second place, we find all of the previous types divided into what are termed *drowned* and *ventilated* turbines. The former are designed to run quite full of water and may be submerged in the tail race, or used with a suction pipe; whilst the latter have ducts to admit air at atmospheric pressure into the wheel, and, consequently, cannot be submerged or used with such a pipe. The energy of the water entering the drowned type is partially potential and partially kinetic, whilst it is wholly kinetic before it enters a ventilated turbine. Hence, these two kinds of motors are sometimes respectively called *reaction* and *impulse* turbines.

The purpose of a suction pipe is to allow the turbine to be placed some distance above the tail race without losing the corresponding head. This suction pipe is merely an air-tight conduit to carry away the used water and must have its lower end below the surface of the tail race; moreover, it must not exceed 20 feet or thereby in vertical height from the turbine to the tail race; otherwise

the water column supported by the pressure of the atmosphere will be broken by the introduction of air into the exhaust conduit.

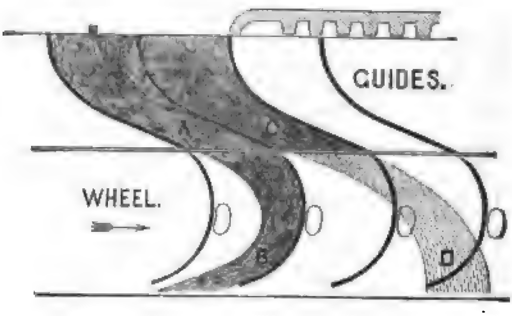
A disadvantage of most turbines of the "drowned" type is, that in regulating its speed and power we cannot gradually cut off the water without a considerable drop in efficiency. We can, however, do so in several steps when divisions are placed in the wheel for this purpose. This is because the guide passages to the wheel must always be quite full of water, which would not be the case if the opening to any particular guide port was only partially closed.

Girard Turbine.—This wheel is named after the French engineer M. Girard, who in 1856 first designed the ventilated and impulse turbine. It can be made with either a vertical or horizontal axis and for both axial and outward flow. The former is used for low falls of from 6 feet and upwards, and the latter for high falls up to 1,000 feet.

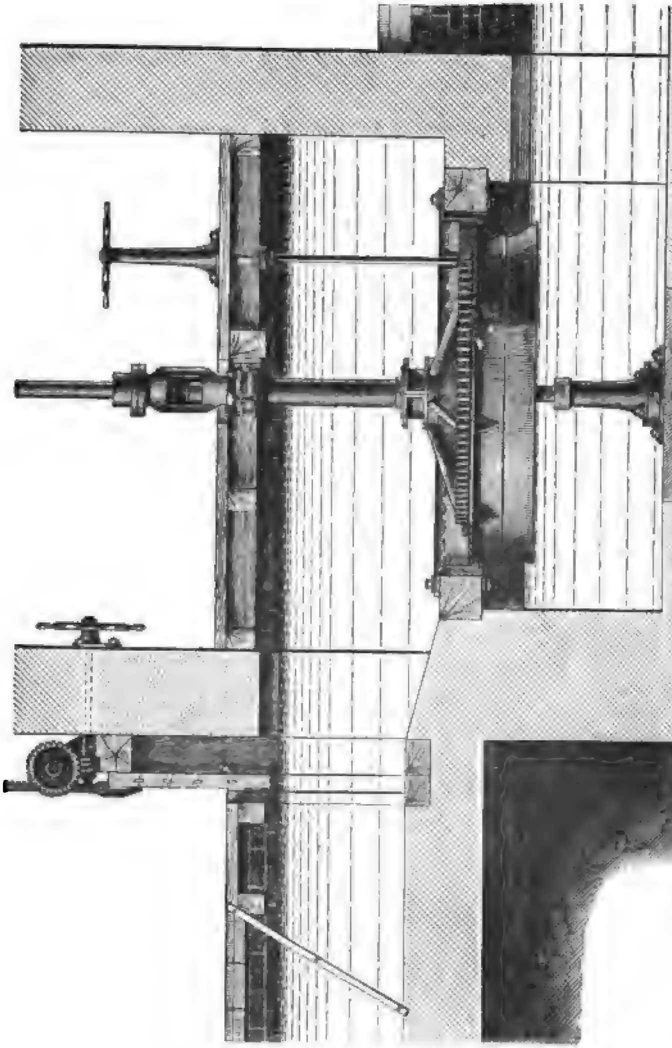


Radial Section. Günther's Axial Flow Girard Turbine.

By referring to the two accompanying figures, which represent radial and circumferential sections of this turbine, it will be seen that the gate for controlling the water supply is placed above the guide ports A, C, through which the water passes and issues with full velocity due to its

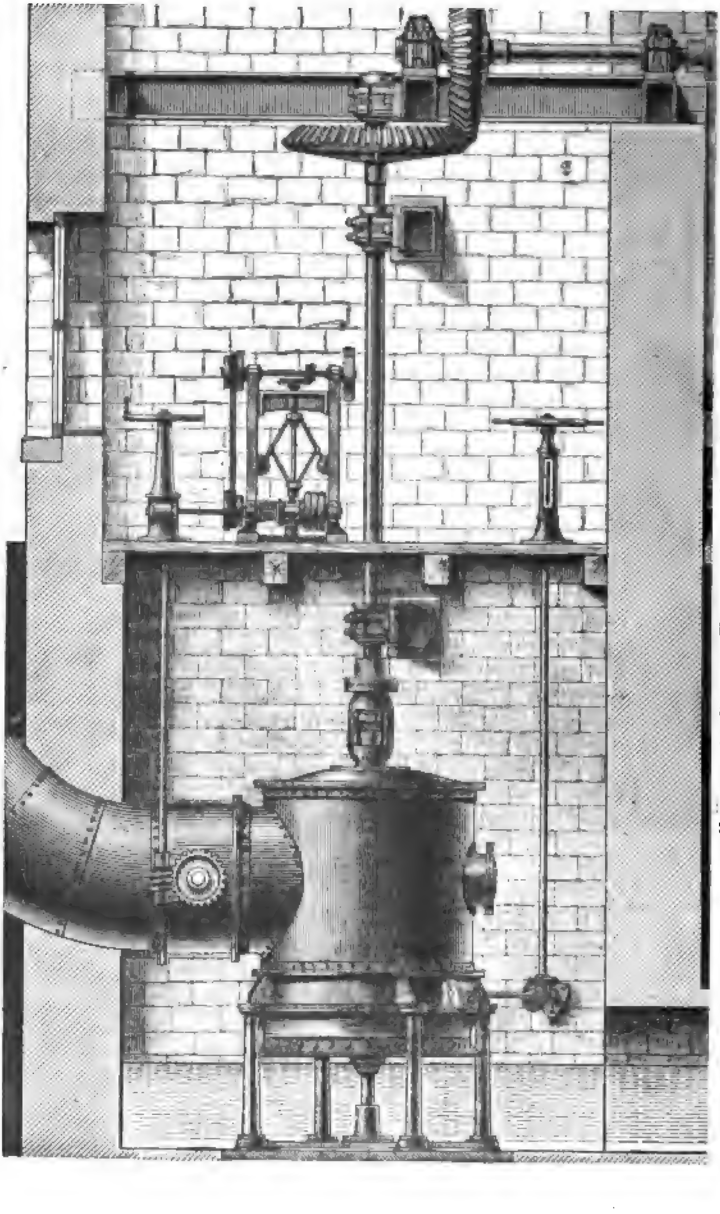


Circumferential Section. Günther's Axial Flow Girard Turbine.



Girard Turbine for Low Falls by Messrs. W. Gunther & Sons, Oldham.

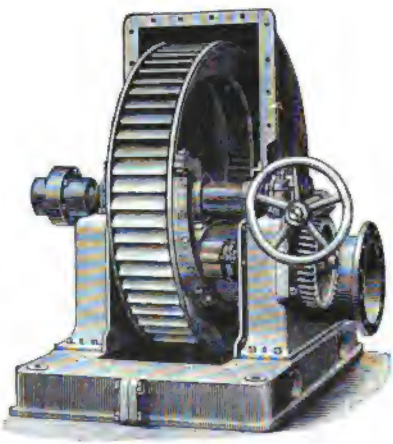
head. It then glides along the concave surfaces of the wheel buckets B, but does not quite fill them. The inclination of the upper edges of the buckets is obtained in the manner explained



GUNTHER'S GIRARD TURBINE FOR LOW FALLS.

for undershot wheels, while that of the lower edges is made as small as possible in order that the water may not leave the vanes with much kinetic energy. To allow of this inclination being smaller than would otherwise be the case, the sides of the wheel are splayed outwards as shown by the radial section in order that the water may spread and not foul the convex surface of the next blade. The path of the water relatively to the moving wheel is shown at B, whereas, the dotted lines through D show the actual motion of the stream as seen from a fixed point. The ventilating holes are clearly shown in both views. These

GÜNTHER'S GIRARD TURBINE FOR LOW FALLS.

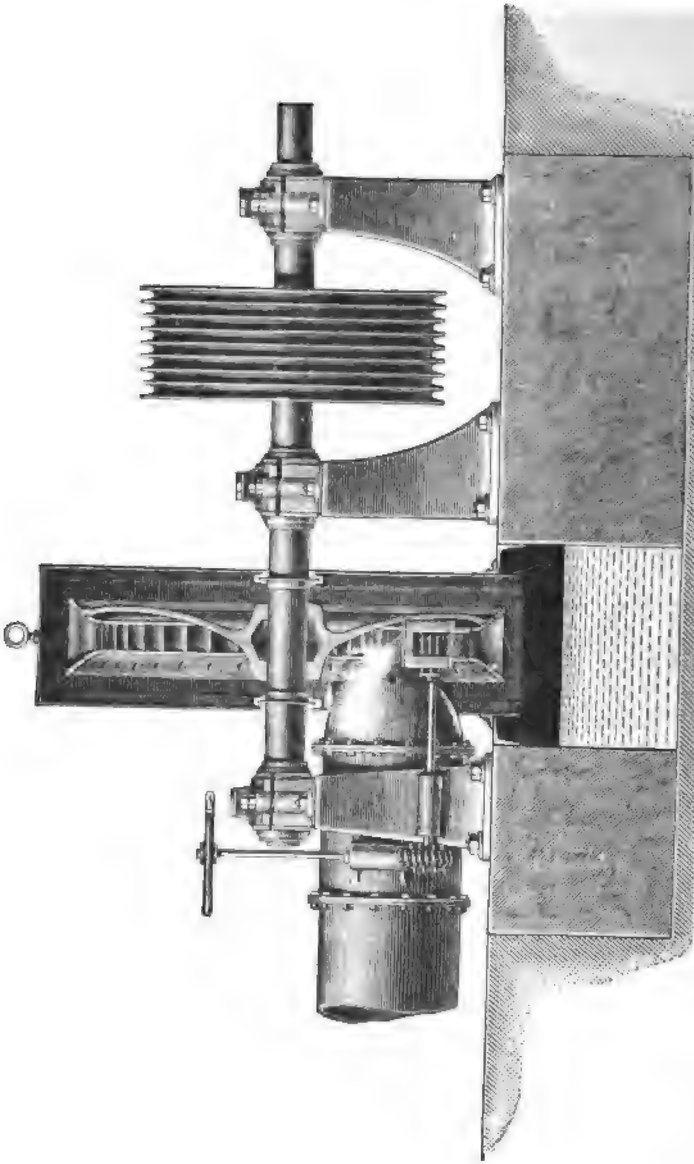


GÜNTHER'S GIRARD TURBINE FOR HIGH FALLS.

admit air to the wheel and prevent the formation of eddies in the empty parts.

The above left-hand illustration is that of a complete turbine, and the two full-page illustrations show how they are fixed in position ready for work. The second of these also indicates how the governor is attached.

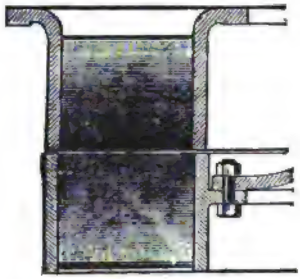
When used for high falls the water is only admitted to a few of the buckets at a time. It is then usually made with outward



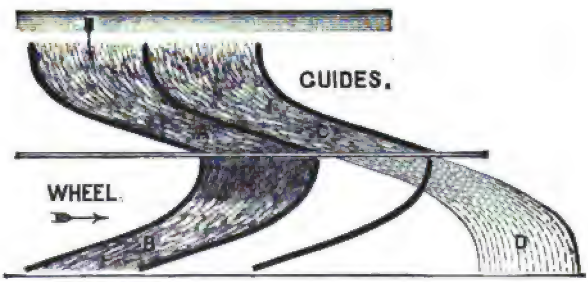
Longitudinal Section of Gunther's Girard Turbine for High Falls.

flow, and mounted on a horizontal axis as illustrated by the sectional drawing facing this and the right-hand figure on the previous page. The pipe for admitting the water and the hand gear for adjusting the flow are clearly visible, whilst a grooved pulley for rope driving is placed on the right. This type of turbine gives a high efficiency at medium as well as at full load.

Jonval Turbine.—This turbine is of the parallel or axial flow type and is designed to be always kept full of water. The accompanying figures give radial and circumferential sections, having the same lettering as the corresponding figures for the Girard turbine. It is usually employed for low and medium falls of from 2 to 40 feet, and can run equally well when completely submerged or when connected to a suction pipe. The adjustment of the water supply is effected by means of a slide or slides which close the guide passages one after another. A turbine of this type has the greatest efficiency when all its passages are full of water, and consequently the size of the wheel depends on the quantity of water to be passed in a given



Radial Section. Günther's Jonval Turbine.

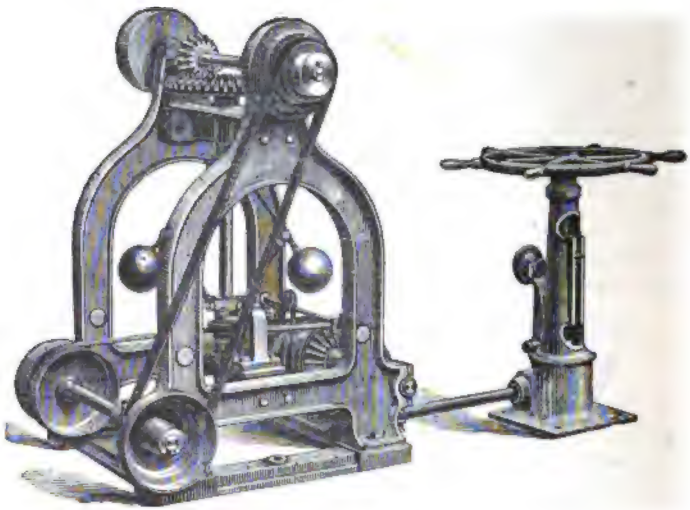


Circumferential Section. Günther's Jonval Turbine.

time. The reason why this class of turbine is not used for small power with high falls is, that very little water would be required, and, therefore, the dimensions of the wheel would be very small —perhaps impracticably small—whilst the speed of rotation would be very great. With the Girard type on the other hand, the efficiency is not reduced by having only one or two of the

buckets in use at one time. We can, therefore, employ as large a wheel as we like, and only use the requisite number of buckets for the required power. This enables us to get a slower speed of rotation which is usually desirable. The Girard type, however, cannot work with a suction tube, and only works well when clear of the tail water. An inch or two of fall, must, therefore, be sacrificed, and this reduces the efficiency with low falls.

Günther's Turbine Governor.—For many purposes the motion of a turbine is so regular that no automatic control is required, but for some classes of machinery a self-acting governor is desirable. The accompanying figure shows the one made by Messrs. Günther & Sons, of Oldham, for this purpose. It consists of an ordinary

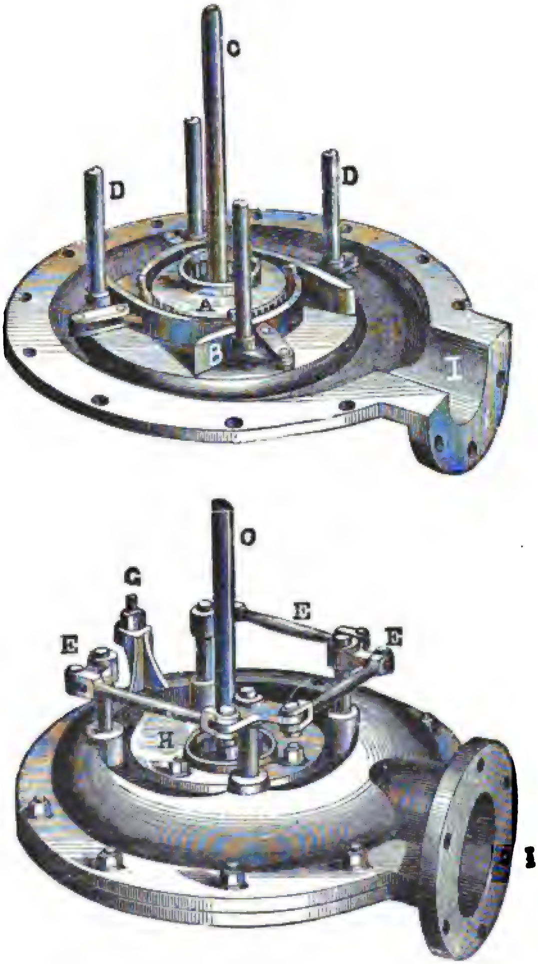


GÜNTHER'S TURBINE GOVERNOR.*

Watt governor (see Lecture XXIII.) which can shift a belt from a loose pulley to either of two fast pulleys connected by bevel gearing to the pillar for adjusting the turbine. At the normal speed the belt is on the loose pulley, but any change of speed causes the belt to be shifted and the vertical shaft is turned more or less in one way or the other according to the required quantity of water. The governor is driven from a belt on the turbine, or by some shaft from it, and can be disconnected from the hand gear by merely freeing a clutch.

* See Notes at end of this Lecture.

Thomson's Vortex Turbine.—Imagine a wheel placed with its axis coinciding with that of the free vortex already considered in the

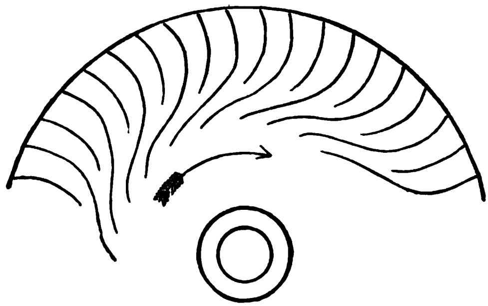


VORTEX TURBINE BY MESSRS. GILBERT GILKES & CO., LTD., KENDAL.

preceding lecture. This wheel will be carried round by the vortex, and if we arrange matters so that the water passes through the

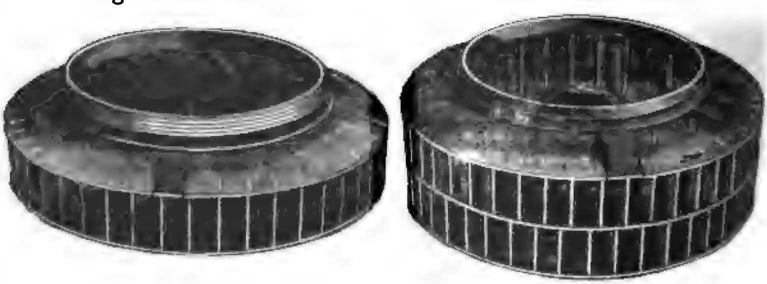
wheel giving up its energy to it while fresh water takes its place, we have the essentials of Professor James Thomson's vortex turbine.

As will be seen from the foregoing illustrations, there are only four guides in the vortex, and by altering the inclination of these, we can adjust the radial component of the motion of the water and the amount of water flowing through the turbine. In the first figure the turbine is represented with its cover removed, and in the second with its case complete. A is the revolving wheel keyed to the shaft C. B is one of the guide blades connected by the bell cranks and shafts D to the outside rods E, which can be adjusted by a screw or by a governor. The shaft runs on a lignum vitæ pivot which is lubricated by the water.



SECTION OF WHEEL OF VORTEX TURBINE.

On account of the constrained motion of the water inside the wheel it requires a large number of guides. For the purpose of reducing the friction and to lessen the loss of area due to the

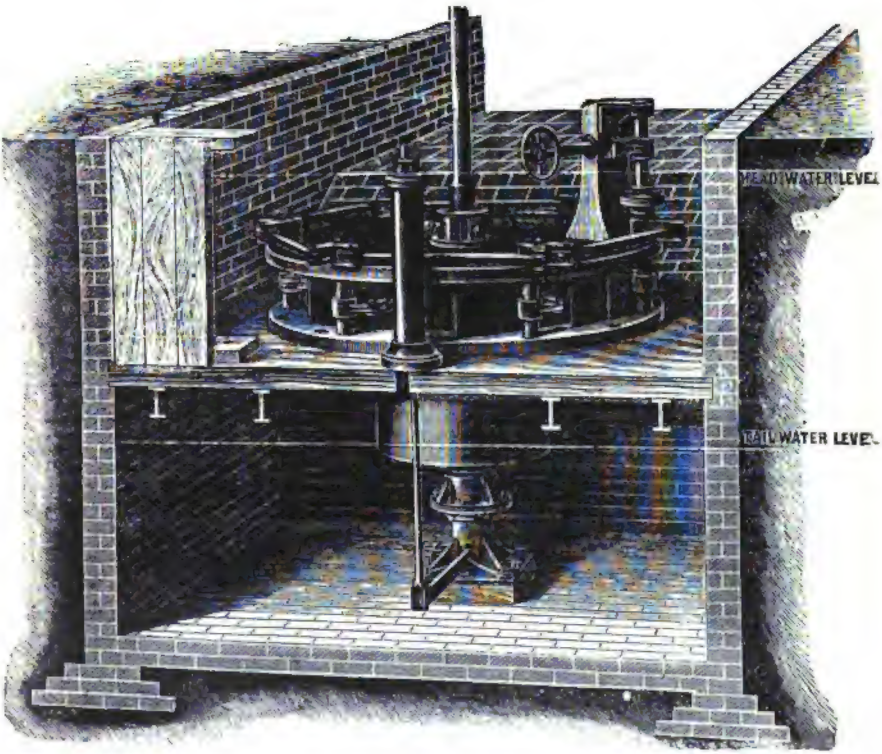


SINGLE AND DOUBLE VORTEX WHEELS.

thickness of the guides, it will be seen from the above section that every second guide is only half the length of its neighbouring one. The wheel is made either single or double. In the latter case, it

has the same efficiency at half gate as at full gate, but being of the inward flow "drowned" type it will have a lower efficiency at other loads. Our next figure illustrates one of these turbines as fixed in position.

One great advantage of an inward flow turbine is that, to a certain extent, it is self-governing. When its velocity increases,



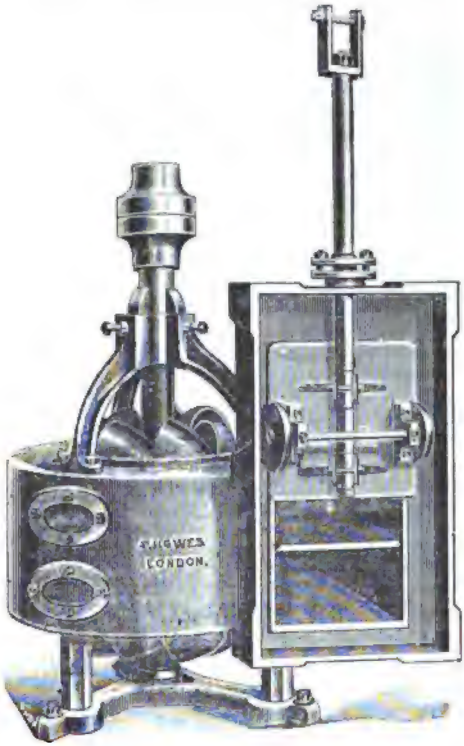
SINGLE VORTEX TURBINE BY GILBERT GILKES & Co., LTD, KENDAL.

so also does the centrifugal force which opposes and consequently reduces the inflow of the water; whilst the opposite action takes place when the turbine is being reduced below its normal speed.

Little Giant Turbine.—We now come to the fourth or mixed flow type of turbine, and as an example we have chosen the "Little Giant" turbine. As may be seen from the illustrations the water

enters at the circumference and passes out at the top and bottom and there is a sluice for regulating the supply. The passage has a division so that water can be entirely shut off from the upper half of the wheel.

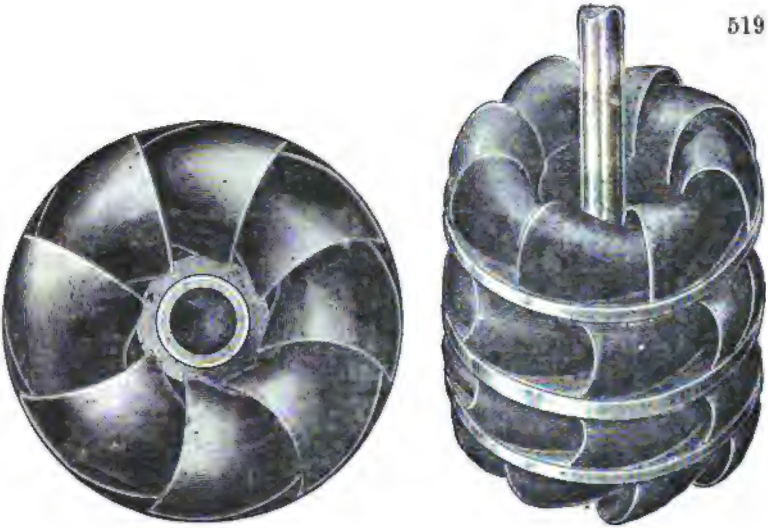
The same firm also makes a special "flume" turbine—*i.e.*, one which is placed directly in the water in the same way as the



LITTLE GIANT TURBINE BY S. HOWES, LONDON.

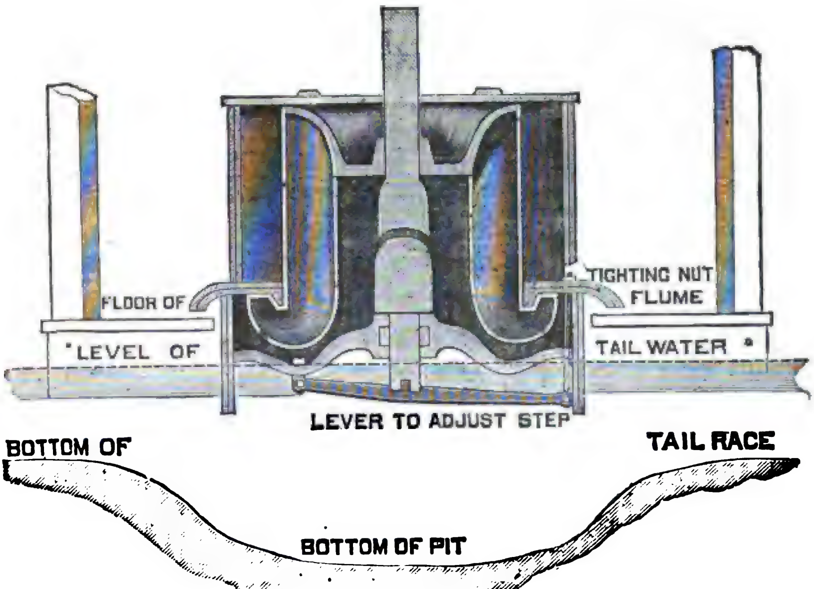
WHEEL FOR LITTLE GIANT SPECIAL FLUME TURBINE.

single vortex turbine shown on the previous page. The sectional view shows a turbine formerly made with a wheel similar to that in the Little Giant Flume Turbine, but with a different casing. In this instance, the pivot for the wheel can be raised or lowered by a lever which is clamped in position by a nut. The wheel is held down by a fixed ring fitting in a groove on the wheel, just above the lower splayed-out part.



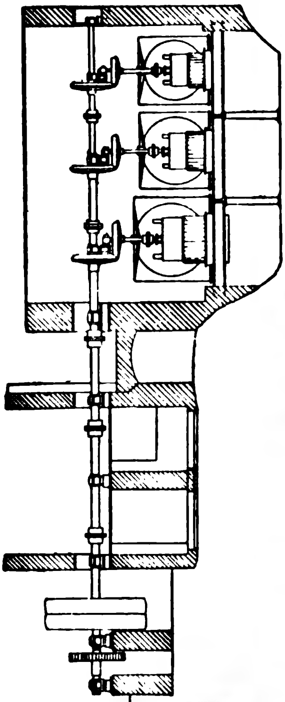
End View. Side View.

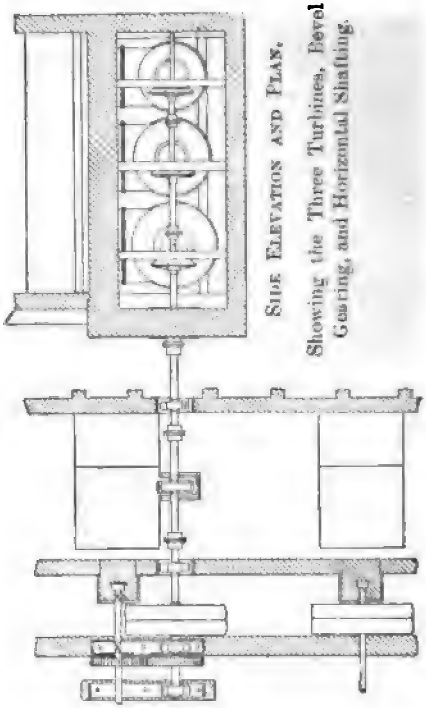
Wheel for Little Giant Turbine.



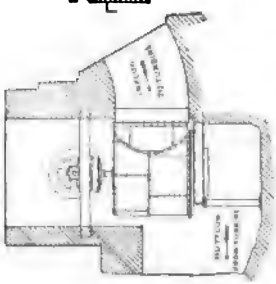
Vertical Section of the Little Giant Turbine.

Stoneywood Paper Works, Turbine Installation.*—This installation consists of three "Hercules" turbines, 54 inches, 48 inches,





Side Elevation and Plan.
Showing the Three Turbines, Bevel Gearing, and Horizontal Shafting.



Vertical Section through Outflow and Inflow.

Stoneywood Paper Works.

"Hercules" Turbine Installation of 1,200 H.P.

* I am indebted to John Turnbull, Jun. & Sons, Glasgow, who put down and set agoing this plant, for the drawings from which the accompanying figures were made.—A.J.

and 42 inches diameter respectively. They are all worked by a fall of 23 feet, and the combined 1,200 H.P. of the three turbines is transmitted by bevel gearing to a horizontal shaft, which revolves at 83 revolutions per minute, as shown by the accompanying views. The power is then conveyed to the various departments of the paper works from this horizontal shaft, principally by belts, but in some cases by gear wheels.

The water is supplied to the turbines through a head race, from the Aberdeenshire river, Don. The sectional area of this race is 22 feet wide by 5 feet 6 inches deep. After passing through the turbines, the water is discharged into a tail race which connects with the river at a lower level.

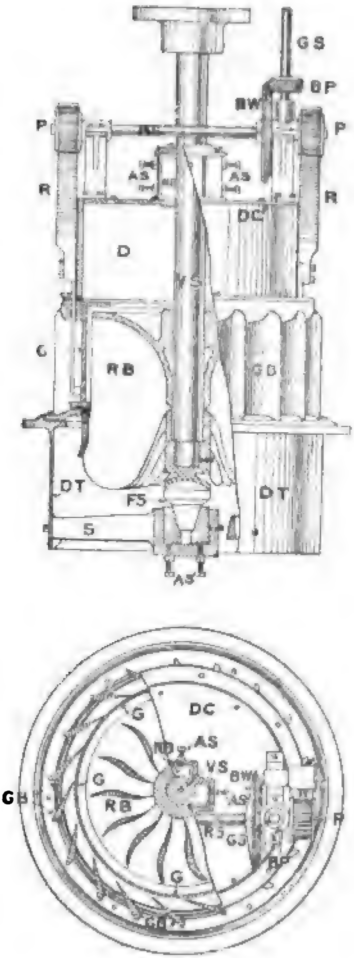
Hercules Turbine.—This low-pressure mixed flow turbine is of American origin and make. Many of these turbines are to be found doing good work in this country under different water heads up to 40 feet. They are made in twenty different graduated sizes, varying from 1 to 2,800 horse, of which the following table shows the capabilities of the former and the latter under the minimum and maximum pressures:—

	Head in Feet.	Horse-Power.	Cubic Feet of Water discharged per Minute.	Revolutions per Minute.
No. 1 Size, . . {	4 40	1·2 35	190 580	266 840
No. 20 Size, . . {	4 40	90 2,830	14,800 46,800	38 117

From the previous and accompanying figures of this turbine, with index to parts, the construction and action will be easily understood. When the gate shaft, G S, is turned in the direction for lifting the gate, it does so through the bevel gear, B P and B W, pinions, P, and racks, R, which are bolted to the upper end of the thin steel cylindrical gate, G. This action permits the water from the source of supply to flow inwards upon the revolving buckets, R B, through the fixed guide blades, G B. Having given up the greater part of its energy to the vertical shaft, V S, the water discharges freely from the draft tube, D T, into and below the surface of the tail race, as shown by the left-hand arrow in the previous figures.

The makers of this turbine and their representatives maintain, that turbines of the "Vortex," and "Little Giant" types, pre-

viously described in this Lecture, cannot possibly give such a high efficiency at part gate as those with fixed guide blades. They also state, that turbines constructed with gates which, when partly closed, alter the angle of the inflowing water as it strikes or presses upon the buckets of the revolver or rotor, are less efficient than the mixed-flow type for the reason, that the angle of direction of the moving water in the latter is not altered at any part of the gate opening.



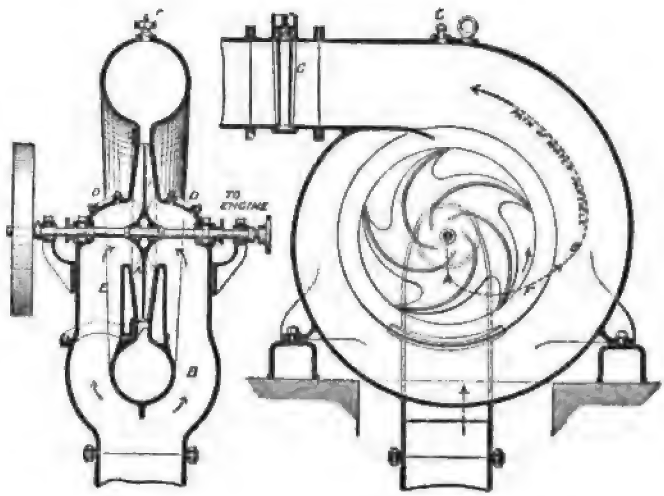
IMPROVED "HERCULES" TURBINE.

INDEX TO PARTS.

G S for Gate Shaft.
B P ,, Bevel Pinion.
B W ,, Bevel Wheel.
R S ,, Rack Shaft.
P ,, Pinions.
R ,, Racks.
G ,, Gate.
G B ,, Guide Blades.
R B ,, Revolving Buckets.
V S ,, Vertical Shaft.
F S ,, Footstep.
S ,, Support for Footstep.
A S ,, Adjusting Screws.
D T ,, Draft Tube.
D ,, Dome.
N B ,, Neck Bush.

Further, that those turbines which are fitted with the cylindrical gate, as herewith illustrated, maintain a higher efficiency at all loads under the maximum power of the turbine than other forms of low-pressure turbines. They guarantee an efficiency of 80 per cent. from half to full gate, and with the larger sizes, they claim 85 per cent. efficiency for ratio of B.H.P. to E.H.P. due to waterhead.

Centrifugal Pumps and Fans.—If certain kinds of turbines be driven by a prime motor the centrifugal force of the fluid carried round with the wheel will cause the fluid to flow outwards in a continuous stream. If the fluid be water the machine is termed a *centrifugal pump;* whereas, if we are dealing with air, it is called a *fan*. Such machines are, therefore, merely reversed turbines; but, in order to get the best results, they must be specially designed, because the best design for a turbine is not by any means the most efficient and suitable for a pump or fan.



CENTRIFUGAL PUMP.

Centrifugal pumps are largely used whenever a quantity of water has to be elevated through a small height, such as in the case of emptying graving docks and sunken vessels, circulating the cooling water through condensers, or dredging sand, gravel, and mud from rivers, whilst centrifrugal fans are employed for ventilating mines, ships, and buildings, as well as for producing an artificial draught to smiths' fires, cupolas, and boiler furnaces, &c.

The illustration* shows a good form of centrifugal pump with curved blades. A is a wheel rotating inside a casing B, thus giving the water which enters at the centre a certain amount of kinetic and pressure energy. F is a small whirlpool chamber which,

* The above figure is reduced from one in Mr. Lineham's *Text-book on Mechanical Engineering* (Chapman Hall & Co.), by kind permission.

as already explained, allows the water to form a free vortex, and converts part of its kinetic energy into energy of pressure. The water moves in a free vortex in the volute-shaped pipe, the path of a partical being shown in the figure by a dotted line and arrows.

The difference of pressure produced by the pump depends upon the density of the fluid in the wheel as well as on the speed of rotation. Consequently, when there is only air in it, the pump is not able to produce a sufficient vacuum to make the water rise into it. In order to get over this difficulty, an ejector G, and a sluice C, are added. When the pump is to be started the sluice is closed and the air exhausted from the pump chamber by a jet of steam being passed through the ejector. The water then rises into the wheel and the sluice is gradually opened as the speed is increased. When the pump is fairly started the steam jet is shut off and the sluice fully opened. Sometimes a non-return valve is placed at the foot of the suction pipe to prevent the pump and pipe emptying when the wheel is stopped. In such a case, the pump is ready to start again at once.

Sometimes centrifugal pumps are made with radial blades. They then require a much larger whirlpool chamber to allow the kinetic energy to change into pressure energy without a serious loss in eddies.

Note 3 on the Second Edition, by Mr. J. Ritchie, Waverley Engineering Works.

Pelton Wheels.—"Your description of the action of a jet on the Pelton bucket is very clear, as it illustrates the flow of the water over the bucket in a way which I have never met with before. It does not, however, in my opinion represent the true direction of the water along the face of the bucket. It would be all right if the buckets were fixed to a chain passing over two pulleys, where they would move away in a straight line from the jet, until completely emptied. They move, however, in a circle, and the centrifugal force brings into action another movement in an outward direction to the point of the bucket, and part of the water is, therefore, discharged over the lip of the bucket, as well as at the sides.*

"The accompanying figure shows a small Pelton wheel bucket, from which I have got very good results. With a 12-inch wheel at 1,350 revolutions per minute and a head 310 feet, it gives 8 horse-power.

"I have never been able to get the very high efficiencies stated by the Pelton Wheel Company. Even with polished bronze wheel buckets, 75 to 80 per cent. is the highest which I have obtained.

Pelton Wheel Regulation.†—"I think something might be added on the subject of regulation. The weak point, of course, is, as you point

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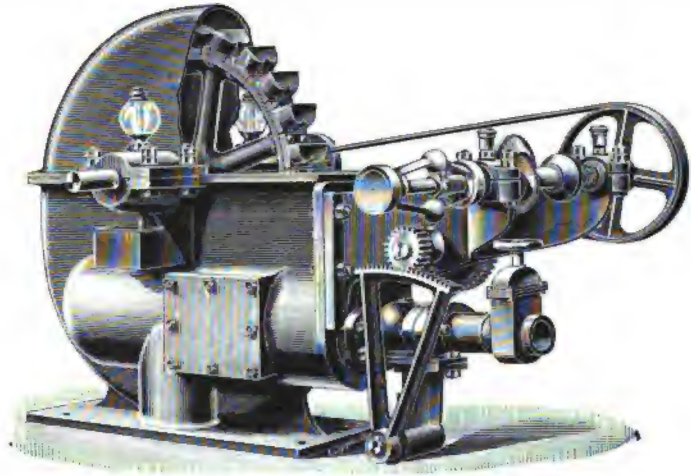
* True; but the three positions of the Vanes in my illustration are supposed to be *very close* to each other.—A.J.

† Students may here refer to a paper on "Experimental and Analytical Results of a Series of Tests with a Pelton Wheel," by William Campbell Houston, B.Sc. See vol. xlvi., part viii., of *Transactions of the Institution of Engineers and Shipbuilders in Scotland*, Glasgow, April, 1903.—A.J.

out, the waste of water by throttling. The nozzle has to be made large enough to do the maximum work, and unfortunately at the same time that the greatest quantity of water is passing, the net or true head is generally the least, since the frictional losses in the pipe are then greatest.

"I have adopted a central spear to reduce the area of the jet and find it to be one of the most efficient methods, since the jet becomes thereby



PELTON WHEEL WITH A GOVERNOR.
(By Carrick & Ritchie, Edinburgh.)

partly annular. The accompanying figure shows the type of this wheel fitted with a regulating nozzle under the control of a governor.

"One of the principal losses with Pelton wheels is due to the bucket striking the jet, whereby a portion of the water is momentarily diverted as the bucket is entering the jet. Many different forms of buckets have been devised to avoid this loss, but without much success.

"In double or treble nozzle wheels it is advisable to allow at least one bucket to intervene between the two that may be under the action of the jets. This is to allow of the action which you describe on p. 507, where the water continues to travel over the bucket, when the jet is acting on the following vane.

"The use of Pelton wheels is rapidly extending for high falls. The efficiency with a good regulating nozzle is as high as a Girard turbine. They are less complicated and cheaper.

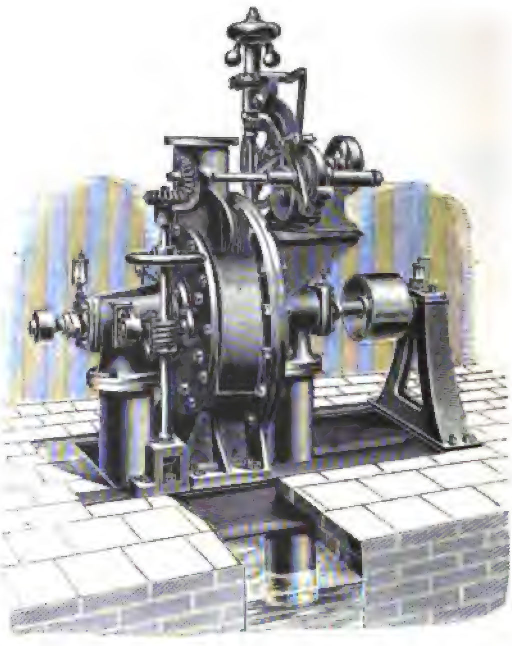
Girard Turbines.—"The description and illustrations are all good. It may be mentioned that outward-flow Girard turbines are very difficult to govern, owing to the centrifugal force acting on the water passing through the wheel, increasing the flow instead of diminishing it, as in the inward flow turbine.

"I have made a great many of the outward-flow Girard type of turbine with horizontal shafts. I have not found the maximum efficiency to be so high as in the case of the inward-flow or pressure and reaction type of

turbines. It is not advisable to use the latter for heads over 200 feet; whereas, I have Girard turbines working under 800 feet. At the same time there is no doubt, that for great variations—say down to one fourth of the supply—there is nothing to beat the Girard turbine, owing to the method of regulation—viz., shutting off the supply from each port, one after the other.

Jonval Turbines.—"I notice that on the Continent the Jonval turbine is being rapidly superseded by mixed-flow turbines. The speed of the former is necessarily slow, owing to its comparatively large diameter, which naturally involves heavier gearing and larger pits, &c., than with the latter.

Turbine Governors.—"The governor shown at pp. 510, 514 is a good one, but it is not quite sensitive enough for electric lighting work. In order to move the belt, the governor itself must be slow and heavy. King's patent governor, which is shown on the high-pressure double-dis-



High-Pressure Double-Discharge Waverley Turbine with King's Governor.

charge Waverley turbine, is a very sensitive high-speed governor. All the work this governor has to do, is to move the cover plate of the pawl wheel.

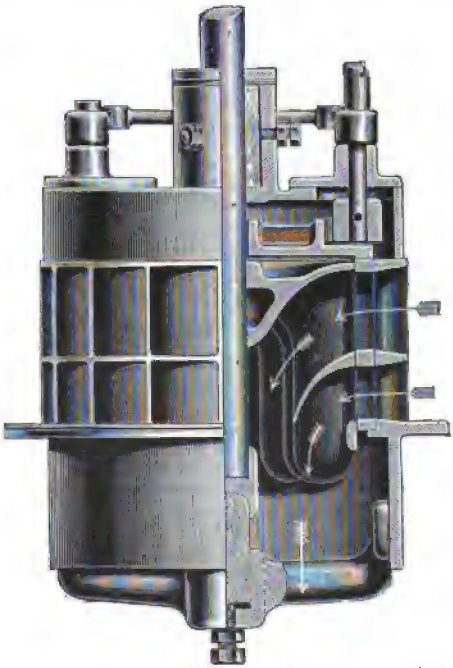
"Owing to water being a non-elastic body, turbines and Pelton wheels are much more difficult to govern than a steam engine. The principal difficulty to contend with is hunting, as the action of the governor, owing to the momentum of the parts, is continued after the reason for the action

ceases, and the governor, therefore, keeps turning 'off' or 'on' the water longer than is necessary. The relay arrangement is calculated to prevent this.

Inward-Flow Turbines.—"Thomson's Vortex turbine is the parent of all the inward and mixed-flow turbines in use at the present day. It has a very good efficiency. The principal objection to it is the choking which takes place at the centre of the wheel. Consequently, it has to be made of large diameter and comparatively shallow, compared with other turbines of equal power; but for many purposes it still holds its own.

Mixed-Flow Turbines.—"I must frankly say I do not think you have been fortunate in selecting for illustration the best type of a mixed-flow turbine. Owing to the attempt which most American manufacturers have made, to get the greatest possible quantity of water through the wheel for a given diameter, the designers of this wheel have introduced such sharp curves, that the frictional losses are great. There are no guides in the proper sense of the term, the casing being in the form of a scroll. The water assumes a circular or semi-vortex motion in the same direction, before entering the buckets of the wheel.

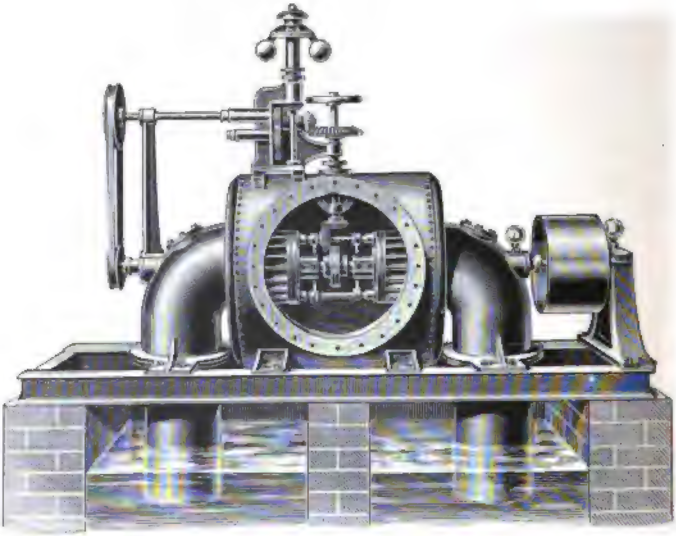
Waverley Mixed-Flow Turbine.—"The usual form of this turbine with a vertical shaft, suitable for fixing in an open flume is shown by the following figure. The special feature of the Waverley turbine is the



WAVERLEY MIXED-FLOW TURBINE WITH DOUBLE BUCKET.
(By Carrick & Ritchie.)

division in the buckets. This division may be placed in any position from one-third to one-half down the bucket, and is specially of advantage where the water supply is variable. The usual form of gate is cylindrical and it is placed between the wheel and the guide ring. The upper portion of the wheel may be entirely shut off in the event of the water being low, and the efficiency of the wheel is thereby increased. In the Waverley wheel about one-half of the water is discharged radially and the other half axially, through the lower part of the buckets.

"The cylindrical gate of the turbine is operated by a pinion and rack.* The accompanying figure shows a duplex turbine, consisting of two wheels



HORIZONTAL DUPLEX WAVERLEY TURBINE.

on one shaft. This arrangement has the advantage of doing away with the end thrust on the shaft as the pressure on the one wheel is balanced by the other.

"This type of turbine is suitable for falls up to 50 feet, but the former figure with King's governor shows a turbine of a somewhat similar construction with a double-discharge pipe, which is built of sufficient strength for falls up to 200 feet. These turbines are very suitable for the direct driving of dynamos. For example, in connection with the electric lighting of Blebo House, near Dairsie, Fifeshire, I used one where the head is only 26 feet, and 10 feet of this head is due to the 'draft or suction pipe.' It has two gun-metal wheels 9 inches diameter, and gives 15 H.P. at 680 revolutions per minute. The lamps are run direct from the dynamo, for there are no accumulators, and the speed of this turbine is entirely controlled by a King's governor.

* This is a similar arrangement to that already described in this Lecture in connection with the "Hercules" turbine.—A.J.

"The forms of the guides and buckets of mixed-flow turbines are not the result of any elaborate 'scientific' calculations, but are based on the general assumption, that the greatest power is got out of the water by turning it as nearly in an opposite direction to its flow, or through 180°. Of course, mechanical difficulties make the latter impossible, but the mean angle through which the water is turned is about 125°. The body of the wheel being a cone, the water is at the same time given a vertical direction, the lower portion of the bucket being curved backwards. The mean angle of discharge is about 20°, the extreme end of the bucket is only about 16°, so that the water in this part of the wheel takes a downward, and also slightly outward direction.

"The best form of buckets and guides are simply the result of innumerable tests, in which the forms which gave the highest results have been retained.

"In mixed-flow turbines it is usual to have an unequal number of buckets to that of the guides. This, with the slight curve given to the edge of the bucket, is found to give a more regular turning moment, than if all the ports and buckets came exactly opposite each other at the same moment. One great advantage of the mixed-flow turbine is the ease with which a draft pipe may be used in connection with it. At Tobermory, Scotland, I put down a horizontal draft or suction pipe with a total length of 90 feet, in order to obtain a final vertical drop of 12 feet into the tail

TURBINE INSTALLATION AT TOBERMORY.
(Showing Long Draft Pipe.)

race of the river. The head due to this long draught pipe is nearly as efficient as if the same head were on the pressure side of the wheel."*

* See my Question 21 on this point at the end of this Lecture.—A.J.

Lecture XXXVI.—Questions.

1. Sketch an undershot wheel. Explain why its efficiency is so low when it has radial blades, and show how the blades should be made to avoid this loss.

2. Water flows radially at 4 feet per second towards a part of a wheel of a centrifugal pump or turbine which is moving at 12 feet per second, find the angle of the vane that the water may enter without shock. If the vane were radial, at what angle ought the water to be guided so that it might enter without shock; its radial component of velocity being the same as before? (S. & A. Adv. Exam., 1898.)

3. Give outline sketches of the common types of water-wheel, and compare their relative advantage.

4. Distinguish between water-wheels and turbines, and explain the advantages of the latter.

5. Sketch the Pelton wheel and describe its action.

6. Sketch and describe a turbine of the Girard type, and mention its advantages and disadvantages.

7. Describe, with sketches, a Jonval turbine, and explain its relative advantages.

8. Sketch the wheel and case of an inward flow turbine for a fall of 50 feet; 8 cubic feet of water per second. Calculate the diameters and breadths of the wheel, the number of revolutions per minute, and the size of the shaft. (S. & A. Hons. Exam., 1897.)

9. The vane of a wheel of an internal flow turbine is normal to the rim which moves at 40 feet per second. Water is guided so as to enter the rim with a radial velocity of 4 feet per second; what must be the angle made by the guide blade with the rim, if the water is to enter without shock? If the circumferential area of all the openings of the rim is 1·5 square feet, what volume of water flows per second? We use convenient but not very exact language when we ask—what is the total tangential momentum per second entering the rim? (S. and A. Adv., 1899.)

10. Sketch an inward flow turbine for a fall of 30 feet and 20 cubic feet of water per second. Give its principal dimensions and speed and the angles of guide blades and vanes What is the kinetic energy of a pound of water just entering the wheel? Neglect losses by friction. (S. and A. H., Part I., 1899.)

11. The radial velocity of water in a centrifugal pump wheel is 2 feet per second. The vane makes an angle of 35° with the circumference; what is the velocity of the water *relatively to the wheel?* The wheel is 2 feet external diameter, and makes 300 revolutions per minute; what is the actual velocity of the water as it leaves the wheel? What is the circumferential momentum of each pound of water? Neglecting radial velocity of water, what is the work done to every pound of water if it enters the wheel with only a radial velocity. and what is the kinetic energy of every pound of water? *Ans.* 3·48 feet per second; 28·63 feet per second; 0·89 lb.-ft.; 27·9 ft.-lbs. (B. of E. Adv. & H., Part I., 1900.)

12. Ten cubic feet of water per second enters a turbine wheel with a tangential velocity of 50 feet per second; it enters without shock, the velocity of the rim of the wheel being 50 feet per second; the water leaves the wheel with only a radial velocity. What energy does the water give to the wheel per second? (B. of E. Adv., 1901.)

13. Water, with a radial velocity of 6 feet per second, is really moving along a vane which makes 30° with the circumference of a centrifugal pump wheel, which lets the water leave the wheel with less tangential velocity than that of the wheel itself. Show on a diagram and state the tangential velocity of the water if that of the wheel is 40 feet per second. If the water before it enters the wheel has a radial velocity of 6 feet per second, and enters without shock, what energy is gained per pound of water? What is the gain of kinetic energy? What difference will friction of the passages make in your result? (B. of E. H., Part I, 1901.)

14. Deduce an expression for the efficiency of a simple hydraulic reaction wheel when you are given the available head and the velocity of the water at the discharging orifice. What will the efficiency be if the head equals 112 feet and the velocity of discharge is 90 feet a second, neglecting friction? (C. & G., 1900, H., Sec. C.)

15. Describe, with sketches, a Pelton waterwheel. In what circumstances is such a waterwheel used? What is likely to be the efficiency of the wheel in ordinary working? (C. & G., 1901, H., Sec. C.)

16. A Pelton wheel, 2 feet in diameter, runs at 600 revolutions a minute, the available pressure at the nozzle being 200 lbs. per square inch, and the available supply 100 cubic feet per minute. Estimate the available horse-power and the greatest possible hydraulic efficiency of the wheel.
(C. & G., 1902, H., Sec. C.)

17. Discuss, in general terms, the various losses in a centrifugal pump, pointing out in particular the effect of the whirlpool chamber and of the angle of discharge of the vanes on the efficiency. In a test of a compound centrifugal pump the pressure in the rising main, measured at the pump, was found to be 55·8 lbs. per square inch above atmosphere, and on the suction pipe, measured at the pump, 6·7 lbs. per square inch below atmosphere. The water raised per minute was 119 gallons, whilst the turning moment transmitted to the pump spindle, as measured by a dynamometer, was found to be 27·3 foot-lbs., the revolutions being 1460 per minute. Find the lift, the horse-power of the pump, and the efficiency.
(C. & G., 1902, H., Sec. C.)

18. Explain what is meant, in referring to hydraulic pressure engines, by the statement that "they contain within themselves their own brakes." It is found that a ram of 10 inches in diameter can lift a load of 20 tons with an uniform velocity of 6 inches per second when supplied with water from an accumulator working at a pressure of 750 lbs. per square inch. Find the coefficient of hydraulic resistance referred to the velocity of the ram, and, assuming this to remain constant at all speeds, find the velocity of the ram when under no load. (C. & G., 1902, H., Sec. C.) (*For answer to first part refer to the Froude Water Dynamometer or Brake described in Vol. I.*)

19. Explain, with the aid of sketches, the various devices employed to make water-pressure engines economical when working under loads less than their maximum. (C. & G., 1902, H., Sec. C.)

20. The following results were obtained from an undershot water-wheel of 15 feet diameter, 4 feet 6 inches wide, and having only a head of water of 3 feet 6 inches:—

Revs. per Minute.	B.H.P.	Revs. per Minute	B.H.P.
7·3	6·68	8·9	6·68
8·3	6·7	9·2	6·48
8·4	6·7	10·0	6·4
8·6	6·73	...	...

Plot the circumferential speeds, and the corresponding B.H.P. as co-ordinates. Then, find a constant for the ratio of the velocity of rim of wheel to its diameter at the most effective speed in this case. Also, find the ratio of the speed of rim of wheel to the velocity of the water due to the above head. Can you refer to any reliable data regarding these two ratios or other applicable ratios for water-wheels?

21. Given two turbines with their feed-water pipes identically the same in every respect. Let each turbine be supplied with water from, say, a height of 20 feet above their respective centres. In case (1), the turbine discharges directly into the air; whereas, in case (2) the bottom end of the discharge pipe is led into the tail race below its surface, and the vertical distance from this surface to the centre of the turbine is 20 feet. Which will yield most power, why, and by how much? Sketch and explain your answer. If the centre of the turbine in case (2) were 30 or more feet above the surface of its tail, what would happen, and why?

LECTURE XXXVI.—A.M.INST.C.E. EXAM. QUESTIONS.

1. What are the two conditions to be observed in designing the angles of the vanes at the inlet and outlet surfaces of a turbine to secure the greatest efficiency? Define an impulse and a pressure turbine. (I.C.E., *Oct.*, 1897.)

2. Describe the construction of any one form of turbine, stating how the power and consumption of water is regulated. Discuss the effect of regulating the flow through the turbine on the efficiency. (I.C.E., *Oct.*, 1897.)

3. Show that the hydraulic efficiency of a turbine which works on a fall of H feet is $\eta = w_1 V_1/g H$, where w_1 is the velocity of whirl, and V_1 the velocity of the wheel at the receiving surface, the water being rejected radially. (I.C.E., *Oct.*, 1897.)

4. In an inward flow turbine the water enters the inlet circumference, 2 feet diameter, at 60 feet per second, and at 10° to the tangent to the circumference. The velocity of flow through the wheel is 5 feet per second. The water leaves the inner circumference, 1 foot diameter, with a radial velocity of 5 feet per second. The peripheral velocity of the inlet surface of the wheel is 50 feet per second. Find the angles of the vanes at the inlet and outlet surface. (I.C.E., *Feb.*, 1898.)

5. Describe some different forms of turbine, and mention any special advantages of each arrangement, or any particular conditions for which each is most suitable. Describe specially the regulating arrangement of each turbine. (I.C.E., *Feb.*, 1898.)

6. Describe any one form of turbine governor. State what are the special difficulties in applying governors to turbines, and point out how the difficulties are met in the governor you describe. (I.C.E., *Feb.*, 1898.)

7. Describe how you would measure the actual power given out by a water-motor, or any other motor, and the power supplied to it, and so arrive at its efficiency. (It may be well to say that if the Candidate has not actually measured the brake-power of a machine, he will be apt to make a mistake in the description.) (I.C.E., *Oct.*, 1898.)

8. A horizontal jet of water impinges on a vane rotating about a horizontal axis above it, when the arm of the vane makes an angle of 120° with the jet, sketch the shape of vane to obtain the maximum amount of energy out of the jet, and indicate the path of the water on leaving the vane. (I.C.E., *Oct.*, 1898.)

9. Deduce the general formula for turbine efficiency. (I.C.E., *Oct.*, 1898.)

10. A vane, circular in section, subtending an angle of 30° at its centre, rotates about that point, in a vertical plane, with a linear velocity of 3 feet per second. A jet of water, 12 square inches in section, impinges upon it tangentially with a velocity of 9 feet per second. Find the power transmitted by the jet to the vane. (I.C.E., *Feb.*, 1899.)

11. Water under a head of 100 feet flows at the rate of 10 feet per second through a horizontal bend-pipe 36 inches diameter and 20 feet radius, subtending an angle of 60° at the centre of the curve; calculate the total pressure tending to force the bend-pipes outward. (I.C.E., *Feb.*, 1899.)

12. Describe the construction of an impulse turbine, and state how it is regulated for supplies below the normal. (I.C.E., *Feb.*, 1899.)

13. Sketch a centrifugal pump and state what you know of the conditions upon which its efficiency depends. (I.C.E., *Oct.*, 1899.)

14. It is desired to utilise a stream of water having a head of 40 feet, measured from the rock over which it falls to the level of the stream below. The flow is exceedingly variable, and while it is desired to make the most of the minimum flow, amounting to about 5 cubic feet per second, advantage must be taken of at least four times that flow when it occurs. (*a*) Between water-wheels and turbines, and their different classes, how would you choose, and why? (*b*) What brake horse-power should you expect to obtain from the minimum flow? (I.C E., *Feb.*, 1900.)

15. Explain, with as few mathematical symbols as possible, the fundamental conditions of mechanical efficiency in turbine action, taking account both of the waste pressure and kinetic energy in the water-discharge and of the frictional wastes. (I.C.E., *Oct* , 1900.)

16. What are the latest developments in the use of centrifugal pumps for force-pumping against a large head? Describe briefly the dynamics of this use of centrifugal pumps, and more particularly in respect of the mode of generation and accumulation of high pressure and in respect of mechanical efficiency. (I.C.E., *Feb.*, 1901.)

17. Sketch good forms of admission and exhaust valves for hydraulic pressure motor engines. (I.C.E., *Feb.*, 1901.)

18. Explain fully how the energy of water is utilised in an over-shot water-wheel, in a Pelton wheel, and in a reaction turbine.

(I.C.E., *Oct.*, 1901.)

19. Indicate how a vane, moving with a velocity of 25 feet per second in a horizontal direction, must be shaped in order to abstract the maximum amount of energy from a jet of water impinging upon it at an angle of 45° to the horizontal with double the above velocity. What pressure would be exerted on the vane per cubic foot of water impinging per second?

(I.C.E., *Oct.*, 1901.)

20. Explain the action of a Girard type of impulse turbine, and indicate how with a vertical shaft the pivot supporting the weight of the turbine wheel can be made easily accessible for lubrication and adjustment.

(I.C.E., *Feb.*, 1902.)

21. In an outward flow impulse turbine the jet of water, moving with a velocity of 150 feet per second, is guided on the wheel vanes at an angle of 75° to the radius, the inside radius of the wheel being 5 feet and the outside radius 6 feet. If the wheel makes 150 revolutions per minute, and the water leaves the wheel with a radial velocity only, draw the shape of the vanes. (I.C.E., *Feb.*, 1902.)

22. Show how the hydraulic efficiency of a turbine may be found from the principle that the angular impulse is equal to the change of angular momentum. (I.C.E., *Feb.*, 1902.)

23. Calculate the maximum possible efficiency of a Pelton wheel in which the bucket velocity is three-quarters of the jet velocity. How much less than this would the actual efficiency be in the case of a wheel of about 40 H.P.? To what, and in what proportion, would you debit the losses?

(I.C.E., *Oct.*, 1902.)

24. Explain briefly the apparatus you would use, and the calculations you would make, to determine the net efficiency of a turbine giving about 20 B.H.P., the shaft being vertical. (I.C.E., *Feb.*, 1903.)

25. For an inward-flow turbine, given the angle of the guide-blades, the speed of the vanes, and the radial velocity of the water at both inlet and

outlet, show how you would find the angles of the vanes at inlet and outlet, and how you would proceed to design them. (I.C.E., *Feb.*, 1903)

26. Obtain a general expression for the hydraulic efficiency of a turbine in terms of the head, the angular velocity of the vanes, the tangential components of the velocity at inlet and outlet, and the radii of the inlet and outlet surfaces. Write down the corresponding formula for a centrifugal pump. (I.C.E., *Feb.*, 1903.)

LECTURE XXXVII.

REFRIGERATING MACHINERY.

CONTENTS.—Refrigeration—Preliminary Considerations—Carbon Dioxide as a Refrigerating Agent—Elementary Refrigerating Apparatus—Simple Refrigerating Machine—Carbon Dioxide Refrigerating Plant—Anhydrous Ammonia as a Refrigerating Agent—De La Vergne's Refrigerating Plant—De La Vergne's Double Acting Compressor—The Linde System of Refrigeration—Apparatus for Transmitting the cold produced to the Chambers requiring Refrigeration—Pneumatic Tools—Questions.

Refrigeration—Preliminary Considerations.—An interesting example of the conversion of heat into work is afforded by a refrigerating machine. The simplest form of machine consists of an air-compression pump driven by a steam engine, or other motive power, in which the pump is water jacketted and the air is cooled under pressure by being passed through a surface condenser where the water abstracts the sensible heat generated by the mechanical work of compression. The air thus cooled, but still under pressure, is conveyed to an air engine and allowed to perform work by expanding against some resistance. A large proportion of the work originally performed during the operation of compression is again given out, with a corresponding reduction in the air temperature. A machine on this principle may be conveniently constructed by arranging the steam, compression and expanding engines to work on one crank-shaft. The expanding air thus assists in the work of compression. After deducting the necessary losses due to cooling, leakage, &c., the work done in the expansion cylinder amounts to about 65 per cent. of the power absorbed by the compression cylinder; the remaining 35 per cent. being supplied by the steam or other prime mover. The air having thus given up its heat, exhausts from the expansion cylinder at a very low temperature, reaching in one authenticated instance as low as $-124°$ F.

The large coal consumption of machines of this class has, however, led to their being superseded, in almost all cases, by machines in which a more direct conversion of heat into work takes place.

If we take any liquid and commence to vaporise it, we find that, it is necessary to maintain a continual application of heat in order to bring about this physical change. The amount of heat necessary to convert a unit weight of a liquid to a unit

weight of gas at the same pressure, is always constant for the same liquid. For example, 1 lb. of water at a temperature of 212° F. and at atmospheric pressure, requires the application of 966·6 British Thermal Units to convert it into 1 lb. of steam at the same temperature and pressure. Conversely, to condense 1 lb. of steam to 1 lb. of water, both being at 212° F. and 14·7 lbs. pressure per square inch, we must abstract from the steam 966·6 thermal units by contact with a cold body. This principle holds good for any liquid.

A refrigerating machine with steam as a working medium, would not be practicable unless the temperature of everything in connection with it was maintained above 212° F., but there are many liquids which have, when compared with water, a very low boiling point; notably ether, sulphurous acid, ammonia, carbon dioxide, and ethylene. Each of these has been employed for purposes of refrigeration with more or less success; and all of them depend on the same principle—viz., the absorption or the giving out of their latent heat in converting the liquid to a gas, or *vice versâ*.

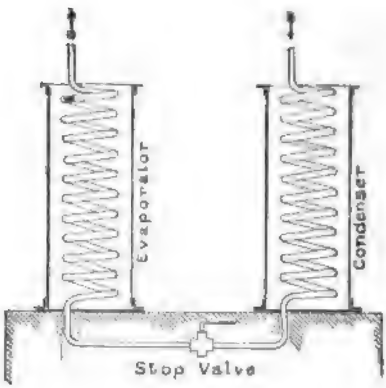
It is not necessary here, to enter into a discussion as to the relative merits of different liquids as refrigerating agents, but in practice, anhydrous ammonia is the agent generally used, and in a lesser degree, carbon dioxide. The necessary machinery is of itself extremely simple, although the details afford scope for a great amount of elaboration and ingenuity.

Carbon Dioxide as a Refrigerating Agent.—Carbon dioxide or carbonic anhydride, which is commercially known as "carbonic acid," is a colourless gas, and quite without odour when pure. It is under all circumstances perfectly innocuous, and has practically no effect on animal tissues or other bodies. It will, however, produce asphyxiation in animals when present in the atmosphere in quantities exceeding 25 per cent. by excluding oxygen from the blood. This gas may be very readily liquefied, either by diminishing its temperature or by increasing its pressure. This fluid has a specific gravity of about ·8, and can only remain in the liquid state when under considerable pressure, the pressure varying with the temperature of the liquid.* The moment the pressure is removed, the heat present in surrounding bodies, at once assists in the evaporation of the liquid carbon dioxide and the bodies themselves are consequently left in a colder condition than before the evaporation took place.

Elementary Refrigerating Apparatus.—Let us consider for a moment an elementary piece of apparatus in which refrigeration

* Carbonic acid gas can only be liquefied by pressure when below 86° F. which is termed its critical temperature.

can take place. If we take two strong coils of piping and surround each with a vessel of water and then connect the two by means of a stop valve at their lower ends, as shown by the accompanying figure, we shall have a simple form of refrigerating machine. Suppose, that when the stop valve is closed, we charge the condenser coil with gas under liquefying pressure, by means of a force pump. The water surrounding the coil will absorb the heat which has been imparted to the gas by compression, and the condensed liquid will gradually accumulate at the bottom of the coil. On opening the stop valve, this liquid will run into the second or evaporating coil, and the pressure here being lower than is necessary for maintaining the liquid state of the material, evaporation will at once commence. The heat necessary for evaporating this liquid is

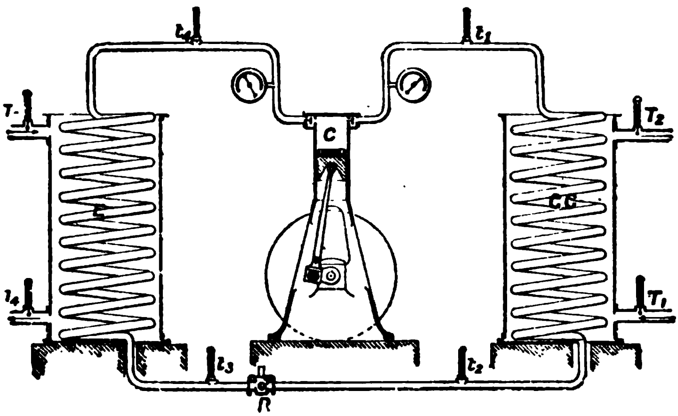


ELEMENTARY REFRIGERATING MACHINE.

absorbed from the water surrounding the evaporating coil, which will thereby become considerably reduced in temperature. To accomplish this result in practice three things are necessary:—(1) A compressor, to raise the pressure of the gas to whatever may be necessary for its liquefaction; (2) a surface condenser, to remove the heat generated by the mechanical work of compression; (3) an evaporating vessel, where the liquid may re-evaporate into a gas, and absorb heat in the operation.

Simple Refrigerating Machine.—The following figure of a simple refrigerating machine will explain the cycle of operations. C is a compressing pump delivering gas under pressure into the condensing coil C C, which consists of a strong worm of iron or copper piping immersed in a tank of water. R is a

regulating-stop valve having a fine adjustment. E is the evaporator which consists of a coil of piping similar to the condensing coils. It is also immersed in a tank containing the water or other fluid to be cooled. The regulator R is closed, as soon as the pressure in the condenser has risen to that at which liquefaction can take place and the gas commences to condense on the inner surface of the coil C C. The drops of liquid descend and accumulate in the lower portion of this coil. The regulator is then opened, with the result, that a small quantity of liquid escapes into the evaporator. Now, since the compressor draws its supply of gas from the evaporator, the pressure in the evaporator must be less than in the condenser. Consequently,



SECTIONAL DIAGRAM OF A SIMPLE REFRIGERATING MACHINE.

INDEX TO PARTS.

C for Compressor.	E for Evaporator.
C C „ Condensing Coils.	t_1 to t_4 „ Thermometers.
R „ Regulating Valve.	T_1 to T_4 „ Thermometers.

the liquid commences to boil, and absorbs heat for its transformation into a gas from the surrounding liquid. The temperature of this liquid is therefore naturally reduced by the operation. The liquid within the coil is entirely re-converted into a gas which ultimately finds its way to the compressor, and thus the cycle of operations is completed.

Suppose four thermometers be inserted into the pipes conveying the gas to and from the condenser and evaporator, as shown at t_1, t_2, t_3, t_4. It will be found that they do not register alike, for t_1 will show the highest temperature, then t_2 and t_3 will be

some degrees lower, and t_4 will be lowest of all. Suppose now, that the liquids in the vessels surrounding C C and E be caused to circulate in the direction of the arrows, and that thermometers T_1, T_2, T_3, T_4, be placed on each of the inlet and outlet pipes. It will be found, that the temperature of the incoming water T_1 is lower than T_2 the temperature of the water going out of the condenser; also, that T_3 the temperature of the liquid entering the evaporator is *higher* than T_4 its temperature as it leaves this vessel. This shows, that with respect to the gas or liquid within the coils of the condenser or evaporator, heat is lost in the condenser and gained in the evaporator. The amount of the former is represented by the difference between T_1 and T_2 multiplied by the weight of water passed through the condenser in pounds, and the latter may be expressed in terms of the difference between T_3 and T_4 multiplied by the weight of the fluid passing through the evaporator, and by the specific heat of this fluid.

If we could construct an ideal machine, in which the liquefaction of the gas was automatic, it would be found that the loss of heat in the condenser, measured in thermal units, was exactly equal to the gain of heat in the evaporator. The sensible heat gained and lost by the fluids surrounding the coils in the condenser and evaporator respectively, would be the exact measure of the latent heat of the refrigerating medium, as abstracted in the condenser and returned in the evaporator. It is, however, necessary to change the physical condition of the gas between the evaporator and condenser, so that it can be liquefied in its passage through the latter vessel. Suppose that the pressures in both evaporator and condenser are the same and constant. In order to ensure condensation and liquefaction in the condenser, its temperature would have to be constantly maintained *below* that of the evaporator, a condition of things which is manifestly impracticable since the evaporator is becoming colder with every repetition of the cycle of operations. This difficulty must therefore be met in another way. If we wish to liquefy any gas, it is necessary to bring its molecules closer together, and this can be accomplished either by *increasing* the pressure or by *decreasing* the temperature of the gas, or both. Now, since it is not in this case practicable to reduce the temperature, the only alternative is to raise the pressure by means of the pump already referred to, which draws the gas from the evaporator and delivers it at an increased pressure into the coils of the condenser. But in order to compress a gas, mechanical work must be performed upon it, and this work re-appears in the form of heat. The temperature of the gas after compression is

therefore considerably higher than it was at the lower pressure on leaving the evaporator. This heat, in addition to the heat imparted in the evaporator, has to be abstracted and carried away by the cooling action of the water of the condenser.

As stated at the beginning of this Lecture, if we convert a unit weight of any liquid into a gas, we require the addition of a definite amount of heat, and to reconvert this gas into a liquid we require the abstraction of the same amount of heat, the amount being constant for any one liquid at a constant pressure and temperature. All gases do not require the same expenditure of energy to raise them to the same pressure, because they vary in what may be called their compressibility, and some gases occupy a smaller volume than others after an equal amount of compression. Carbon dioxide, for example, according to Regnault, only requires about 75 per cent. of the work necessary to produce the same amount of compression, as air or hydrogen. We can, by experiment, readily determine the exact pressure at which liquefaction will take place at any temperature; and knowing this, the machine can be designed of suitable strength to withstand the necessary pressure.

Owing to the difference in the power required to increase the pressure of different gases, it follows that the amount of heat imparted during compression must with some gases be greater than with others. This fact is of great importance in the selection of a suitable gas, and particularly so if cooling water be scarce. But whatever gas be employed, the pressure necessary to liquefy it must always be increased to a greater or less extent as the temperature of the cooling water rises.

Having considered the principles upon which an evaporative refrigerating machine depends for its action, we are now in a position to examine into the actual question of the interchangeability of heat and work. We can moreover at once establish a coefficient of efficiency for any refrigerating material.

Let, L = Latent heat of evaporation of the refrigerating medium in B.T.U.

And, H = Heat imparted during compression in B.T.U.

It follows, from what has been said, that the coefficient of efficiency will be $L \div H$, and, neglecting external losses, wL will represent the heat abstracted in the evaporator, whilst $H + wL$ will equal the heat added to the cooling water of the condenser, where w, is the weight in lbs. of the gas entering the condenser in a given time. It is, of course, impossible that a machine could work under such ideal conditions as we have assumed, since there must always be the effect of the high or

low temperature of surrounding bodies to determine whether there will be a loss or gain of heat in one or other of the parts of the machine. For instance, it is almost certain that the evaporator will be colder than the atmosphere; in that case, no matter how carefully it may be insulated, there will be some conduction of heat and the net quantity of the heat abstracted from the liquid to be cooled can only be $(w\,L - x)$, where x, is the amount of heat derived from outside sources. The amount of heat imparted to the cooling water in the condenser will therefore be $H + w\,L \pm y$; where y, is the heat lost or gained in the condenser due to the difference in temperature between it and its surroundings.

There is still another correction to be made to the above formula. When, as is usual, the evaporator is maintained at a very low temperature, a certain amount of heat must be imparted to it by the refrigerating liquid itself, as it is entering the evaporator in a comparatively warm condition. Thus, supposing there be t degrees difference in temperature between the condenser and evaporator, a unit weight of the refrigerating liquid will as it were import into the evaporator $t\,s$ thermal units; where s is the specific heat of the liquid in question. Therefore, if W be the weight of refrigerating liquid passing into the evaporator in a given time, the heat abstracted in the evaporator will be represented by the expression:—

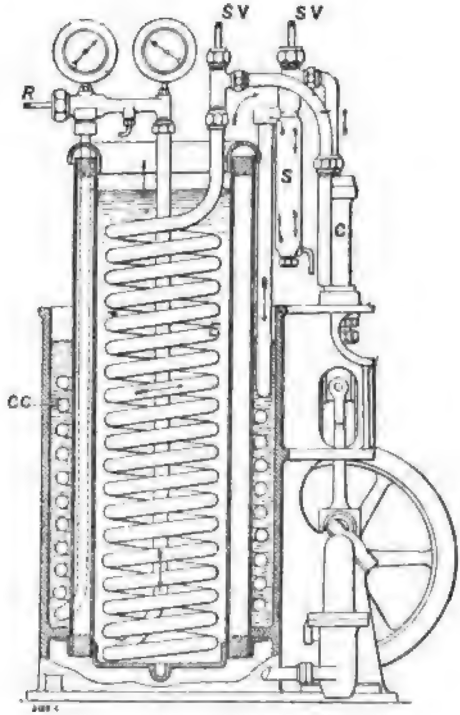
$$w\,L - x - W\,t\,s.$$

Of course, $W\,t\,s$ will not in practice amount to a great deal; but, as Professor Linde has pointed out, it must not be neglected in an exact calculation of the work performed by any refrigerating machine. If there be no leakage, then on the average W will be the same as w.

These formulæ cannot be applied with absolute certainty in practice, owing to the impossibility of making all the necessary corrections due to the gain or loss of heat in the various parts of the machine, and owing to the friction of the gas in constricted passages. But, with care, this gain and loss of heat can be very nearly accounted for in an ordinary machine, as manufactured for commercial purposes and working under the conditions of everyday practice.

Carbon Dioxide Refrigerating Plant.—One method of cooling buildings, &c., on a large scale, is to employ a strong brine obtained by dissolving sodium chloride or common salt in water. This brine is first cooled by passing it through the evaporator of a refrigerating machine, and then circulating it in pipes placed within the chambers which it is desired to cool.

In the accompanying figure all the essential parts are shown of a small refrigerating machine as manufactured by Messrs. J. & E. Hall, of Dartford, for cooling small provision stores, dairies, &c., where the pump may be conveniently driven by a belt. In larger machines, the evaporator is contained in a separate



HALL'S CARBON DIOXIDE REFRIGERATING PLANT.

INDEX TO PARTS.

C	for	Compressor.	G	for	Gauges.
S	,,	Separator.	E	,,	Evaporator.
CC	,,	Condensing Coils.	SV	,,	Stop Valves.
R	,,	Regulator.			

vessel and the compressor is driven by a compound- or triple-expansion engine; but, for the sake of compactness in this case, the evaporator is placed within its condenser, and the intervening space between them is carefully insulated by means of some non-conductor, such as hair felt or slagwool. The coils of

piping which form the condenser and evaporator are welded into continuous lengths and so connected that all joints shall occur in accessible positions. These and all the other gas joints are made by inserting copper rings turned from the solid metal between a pair of flanges or union coupling. This form of joint has been found very satisfactory. The condenser casing, Corliss frame and bearings for the compression pump, &c., are all made of cast iron in one casting. The compressor C is made of special hard bronze in order to ensure freedom from spongy places, while the suction and delivery valves are identical in shape and size so that they may be interchangeable. The compressors for larger machines are bored out of solid steel forgings. This ensures strength together with sound material. A true bore is also provided for the smooth working of the cupleathers with which the pistons are packed. The gland is made gas-tight by means of two U-leathers fitted over the compressor-rod and glycerine is forced between them under a somewhat greater pressure than that in the compressor. Any leakage which takes place is therefore of glycerine—outwards (which can be collected and used over again) and inwards—which both lubricates the interior of the compressor and fills up the clearance spaces, thereby increasing the efficiency of the machine. The superior pressure of glycerine in the gland is obtained by utilising the pressure in the condenser acting through a small intensifier, similar to those in use in hydraulic installations. Any glycerine which passes into the compresser, beyond what is necessary to fill up the clearance spaces is discharged with the gas through the delivery valves. In order to prevent this glycerine passing into the condenser coils, all the gas is delivered into a separator S and caused to impinge against the sides of this vessel. The glycerine adheres to its sides and drains to the bottom from which it may be drawn off from time to time, thus permitting the dry compressed gas to pass away by an opening at the top of the separator to the evaporator E.

One feature of these machines is the safety valve, which is fitted to the gas circuit immediately above the compressor, so that no harm can be done to the machine even if carelessly started with the stop valves closed. It consists of an ordinary spring safety valve, beneath which is a thin copper disc, designed to burst at a certain pressure. This disc can be made perfectly gas-tight, which could not be so easily accomplished by the spring safety valve alone. The latter only comes into play in the event of a rupture of the copper disc.

Anhydrous Ammonia as a Refrigerating Agent.—The most important advantages possessed by anhydrous ammonia as an

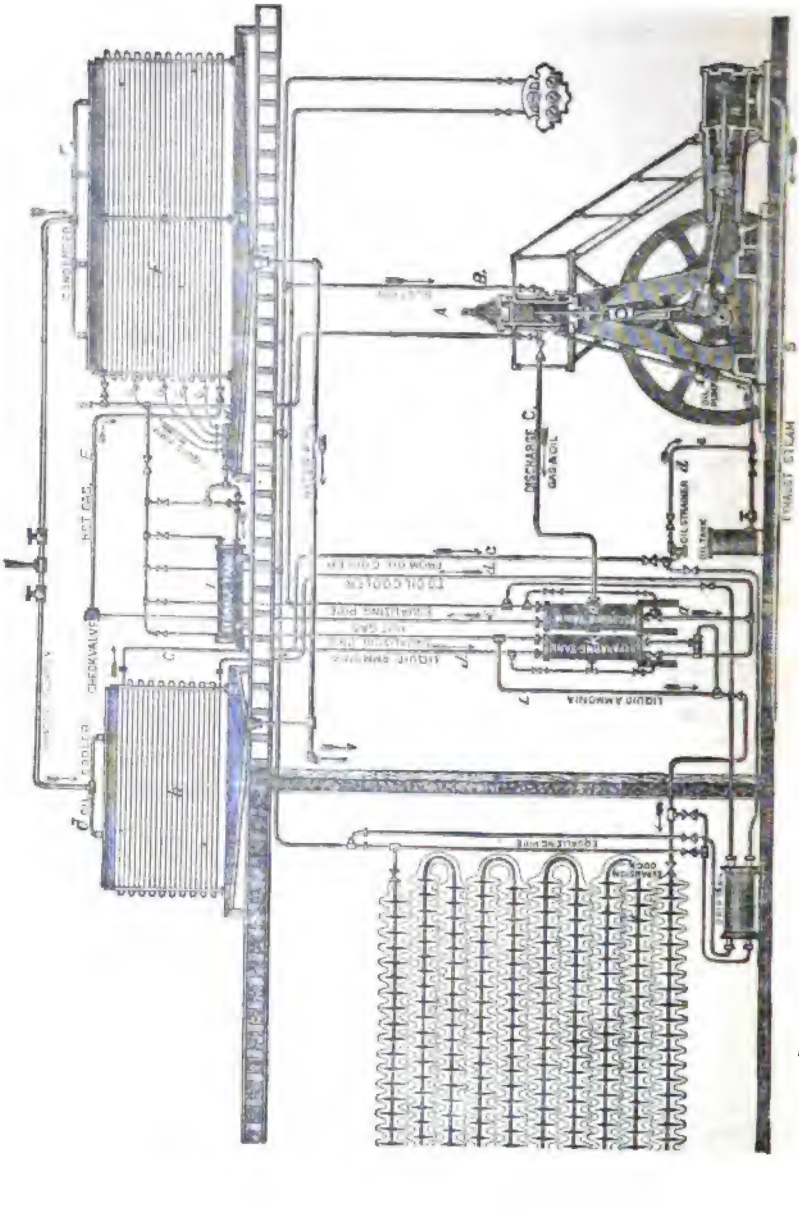
agent for cooling purposes are, its freedom from the danger of explosion, its great latent heat and low pressure of vaporisation.* The latent heat of vaporisation of 1 lb. of carbonic acid at 0° F. is 123 units, and of ammonia 555 units, while the respective pressures in lbs. per square inch, at the same temperature, are 310 lbs. in the case of carbonic acid and only 30 lbs. with ammonia. It follows, therefore, that a carbonic acid plant must be constructed to deal with pressures of about 1,000 lbs. per square inch as against only 150 lbs. or so, in an ammonia machine. The exact pressures in each case are directly proportional to the temperature of the condensing water.

As already stated, this agent is more commonly used than any other, and in the United States of America, where refrigeration is applied to an extent unknown elsewhere, the machine generally employed is on the ammonia compression principle. It is similar to the carbonic acid machine in so far as the complete system consists of (1) a compression, (2) a condensing, and (3) an expansion part; moreover, the cycle of operations is exactly the same.

De La Vergne's Refrigerating Plant.—There are many different kinds of ammonia machines in use, but a general description of one of the best known and most extensively applied—viz., the "De La Vergne" as manufactured by Messrs. L. Sterne & Co., Ltd., of the Crown Iron Works, Glasgow, may be taken as a typical example.

In the following figure, A represents the ammonia compressor driven by a steam engine R. The gas which is returned from the expansion coil N, placed in the cooling chamber, enters the cylinder A by the pipe B, and after being compressed therein it is discharged, through the pipe C into a pressure tank D, together with a certain amount of sealing oil. Here, the oil, being heavier than the ammonia gas, naturally falls to the bottom, and the hot ammonia passes from the top of this tank by a pipe E to the condensers F: where, the cooling action of cold water trickling over the pipes causes the gas to liquefy. It then passes through pipes G, G to a header H, and from thence, to a storage tank I, which is simply a receptacle for holding a reserve supply of liquid ammonia. From this tank

* A liquid with a high latent heat of evaporation need not necessarily be a good refrigerating agent, and *vice-versâ*. What is required is, that its specific heat should be low in proportion to the latent heat of evaporation. Or, we require as great a *difference* as possible between the latent heat of evaporation and the specific heat of the liquid multiplied by the range of temperature in the condenser and refrigerator.

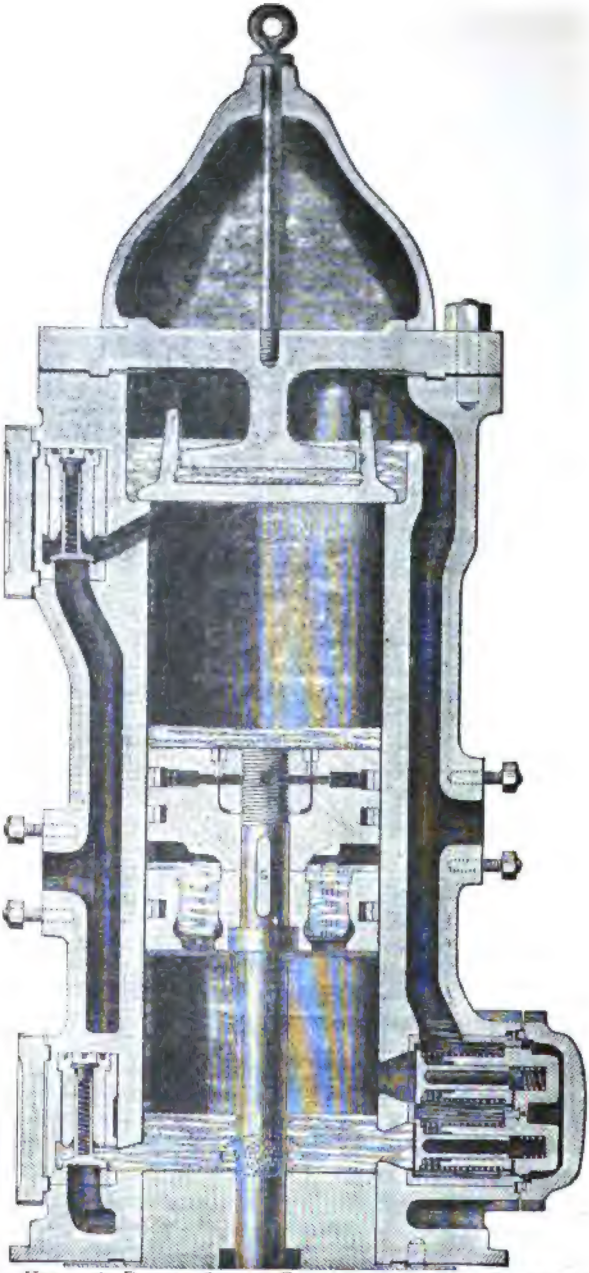


De La Vergne Refrigerating Plant. By L. Sterne & Co., Crown Iron Works, Glasgow.

it is conveyed by a pipe J to a separating vessel K, where any particles of oil that may have been carried over with the liquid are finally separated and the pure liquid ammonia is free to leave it by a pipe L to the expansion coils in the chambers to be cooled. The cock for admitting the ammonia to these coils can be regulated to any degree of minuteness. It thus serves to separate the high pressure from the low pressure part of the apparatus. Hence, the liquid ammonia on passing the expansion cock enters the cooling coils, which are maintained at a low pressure by the pumping action of the compressor. Here, it immediately flashes into gas and by abstracting from its surroundings the heat necessary to cause this change, the temperature of the room is lowered to any desired extent. After having thus done its work in the cooling chamber, the gas is returned to the compressor by a pipe B, to again undergo the same cycle of operations.

The sealing oil passes from the bottoms of the pressure and separating tanks D and K, by the pipes *a* and *d* to the oil cooler *b*; thence, by pipe *c* to the oil strainer *d* and the pipe *e* to the oil pump *f*; by which, it is again circulated through the compressor A.

De La Vergne's Double-Acting Compressor.—The accompanying figure is a section through a "De La Vergne" double-acting compressor, and shows the use of the oil seal. In all ammonia compressors, a certain amount of oil is required for lubricating purposes, and if the compressor be arranged in the ordinary way, the discharge valves at the lower end are placed either on the bottom or at the side, with the result that the oil is discharged *before* the gas. The oil ought, however, to be discharged *after* all the gas is gone; otherwise, re-expansion takes place which would entail a loss of efficiency. In the "De La Vergne" compressor this difficulty has been avoided in the following manner:—At the lower right-hand end of the compressor, two discharge valves are fitted into a side pocket, with the one fair above the other. On the down stroke, either of the valves or both may open until the piston covers the upper one, when only the lower valve is open to the condenser. In the further course of the piston and as soon as the lower valve is also closed, the upper one comes into direct communication with an annular chamber in the piston. This chamber has valves in its bottom side which open into it, as soon as all other inlets on the lower side of the piston are closed. The gas, therefore, first leaves the compressor and then the oil follows, thus permitting no gas to remain in the lower side after the completion of the down stroke. The effect of the oil seal is to make the compressor



De La Vergne's Double-Acting Compressor for his Refrigerator.

work with practically no clearance and thus a maximum of efficiency is obtained. The oil also serves to carry away a considerable amount of the heat of compression and to seal all the valves and stuffing-boxes.

Attention may now be drawn to a few of the details of the above plant. In the first place, it will be noticed that the ammonia condenser is not of the ordinary type where the coils containing the gas are usually submerged in a water tank, but they are of the open or atmospheric type. Here, water is kept constantly trickling over the condenser pipes, and the cooling action is therefore considerably assisted by the evaporation thereof from the surface of the pipes, which enables a maximum of condensation to be effected with a minimum of water supply. It also leaves all the pipes of the condenser open for examination and cleansing. This style of condenser is now coming into extensive use for the condensation of steam in large factories. In the second place, it will be seen that the refrigerating or cooling effect is caused by the direct expansion of the ammonia in pipes placed in the chamber to be cooled. This does away with the unavoidable loss of efficiency due to the use of a supplementary medium such as brine. It, however, necessitates very careful coupling up and jointing all the expansion coils, in order to prevent any leakage of the ammonia gas; more especially, in the case of a large plant where there may be as much as ten or more miles of piping in these cooling coils. In practice, however, these details have been so carefully worked out, that many hundreds of miles of such piping are constantly at work without giving the slightest trouble. Consequently, the old-fashioned method of brine circulation is not now so generally employed except on board ship, where there is a possibility of undue rocking or straining of the pipes and where it is considered advisable to use something that would cause no disagreeable odour in case of a broken pipe or joint.

In applying the refrigerating machine to the manufacture of ice, the simplest method is to place the expansion or cooling coils in a tank filled with brine or other non-congealable liquid, while the water to be frozen is placed in moulds of suitable size, which are then inserted into this brine until frozen. The purpose served by the brine in this case is to convey the heat from the water to the cooling pipes. It is therefore generally kept in slow circulation in order to ensure that the temperature shall be as uniform as possible throughout the tank. If ordinary well water be placed in the moulds, the resulting ice will contain so much air that it will be turned out of a milky white and opaque colour; but if the water whilst in the process of freezing be kept

in slow motion by means of agitators, this air escapes, and a clear glassy ice is the result.

Another method of obtaining clear ice is to use distilled water. This has also the advantage of getting rid of any objectionable matter which might be in solution.

The Linde System of Refrigeration.—This system was first introduced into Germany in the year 1875 by Professor Linde, who was then one of the teaching staff at Munich University. In this country, however, prior to 1888, the principal cold-producing machinery, as manufactured for both land and marine purposes, was the simple cold air machine, in which refrigeration is produced by the compression, cooling when under compression by means of water and subsequent expansion of ordinary atmospheric air. These machines, although simple in construction and giving very good results, possess the disadvantage of requiring a large amount of power to work them in comparison with those employing more efficient refrigerating agents. Consequently, the former method has now very largely given way to one or other of the latter, of which the Linde system is one of the most successful, seeing that over 3,000 of these machines have been constructed up to September, 1897, representing an output, as rated by the capability of producing 69,200 tons of ice every twenty-four hours. In America the largest machine turned out upon any system is rated at 500 tons of ice per twenty-four hours.

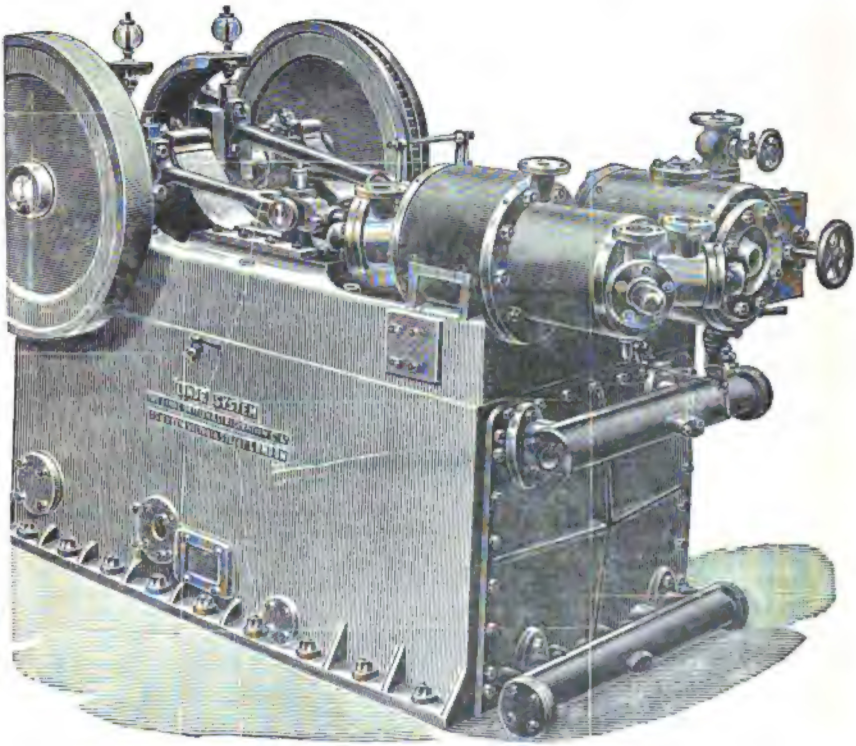
The Linde System of Refrigeration is identical in principle, and only differs in mechanical details from the De La Vergne previously described. It is, therefore, based on the evaporation of liquid anhydrous ammonia and the subsequent liquefaction thereof by means of mechanical compression, together with the cooling of the vapour thus formed, so as to enable it to be used over and over again. As will be seen from the accompanying illustration, the self-contained motive-power plant, as chiefly used on board ship, consists of a horizontal steam engine on the right, with a horizontal duplex compressor pump to the left, and an ammonia condenser in the sole plate. As far as the compressor is concerned, the chief differences between the Linde and the De La Vergne systems are :—

(1) That in the former a horizontal compressor is used instead of a vertical one in the latter case.

(2) That a special oil (not susceptible to change at any temperature attainable by the machine, which does not contain any acid or other deleterious matter, and which does not saponify when brought into contact with ammonia is used solely for lubricating purposes. Whereas, the oil used in the De La

Vergne system serves not only as a lubricant to the working parts, but also to partly carry away the heat of compression, and, further, to fill up the clearance spaces, as well as to seal the valves, glands, &c., so as to prevent the escape and consequent inconvenient smell of ammonia.

(3) In the Linde system a small quantity of liquid ammonia is introduced into the compressor during each suction stroke for



SHIP REFRIGERATING MACHINERY ON THE LINDE SYSTEM.
BY THE LINDE BRITISH REFRIGERATING COMPANY.

the purpose of cooling the vapour of ammonia. This liquid ammonia evaporates during compression, and thus the heat due to compression, which would otherwise appear as sensible heat, is thereby absorbed and rendered latent in producing the change in the physical state of the liquid. The curve of compression is thus kept down as nearly as may be to the isothermal line, and

the power required for compression is to this extent correspondingly reduced.

Apparatus for Transmitting the Cold Produced to the Chambers requiring Refrigeration.—The following general principles are adopted for transferring the cold generated by the refrigerating machinery to the chambers or rooms requiring to be cooled.

First Method.—An uncongealable solution of salt (chloride of sodium) in water is reduced by the refrigerating machine to a low temperature, and this liquor acts as transmitter of cold in one of the following methods:—

(*a*) The cold brine is constantly circulated from the brine refrigerator through pipes placed in the refrigerated chambers, and returned to the brine cooler. The result is that not only is heat abstracted from the air of the refrigerated rooms, but also a large degree of the moisture which may be present in them. This moisture is condensed on the exterior of the brine pipe systems either in the form of condensed water or hoar-frost. Suitable drip trays are provided, in order to prevent this moisture from falling upon the contents of the rooms. The circulation of air with this system is a moderate one, being produced merely by the differences between the temperatures prevailing near the brine-pipes and those in the lower parts of the rooms.

(*b*) The brine is cooled in a shallow rectangular open tank containing the evaporator coils. On the tank is mounted a number of slowly revolving transverse shafts, and on each shaft is fixed a number of parallel discs, partly immersed in the brine, the entire apparatus being placed in an insulated passage through which an air current is continually passed by a fan, in a direction parallel to the revolving discs. It will be seen, that as the discs revolve and are kept covered by a film of the refrigerated brine, the air passing between the disc-spaces becomes cooled, and produces a low temperature in any chamber or room into which it may be conducted through properly arranged air-trunks. As a rule the air is always taken back from the cold rooms, passed over the discs and returned to the cold rooms, and any required amount of fresh air is introduced by means of adjustable openings in the air-trunks, communicating with the outer atmosphere. In this instance, also, moisture may be removed from the refrigerated rooms and deposited in the brine contained in the trough. No accumulation of frost can take place, and the refrigerated surfaces are always perfectly active. The circumstance of all moisture being deposited in the brine necessitates either a periodical loss of the same or its re-concentration. The fan produces a very effective air circulation within the rooms to

be cooled. This in most cases is extremely desirable, and, as will be readily understood, produces the most beneficial results.

Second Method.—Instead of using an uncongealable liquid as a bearer of cold, the refrigerator coils (in which the vaporisation of the liquid anhydrous ammonia takes place) are sometimes constructed with extra large surfaces, and placed either in the upper part of the rooms to be cooled, or in a separate chamber. In the latter case, a fan constantly circulates the air between this chamber and the refrigerated rooms. This is the system generally adopted on board ships, and has been found to be in all respects most satisfactory. In cases where the air temperature is not sufficiently high to cause a complete removal of the snow deposited on the ammonia-coils, the snow is thawed by the ammonia vapours themselves, the evaporator-coils being for the time used as a condenser. Occasionally the snow is thawed by a current of hot air taken from the outside.

Although all of these methods have been applied on an extensive scale, the system most strongly recommended in cases where its application is possible is the combination of revolving discs immersed in brine. There are no brine or ammonia pipes in the rooms; whilst the rapid air circulation by the fan is easily managed, and has been found in most cases to be requisite for obtaining a satisfactory result as to purity, dryness, and equable temperature in all the rooms.

Where circumstances require the refrigerated rooms to be at a distance from the refrigerating machine, it is generally most convenient to place bundles of brine pipes in each room; but even in such a case, in the event of a small amount of motive power being available close to such rooms, the system of revolving discs and fans can be readily applied, the brine being cooled in a refrigerator near the compressor, and conveyed to and from the disc tanks through insulated pipes.

A large beef-chilling plant on the Linde system was erected in the beginning of 1890 at the Woodside lairage of the Mersey Dock and Harbour Board. It is capable of chilling 660 carcases of beef, each weighing about 9 cwts., from 90 degrees Fahr. to 33 degrees in 17 hours. It consists of a horizontal compound tandem jet-condensing steam engine, which drives a double-acting Linde compressor at the rate of about 65 revolutions per minute, when supplied with steam at 120 lbs. pressure from a marine type boiler. The air-cooling apparatus consists of two disc tanks, placed above the chill rooms, at one end. Each disc system has its own fan, which draws the air from the top of each of the chill rooms, passes it over the discs, and drives it into the rooms at the opposite end to that from which it is withdrawn.

The ammonia condenser is placed in the compressor room, and is supplied with cooling water by a pump which takes its supply from a well, fed with the drainage water from the Mersey Tunnel. After passing through the ammonia condenser the water is used in the condenser of the steam engine. There are six chill rooms, each about 55 ft. long by 14 ft. wide, and about 13 ft. high. The walls of the rooms are built of brick, with air spaces. The floors are cement, and the ceilings are timber, covered with a layer of fine ashes. The air-cooling apparatus is contained in an insulated casing, which is so arranged as to cause the air to come in contact with the cooled surfaces of the discs.

The following is a list of some of the books and papers on Hydraulics and Hydraulic Machinery:—

Hydraulic Machinery, by R. Gordon Blaine. (E. & F. N. Spon, Ltd., London, 1897.)

Recent Hydraulic Experiments. Paper by Major A. Cunningham. Proc. Inst. C.E., vol. lxxi., p. 1.

Hydraulic Appliances at the Forth Bridge Works. Paper by E. W. Moir. Proc. Inst. C.E., vol. xci., p. 402.

The 160-Ton Hydraulic Crane at Malta Dockyard Extension Works. Paper by C. & C. H. Colson. Proc. Inst. C.E., vol. cxiv., p. 289.

On Machine Tools and other Labour-Saving Appliances Worked by Hydraulic Pressure. Paper by R. H. Tweddell. Proc. Inst. C.E., vol. xxiii., p. 64.

The Barry Dock Works, including the Hydraulic Machinery and the Mode of Tipping Coal. Paper by J. Robinson. Proc. Inst., C.E., vol. ci., p. 129.

The Distribution of Hydraulic Power in London. Paper by E. B. Ellington. Proc. Inst. C.E., vol. xciv., p. 1.

Hydraulic-Power Supply in London. Paper by E. B. Ellington. Proc. Inst. C.E., vol. cxv., p. 220.

Forging by Hydraulic Pressure. Paper by R. H. Tweddell. Proc. Inst. C.E., vol. cxvii., p. 1.

Hydraulic-Power Supply in Towns. (Glasgow, Manchester, Buenos Ayres, &c.) By Edward B. Ellington, M. Inst. M.E. (Proc. Inst. Mechanical Engineers, Glasgow Meeting, 1895).

Water-Pressure Engines for Mining Purposes. Paper by H. Davey, M. Inst. C.E. Proc. Inst. M.E., 1880, p. 245.

Hydraulic Flanging of Steel Plates. Paper by Easton & Anderson. Proc. Inst. M.E., 1882, p. 518.

Portable Hydraulic Drilling Machine. Paper by M. Berrier-Fontaine. Proc. Inst. M.E., 1887, p. 72.

Lifts in the Eiffel Tower. Paper by A. Ansaloni. Proc. Inst. M.E., 1889, p. 350.

Hydraulic Packing Presses. Paper by C. Hopkinson. Proc. Inst. C.E., vol. xcix., p. 275.

The Treatment of Steel by Hydraulic Pressure and the Plant Employed for the purpose. Paper by W. H. Greenwood. Proc. Inst. C.E., vol. xcviii., p. 83.

Hydraulic Propulsion. Paper by S. W. Barnaby. Proc. Inst. C.E., vol. xxvii., p. 83.

A Hydraulic Pumping Engine. Paper by H. D. Pearsall. Proc. Inst. C.E., vol. cvi., p 292.

Hydraulic Work in the Irawadi Delta. Paper by R. Gordon. Proc. Inst. C.E., vol cxiii., p. 276.

Centrifugal Pumps, Turbines, and Water Motors, by C. H. Innes, M.A. (The Technical Publishing Co., Ltd., Manchester.)

Lifting and Hauling Appliances in Portsmouth Dockyard. Paper by J. T. Corner. Proc. Inst. M.E., 1892, p. 295.

Hydraulic Power and Hydraulic Machinery, by Henry Robinson. (Chas. Griffin & Co., London, 1896.)

Hydraulic Motors, Turbines, and Pressure Engines, by G. R. Bodmer. (Whittaker & Co., London, 1889.)

A Treatise on Hydraulics, by M. Merriman. (John Wiley & Sons, New York ; Chapman & Hall, London, 8th Edition, 1903.)

Hydrostatics and Elementary Hydrokinetics, by G. M. Minchin (Oxford Press, Oxford, 1892.)

Hydraulic Machinery for Glasgow Harbour Tunnel, as Constructed and Erected by the Otis Elevator Co., New Yonkers, New York. See *Engineering,* May and June, 1895.

Paper on *Refrigerating Apparatus,* by Prof. Carl Linde, of Munich. *Journal of the Society of Arts,* 9th March, 1894.

Paper on *Experiments on a Two-Stage Air Compressor,* by John Goodman, Wh.Sc. Proc. Inst. C.E., vol. cxxviii., and also *The Practical Engineer,* 30th July, 1897.

Compend of Mechanical Refrigeration, by J. E. Siebel. (H. S. Rich & Co., Chicago, 1895.) In this book there are several important tables and other valuable information regarding refrigerators.

Adaptation of Hydraulic Power in the Manufacture of Iron and Steel. Paper by James L. Biggart. *Journal of the West of Scotland Iron and Steel Institute,* 1893.

Application of a System of Combined Steam and Hydraulic Machinery to the Loading, Discharging, and Steering of Steamships. Paper by A. Betts Brown, F.R.S.E. Proc. Inst. Nav. Arch., 1890.

The Theory and Practice of Mechanical Refrigeration. Paper by T. R. Murray, Wh. Sch. Proc. Inst. Eng. and Shipbuilders Scot., 1897-98.

The Mechanical Production of Cold. Howard Lecture by Prof. J. A. Ewing, F.R.S. Delivered Feby. 4, 1897, before The Society of Arts, London. See their *Journal* or *The Practical Engineer* for Dec., 1897, and Jan., 1898.

Pumping Machinery, by Henry Davey, M.Inst.C.E. (Charles Griffin & Co., London, 1903.)

Pneumatic Tools.—The pneumatic tools of the present day, form most useful appliances in up-to-date engineering shops. They are adaptable to many kinds of work of which the following are a few examples:—

Classes of Pneumatic Hammers.—These may be divided into two classes—(*a*) the *valveless hammer*, and (*b*) the *valve hammer*.

(*a*) The *valveless hammer*, as its name implies, has no separate valve, but its striking piston or plunger does duty as a valve, and regulates the proper admission of air to alternate ends of the working cylinder.

(*b*) The *valve hammer* has a reciprocating valve placed either horizontally or vertically, which controls the admission of the air to the working cylinder and also the exhaust therefrom. The valve hammer is the type in general use throughout engineering shops and shipyards, since the valveless hammer is too rapid and light in its blow. The estimated number of strokes per minute of the "valve hammer" is from 700 to 2,000; whereas, in the case of the "valveless hammer," they may vary from 10,000 to 18,000 strokes per minute.

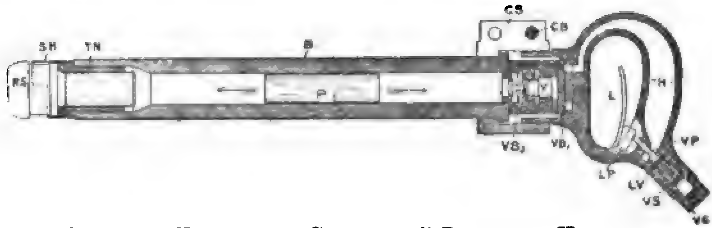
Uses and Capabilities of Pneumatic Hammers.—When used for caulking, they will perform the work of three to four skilled hand caulkers. When used in the foundry for scarfing or dressing castings, they will do the work of from four to five men. In general chipping, they will do the work of at least three men. They are also used in stone dressing, lettering, and carving, with considerable saving in time and cost.

The Cleveland Hammer.*—A further development of pneumatic chipping and caulking hammers, is found in the hand pneumatic riveting hammer. We have here chosen to illustrate and describe the "Cleveland Hammer," which is very strong and at the same time simple in construction, for it has only one solid steel valve.

Action of the Cleveland Hammer.—Referring to the diagram, with index to parts, of this hammer, it will be seen, that it consists of the handle, H, which is connected to the barrel, B, by the coupling sleeve, C S. Inside this barrel is placed the plunger, P, whilst between the end of the barrel and the handle is secured a valve block button, $V\,B_1$, and valve block, $V\,B_2$, in which the valve, V, works. Compressed air is admitted to the interior of the hammer by the action of the trigger or latch, L, upon the latch valve pin, V P, which in turn acts upon the latch valve, L V, and valve spring, V S. The compressed air

* We are indebted to Messrs. John Turnbull, Jun., & Sons, Hydraulic and Pneumatic Engineers, Glasgow, for the original figures from which the illustrations in this article were produced.

when admitted, passes through a hollow chamber in the handle, H, and acts upon the valve, V, thus rapidly moving the plunger, P, backwards and forwards. A rivet set, R S, is placed in the end of the hammer or tool nose, T N, and held by a spring



SECTIONAL VIEW OF A "CLEVELAND" PNEUMATIC HAMMER.

INDEX TO PARTS.

H	for	Handle.	C S	for	Coupling sleeve.
V P	,,	Latch valve pin.	B	,,	Barrel.
V C	,,	Latch valve casing.	P	,,	Plunger.
V S	,,	Latch valve spring.	T N	,,	Tool nose.
L V	,,	Latch valve.	V B_2	,,	Valve block.
L P	,,	Latch pin.	S H	,,	Spring holder-on.
L	,,	Latch.	R S	,,	Rivet set.
V B_1	,,	Valve block button.	C B	,,	Cap bolt.
V	,,	Valve.			

holder-on, S H. The rivet set, R S, must be placed hard on the rivet, before the compressed air is admitted, otherwise the plunger and rivet set would be blown out of the hammer.

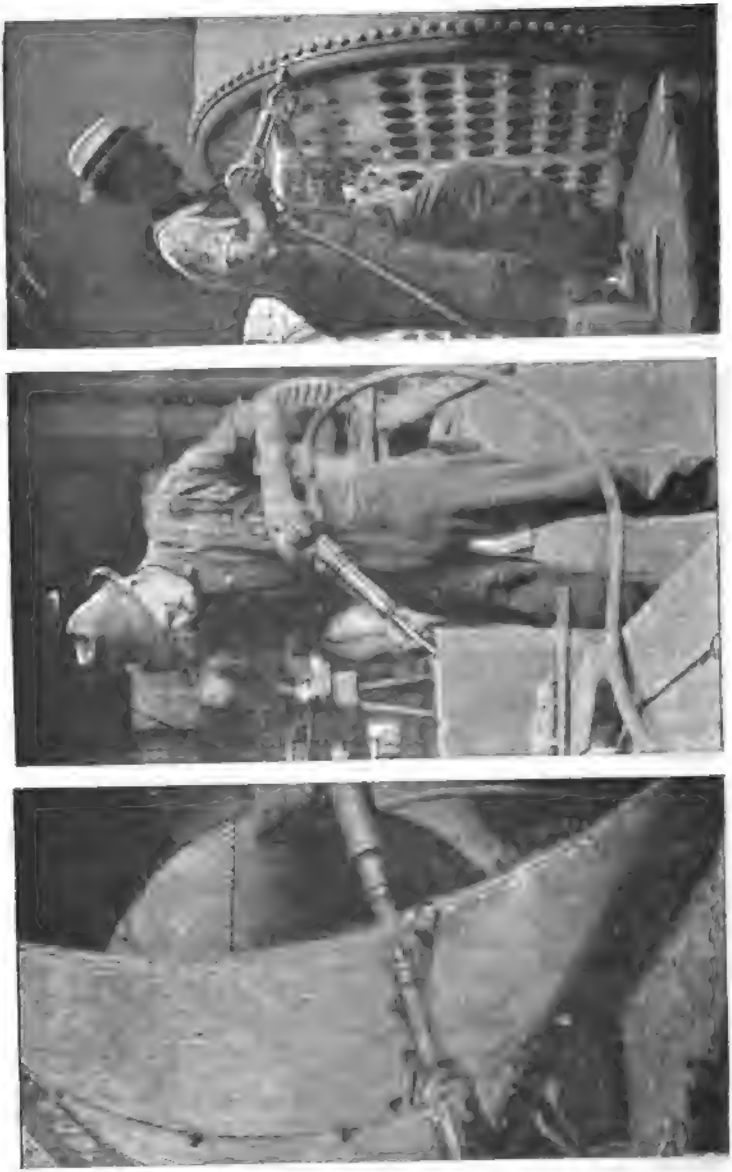
Capabilities of the Cleveland Hammer.—With one of these riveting hammers a man can fix and finish 100 of ½-inch rivets, 90 of ⅝-inch, 80 of ¾-inch, and 70 of ⅞-inch per hour.

Sizes and Speeds of Cleveland Hammers.—These hammers are made in thirteen sizes to take in all classes of riveting work. The strokes of the various sizes are as follows:—

5-inch stroke,	. . . . . .	800 strokes per minute.
6 ,, ,,	. . . .	775 ,, ,,
7 ,, ,,	. . . .	750 ,, ,,
8 ,, ,,	. . . .	725 ,, ,,
9 ,, ,,	. . . .	700 ,, ,,

This allows for a variety of work ranging from ¼-inch cold rivets up to and including 1¼-inch hot rivets.

Pneumatic Holder-up.—When riveting with pneumatic hammers, it is always best to use a pneumatic holder-up, so that the head of the rivet shall be kept close up to the plate. Owing to the rapid succession of blows given by these hammers, the ordinary hand dolly-bar is not so suitable.



RIVETING, CHIPPING, AND CAULKING WITH THE CLEVELAND PNEUMATIC HAMMER.

The pneumatic holder-up consists of a piston placed inside a cylinder, into which compressed air is admitted. The end of the piston being kept up to the rivet head by the air pressure in its cylinder.

Pneumatic Plant.—The following plant would be required for working a number of these hammers :—

1. Air compressor, either belt, steam, or electrically driven.

2. An air receiver, to hold not less than the output of the air compressor for one minute. This receiver forms an important part of the air compressing plant. It receives the air in pulsations from the air compressor, until a pressure of about 100 lbs. per square inch is obtained, and then delivers it to the tools in a steady, even flow. It also receives any moisture which may be in the compressed air, and provision must be made in the receiver for drawing off this moisture.

3. Lines of wrought-iron piping, for conveying the compressed air to convenient parts of the workshop. These pipes should be fitted with air strainers, in order to clear the compressed air, as far as possible, from grit, &c.

4. Lengths of flexible tubing or rubber-lined hose pipes, to form the connection between the line of wrought-iron air pipes and the pneumatic tools.

5. Pneumatic tools to perform the work desired.

Advantages of Pneumatic Plant for Riveting, Chipping, and Caulking.—1. This system is perhaps the best of all when applied to suitable machines, since compressed air, on account of its comparatively low pressure and weight, is more easily distributed than water.

2. There is not much risk from freezing, which is an advantage where plant, pipes, and tools are of necessity exposed to cold weather. In cases, however, where the compressed air had been allowed to contain a certain percentage of moisture in it, there has been a little trouble in the winter time, due to ice forming in the pipes.

3. The cost of a complete pneumatic plant, is said to be not quite so much as that for a corresponding hydraulic one.

4. As previously mentioned in this article, much more and quite as good work can be done, than by ordinary hand riveting, chipping, and caulking.

Lecture XXXVII.—Questions.

1. Explain the fundamental principles upon which a refrigerating machine works. Note specially what becomes of the different quantities of heat generated and absorbed.

2. Sketch the essential parts of a refrigerator, and describe its action.

3. What are the advantages which a vapour possesses over a permanent gas, such as air, for refrigerative purposes?

4. What are the requirements of an economical medium for use in a refrigerator?

5. Sketch and explain the plant required for producing cold by means of carbon dioxide.

6. Explain the reasons that have led to the adoption of anhydrous ammonia in most modern refrigerators, and mention some of the properties of this vapour.

7. Sketch and describe any well-known arrangement for refrigerating, using anhydrous ammonia.

8. Explain and illustrate some of the ways of communicating cold to a chamber from a refrigerator.

9. Sketch a longitudinal section of a pneumatic hammer and describe concisely its construction, action, and uses. State the advantages which are claimed for this hammer in riveting, chipping, and caulking work. Compare the work done by it with ordinary hand riveting, chipping, and caulking.

APPENDICES TO VOL. II.

APPENDIX A.

Tables of Constants, Logarithms, Antilogarithms and Functions of Angles.

APPENDIX B.

1. Board of Education Advanced and Honours Part I. Examination Papers, with the General Instructions and Regulations for May, 1903.
2. Miscellaneous Science and Art Questions, 1896 to 1898.

APPENDIX C.

1. City and Guilds of London Institute—Honours Grade Examination Paper in Mechanical Engineering, with Instructions for the Technological Examinations, 1903.
2. Miscellaneous Questions from the Board of Education, and City and Guilds of London Institute Examinations, 1899 to 1902.

APPENDIX D.

1. Extracts from the present and the revised Rules with Syllabus of the Examinations for The Election of Associate Members into The Institution of Civil Engineers.
2. Miscellaneous Questions from The Institution of Civil Engineers' Examinations, 1897 to 1903.

APPENDIX A.

EXAMINATION TABLES.

USEFUL CONSTANTS.

1 Inch = 25·4 millimetres.

1 Gallon = ·1605 cubic foot = 10 lbs. of water at 62° F.

1 Knot = 6080 feet per hour.

Weight of 1 lb. in London = 445,000 dynes.

One pound avoirdupois = 7000 grains = 453·6 grammes.

1 Cubic foot of water weighs 62·3 lbs.

1 Cubic foot of air at 0° C. and 1 atmosphere, weighs ·0807 lb.

1 Cubic foot of Hydrogen at 0° C. and 1 atmosphere, weighs ·00557 lb.

1 Foot-pound = $1·3562 \times 10^7$ ergs.

1 Horse-power-hour = 33000 × 60 foot-pounds.

1 Electrical unit = 1000 watt-hours.

Joule's Equivalent to suit Regnault's H, is $\begin{cases} 774 \text{ ft.-lbs.} = 1 \text{ Fah. unit.} \\ 1393 \text{ ft.-lbs.} = 1 \text{ Cent. ,,} \end{cases}$

1 Horse-power = 33000 foot-pounds per minute = 746 watts.

Volts × amperes = watts.

1 Atmosphere = 14·7 lb. per square inch = 2116 lbs. per square foot = 760 m.m. of mercury = 10^6 dynes per sq. cm. nearly.

A Column of water 2·3 feet high corresponds to a pressure of 1 lb. per square inch.

Absolute temp., t = $\theta°$ C. + 273°·7.

Regnault's H = 606·5 + ·305 $\theta°$ C = 1082 + ·305 $\theta°$ F.

$p\,u^{1.0646} = 479$

$$\log_{10} p = 6·1007 - \frac{B}{t} - \frac{C}{t^2}$$

where $\log_{10} B = 3·1812$, $\log_{10} C = 5·0871$,

p is in pounds per square inch, t is absolute temperature Centigrade, u is the volume in cubic feet per pound of steam.

$$\pi = 3·1416 = \frac{22}{7} = \frac{355}{113} = \sqrt{3} - \sqrt{2}.$$

One radian = 57·3 degrees.

To convert common into Napierian logarithms, multiply by 2·3026.

The base of the Napierian logarithms is ϵ = 2·7183.

The value of g at London = 32·82 feet per sec. per sec.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
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·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
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·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
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·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
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·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
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·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
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·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

Angle.	Radians.	Sine.	Tangent.	Co-tangent.	Cosine		
0°	0	·0	0	∞	1	1·5708	90°
1	·0175	·0175	·0175	57·2900	·9998	1·5533	89
2	·0349	·0349	·0349	28·6363	·9994	1·5359	88
3	·0524	·0523	·0524	19·0811	·9986	1·5184	87
4	·0698	·0698	·0699	14·3006	·9976	1·5010	86
5	·0873	·0872	·0875	11·4301	·9962	1·4835	85
6	·1047	·1045	·1051	9·5144	·9945	1·4661	84
7	·1222	·1219	·1228	8·1443	·9925	1·4486	83
8	·1396	·1392	·1405	7·1154	·9903	1·4312	82
9	·1571	·1564	·1584	6·3138	·9877	1·4137	81
10	·1745	·1736	·1763	5·6713	·9848	1·3963	80
11	·1920	·1908	·1944	5·1446	·9816	1·3788	79
12	·2094	·2079	·2126	4·7046	·9781	1·3614	78
13	·2269	·2250	·2309	4·3315	·9744	1·3439	77
14	·2443	·2419	·2493	4·0108	·9703	1·3265	76
15	·2618	·2588	·2679	3·7321	·9659	1·3090	75
16	·2793	·2756	·2867	3·4874	·9613	1·2915	74
17	·2967	·2924	·3057	3·2709	·9563	1·2741	73
18	·3142	·3090	·3249	3·0777	·9511	1·2566	72
19	·3316	·3256	·3443	2·9042	·9455	1·2392	71
20	·3491	·3420	·3640	2·7475	·9397	1·2217	70
21	·3665	·3584	·3839	2·6051	·9336	1·2043	69
22	·3840	·3746	·4040	2·4751	·9272	1·1868	68
23	·4014	·3907	·4245	2·3559	·9205	1·1694	67
24	·4189	·4067	·4452	2·2460	·9135	1·1519	66
25	·4363	·4226	·4663	2·1445	·9063	1·1345	65
26	·4538	·4384	·4877	2·0503	·8988	1·1170	64
27	·4712	·4540	·5095	1·9626	·8910	1·0996	63
28	·4887	·4695	·5317	1·8807	·8830	1·0821	62
29	·5061	·4848	·5543	1·8040	·8746	1·0647	61
30	·5236	·5000	·5774	1·7321	·8660	1·0472	60
31	·5411	·5150	·6009	1·6643	·8572	1·0297	59
32	·5585	·5299	·6249	1·6003	·8480	1·0123	58
33	·5760	·5446	·6494	1·5399	·8387	·9948	57
34	·5934	·5592	·6745	1·4826	·8290	·9774	56
35	·6109	·5736	·7002	1·4281	·8192	·9599	55
36	·6283	·5878	·7265	1·3764	·8090	·9425	54
37	·6458	·6018	·7536	1·3270	·7986	·9250	53
38	·6632	·6157	·7813	1·2799	·7880	·9076	52
39	·6807	·6293	·8098	1·2349	·7771	·8901	51
40	·6981	·6428	·8391	1·1918	·7660	·8727	50
41	·7156	·6561	·8693	1·1504	·7547	·8552	49
42	·7330	·6691	·9004	1·1106	·7431	·8378	48
43	·7505	·6820	·9325	1·0724	·7314	·8203	47
44	·7679	·6947	·9657	1·0355	·7193	·8029	46
45	·7854	·7071	1·0000	1·0000	·7071	·7854	45
		Cosine	Co-tangent	Tangent	Sine	Radians	Angle

APPENDIX B.

1903 MAY EXAMINATION ON SUBJECT VIIA. APPLIED MECHANICS.

BY THE BOARD OF EDUCATION SECONDARY BRANCH, SOUTH KENSINGTON, LONDON.

GENERAL INSTRUCTIONS.

If the rules are not attended to, your paper will be cancelled.

Immediately before the Examination commences, the following REGULATIONS *are* TO BE READ TO THE CANDIDATES.

Before commencing your work, you are required to fill up the numbered slip which is attached to the blank examination paper.

You may not have with you any books, notes, or scribbling paper.

You are not allowed to write or make any marks upon your paper of questions, or to take it away before the close of the examination.

You must not, under any circumstances whatever, speak to or communicate with one another, and no explanation of the subject of examination may be asked or given.

You must remain seated until your papers have been collected, and then quietly leave the examination room. None of you will be permitted to leave before the expiration of one hour from the commencement of the examination, and no one can be re-admitted after having once left the room.

Your papers, unless previously given up, will all be collected at 10 o'clock.

If any of you break any of these rules, or use any unfair means, you will be expelled, and your paper cancelled

Before commencing your work, you must carefully read the following instructions:—

Candidates who have applied for examination in the Elementary Stage must confine themselves to that stage. Candidates who have not applied to take the Elementary Stage may take the Advanced Stage, or Honours (Part I.), or, if eligible, Honours (Part II.), but they must confine themselves to one of them.

Put the number of the question before your answer.

You are to confine your answers *strictly* to the questions proposed.

Such details of your calculations should be given as will show the methods employed in obtaining arithmetical results.

A table of logarithms and functions of angles and useful constants and formulæ is supplied to each candidate. (See end of Appendix to this Book.)

The examination in this subject lasts for three hours.

ADVANCED STAGE.*

Instructions.

Read the General Instructions.

You may only answer *eight* questions, two of which must be Nos. 21 and 22.

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21. Describe, with sketches, only *one* of the following:—(*a*) The mechanism of a Radial Drilling Machine, how the power is transmitted to the drill, and how an automatic feed is given to it. (*b*) The mechanism of a machine for grinding cylindrical surfaces to a truly circular form. (*c*) Any pneumatic hand tool. (B. of E. Adv., 1903.)

22. Answer only *one* of the following:— (*a*) Describe an experiment by which you could determine E, Young's Modulus of Elasticity, by stretching a steel wire. (*b*) Describe an experiment to measure how the kinetic energy of a flywheel depends upon its speed. (B. of E. Adv., 1903.)

23. In a crane an effort of 122 lbs. just raises a load of 3,265 lbs. What is the mechanical advantage? If the efficiency be 60 per cent. what is the velocity ratio? Would this crane overhaul? (B. of E. Adv., 1903.)

24. A tramcar, weighing 15 tons, suddenly had the electric current cut off. At that instant its velocity was 16 miles per hour. Reckoning time from that instant, the following velocities, V, and times, t, were noted:—

V—Miles per hour	16	14	12	10
t—Seconds	0	9·3	21	35

Calculate the average value of the retarding force, and find the average value of the velocity from $t = 0$ to $t = 35$. Also find the distance travelled between these times. (B. of E. Adv., 1903.)

25. A projectile has Kinetic Energy = 1,670,000 foot-pounds at a velocity of 3,000 feet per second. Later on its velocity is only 2,000 feet per second; how much Kinetic Energy has it lost? What is the cause of this loss of energy? Calculate the kinetic energy of rotation of the projectile if its weight is 12 lbs., and its radius of gyration is 0·75 inch, and its speed of rotation is 500 revolutions per second. (B. of E. Adv., 1903.)

26. If a shaft 4 inches in diameter will safely withstand a torque of 120,000 pound-inches, what torque would a 9-inch shaft take? What H.P. would the former shaft transmit at 200 revolutions per minute, and what would the latter transmit at 50 revolutions per minute? What do you mean by shear stress in a shaft? (B. of E. Adv., 1903.)

27. Water at a pressure of 700 lbs. per square inch is supplied to a hydraulic crane, and 11 cubic feet are used in lifting 15 tons through a height of 18 feet. How much energy has been given to the crane? How much energy has been wasted? (B. of E. Adv., 1903.)

28. The rim of an inward flow turbine moves at a speed of 30 feet per second, and the vanes are there at right angles to the rim. Water enters the rim with a radial velocity of 5 feet per second. If the water is to enter without shock, what must be the angle between the rim and the

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* Students should refer to my *Elementary Manual*, as well as the other Volume of this *Text-Book on Applied Mechanics*, before answering some of these questions.

guide blades? Find the weight of water entering per second if the circumferential area of all the openings of the rim is 2·4 square feet.

(B. of E. Adv., 1903.)

29. What are the functions of the top and bottom booms, and of the diagonal pieces of a railway girder? Why are the booms usually larger in section towards the middle of the girder, and the diagonal pieces larger towards the ends of the girder? (B. of E. Adv., 1903.)

30. A weight of 10 lbs. is hung from a spring, and thereby causes the spring to elongate to the extent of 0·42 feet. If the weight is made to oscillate vertically, find the time of a complete vibration. (Neglect the mass of the spring itself.) (B. of E. Adv., 1903.)

31. A heavy chain is supported by its ends, A and B, which are 12 feet above the lowest part of the chain. The horizontal distance between A and B is 66 feet, and the weight of the chain is 20 lbs. per foot of its horizontal projection. Draw out to scale (10 feet to the inch) the shape of the chain, and find the force in the chain at the lowest point. What is the maximum force in the chain? (B. of E. Adv., 1903.)

32. Electric current is supplied to a certain motor plant at 220 volts, and 150 amperes are taken. What H.P. does this represent? How much would it cost if used for an average of 6·5 hours per day for a whole year of 313 days? The power is supplied at 2·24d. per H.P.-hour (*i.e.*, 3d. per B.T.U.). (B. of E. Adv., 1903.)

HONOURS—PART I.*

INSTRUCTIONS.

Read the General Instructions.

You may only answer *eight* questions, two of which must be Nos. 41 and 42.

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41. Describe, with sketches, only *one* of the following:—(*a*) The mechanism of a radial drilling machine, how the power is transmitted to the drill, and how an automatic feed is given to it. (*b*) The mechanism of a machine for grinding cylindrical surfaces to a truly circular form. Describe how such a machine is used to grind up the commutator of a direct current generator. (*c*) Any pneumatic hand tool.

(B. of E. H., Part I., 1903.) (*See Index, Vol. II.*)

42. Answer only *one* of the following:—(*a*) Describe an experiment by which you could determine E, Young's modulus of elasticity, by bending an iron bar. What shape of bar would you use, and why? How would you attach the load, and how support the beam? How would you ascertain the deflection of the beam? (*b*) Describe an experiment to measure how the kinetic energy of a flywheel depends upon its speed. If the time the flywheel took to come to rest were measured, how could you calculate the average H.P. lost in friction? (B. of E. H., Part I., 1903.)

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* Students should refer to my *Elementary Manual*, as well as the other volume of this *Text-Book on Applied Mechanics*, b fore answering some of these questions

43. In a crane an effort of 122 lbs. just raises a load of 3,200 lbs. What is the mechanical advantage? If the efficiency be 60 per cent. what is the velocity ratio? Would this crane overhaul? If the efficiency had been 40 per cent. would the crane overhaul? Give clear reasons for your answer.
(B. of E. H., Part I., 1903.)

44. A tramcar, weighing 15 tons, suddenly has the electric current cut off. At that instant the speed of the car was 16 miles per hour. Reckoning time from that instant, the following velocities, V, and times, t, were noted:—

V—Miles per hour	16	14	12	10
t—Seconds	0	9·3	21	35

Calculate the average value of the retarding force, and find the average velocity from $t = 0$ to $t = 35$. Also find the distance travelled between these times. If the law of resistance be:—

$$F \text{ (lb.)} = a + bV + cV^2,$$

where V is in miles per hour as before, indicate the method by which values of a, b, and c could be found from the above observations.
(B. of E. H., Part I., 1903.)

45. A flywheel weighs 5 tons and has a radius of gyration of 6 feet. What is its moment of inertia? It is at the end of a shaft 10 feet long, the other end of which is fixed. It is found that a torque of 200,000 lb.-feet is sufficient to turn the wheel 1°. The wheel is twisted slightly and then released; find the time of a complete vibration. How many vibrations per minute would it make? (B. of E. H., Part I., 1903.)

46. Describe a Naval mounting for a heavy gun. How is the recoil of the gun taken up after firing, and how is it brought back to the firing position? What is parabolic rifling, and what are its disadvantages?
(B. of E. H., Part I., 1903.)

47. Power is transmitted from one shaft to another by means of belting. How is the amount of power capable of being transmitted affected by, (1) the angle of lapping, (2) the weight of the belting, (3) journal friction, (4) the stiffness of the belt, (5) elastic slip, (6) centrifugal tension in the belt?
(B. of E. H., Part I., 1903.)

48. Give the theory of the laterally loaded strut. What is the effect of eccentric loading, and what is the effect of non-uniformity of material? When is this theory useful? (B. of E. H., Part I., 1903.)

49. State the principles employed in balancing the moving parts of vertical steam engines with one, two, or three cranks. What is the Yarrow-Schlick-Tweedy method? (B. of E. H., Part I., 1983.) (*See 14th Edition of my "Textbook on Steam and Steam Engines."*)

50. Describe the method of manufacture of Portland cement. How is the finished material tested? (B. of E. H., Part I., 1603.) (*See Rankine's "Civil Engineering."*)

51. What is the meaning of the term "gyrostatic action"? How do you explain its effects? (B. of E. H., Part I., 1903.) (*See Index, Vol. II.*)

52. Prove the formula you use for calculating the deflection of a beam of rectangular cross-section. A beam 1 inch wide and 1·5 inch deep, is placed upon knife edges 4 feet apart, and the overhanging ends are loaded with weights of 100 lbs. each placed 18 inches beyond the points of support (*i.e.*, the loads themselves are 7 feet apart), what is the upward deflection of the middle point of the beam (E = 30,000,000)? What is the shearing force between the knife edges? (B. of E. H., Part I., 1903.)

SCIENCE AND ART DEPARTMENT'S ADVANCED AND HONOURS EXAMINATIONS IN APPLIED MECHANICS FOR 1896, 1897, AND 1898 NOT INCLUDED AT THE END OF ANY OF THE LECTURES IN EITHER VOLS. I. OR II.

1. In a vernier calliper, the bar of the instrument is divided into inches, and each inch is sub-divided into 40 equal divisions. On the sliding jaw of the instrument is carried a vernier whose length is equal to 24 of the small divisions on the bar of the calliper (the vernier therefore measures $\frac{24}{40}$ inch in length), and the vernier scale is divided into 25 equal divisions. When the sliding jaw is brought into close contact with the fixed limb of the calliper, the zero line on the vernier then coincides with the zero line on the bar; what would then be the distance between the first line from zero on the vernier and the first line of the scale on the bar of the calliper? Sketch and describe the construction of the instrument and the method of taking outside measurements with it. What would be the exact position of the vernier on the bar of the instrument when the two jaws of the callipers are separated by a distance of 0·782 inch? (S. & A. Adv. Exam., 1896.)

2. What are the differences in the methods of working of a milling machine and of a planing machine, as arranged for tooling flat surfaces? What are the advantages of milling over planing? Sketch in front and end elevation the cutter or mill for tooling a flat surface, and give any details you can as to the best form for the teeth, and say why for cutting metals the cutting speeds of milling tools can be made greater than those of ordinary planing tools. (S. & A. Adv. Exam., 1896.)

3. Compare the physical qualities of cast iron and wrought iron, and of these with mild steel such as is used for boiler construction; also compare them with the steel used for turning tools. Give a numerical statement of the relative powers of these four varieties of iron to resist tensile, compressive, and torsional stresses respectively. What are the fundamental differences in chemical composition between cast iron, wrought iron, and mild steel? (S. & A. Hons. Exam., 1896.)

4. Show clearly why, under ordinary conditions, a worm wheel should not be employed to drive a worm, and state also under what conditions such a method of driving becomes possible. In large horizontal boring machines, the boring bar that carries the boring head is slowly revolved by a large worm wheel, which is itself driven by a worm rotated either by suitable pulleys and belting from the main driving shaft of the shop, or by a small engine coupled direct on to the worm shaft. Sketch the boring bar, with the boring head, as also the driving gear, and show how the boring head is traversed along the bar. Why is worm gearing used for driving these heavy machines? (S. & A. Hons. Exam., 1896.)

5. Describe and show, with the necessary sketches, the driving arrangement of the Whitworth double-geared slotting machine. Show and describe clearly how the upward or return stroke of the tool is made more quickly than the downward or cutting stroke. Show also how the length of the stroke is varied; how the height to which the ram can be lifted is adjusted to suit the varying depths of work on the table; and, lastly, indicate how the back gear is thrown in and out of gear. (S. & A. Hons. Exam., 1896.)

know most. Give your own notion of what probably occurs in the material, and your reasons, although there is no satisfactory explanation. (S. & A. Adv. Exam., 1897.)

7. A steam piston of a bull engine 30 ins. diameter, the pressure underneath exceeds the pressure above by the amount p lb. per square inch. p is given in the following numbers for every 6 inches of stroke (but the piston may not go so far as stated), beginning with the bottom position, 100, 98, 97, 96, 95, 79, 67, 57, 49, 43, 39, 35, 31, 28, 25, 23, 21. The total weight lifted is 15 tons, find the velocity at various points of the stroke and the length of the stroke.* (S. & A. Hons. Exam., 1897.)

8. A bicycle rider weighing 127 lbs., his bicycle being 33 lbs., finds that on a descent of 1 in 80 with feet off pedals he is just able to get on very slowly but steadily; call his speed 0. On a long slope of 1 in 40 his steady speed is 10 miles per hour; on a long slope of 1 in 20 his steady speed is 20 miles per hour. In all three cases his feet are off the pedals. Find the resistance to motion on the same kind of level road at each of these speeds. Neglect the extra resistance due to pedalling, and find a, b, and c in $F = W(a + bv + cv^2)$, the law of resistance on a level road at the speed v if W is the total weight. At 12 miles per hour going up and going down an incline of 1 in 60, what are the horse-powers exerted by a rider of 150 lbs. on the same bicycle on the same kind of road?* (S. & A. Hons. Exam., 1897.)

9. A link has motion parallel to a plane. Given the velocities and accelerations of two pins, how do we draw the velocity and acceleration diagrams which give at once the velocity and acceleration of any point in the link? Prove your two constructions to be correct.† (S. & A. Hons. Exam., 1897.)

10. Answer only one of the following a, b, c, or d:—a. How is cement made? What is your notion of what occurs when it sets and gradually hardens? What is the effect of the addition of sand? b. Choose some cast-iron object and explain why, and where, it may have initial strains and weakness. c. Describe very briefly the differences in composition, properties, and uses of cast iron, wrought iron, mild steel, tool steel, and any three other metals or alloys used in mechanical construction. d. What is a chilled casting? a malleable casting? how is each produced? Describe how wrought iron is case-hardened. (S. & A. Adv. Exam., 1898.)

* See Prof. Perry's *Applied Mechanics.* Chapter xi.
† See Kennedy's *Mechanics of Machinery.*

APPENDIX C.

City and Guilds of London Institute.

DEPARTMENT OF TECHNOLOGY.

TECHNOLOGICAL EXAMINATIONS, 1903.

46.—MECHANICAL ENGINEERING.*

HONOURS GRADE (Written Examination).*

Wednesday, April 29th, 7 *to* 10.

INSTRUCTIONS.

The Candidate for Honours must have previously passed in the Ordinary Grade, and is required to pass a Written and Practical Examination. He is requested to state, on the Yellow Form, whether he has elected to be examined in A, Machine Designing, or in B, Workshop Practice (*a*) *Fitting*, (*b*) *Turning*, (*c*) *Pattern-making*. Candidates in Machine Designing must forward their work to London not later than May 6th, and in Workshop Practice not later than May 13th.

The number of the question must be placed before the answer in the worked paper.

The Candidate is at liberty to use divided scales, compasses, set squares, calculators, slide rules, and mathematical tables.

Five marks extra will be awarded for every answer worked out with the slide rule, provided the method of working is explained.

The maximum number of marks obtainable is affixed to each question.

Three hours allowed for this paper.

The Candidate is not expected to answer more than *eight* of the following questions, which must be selected from *two* sections *only*.

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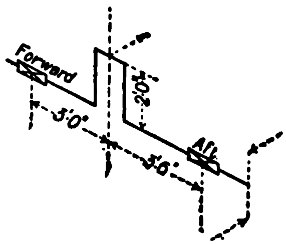
* The questions in Sections A and C may be answered from Vols. I. and II. of my *Text-Book on Applied Mechanics*. Section B is printed in the fourteenth edition of my *Text-Book on Steam and Steam Engines*, and students may also refer to that book for help with question 2 of Section A. The 1903 Questions for the Ordinary Grade are printed at the end of the fifth edition, 1903 issue, of my *Elementary Manual on Applied Mechanics*.

1. The legs of a tripod are 15, 16·5, and 18 feet long respectively, and the lengths of the lines joining the feet of the legs are each 20 feet. Find the force along each leg when a weight of 10 tons is suspended from the apex. Graphical constructions may be used. (*35 marks.*)

2. The crank-shaft of a single cylinder engine is as shown in the figure. The work is absorbed at one end and the bearings may be assumed to exercise no constraint on the shaft. When the connecting-rod is perpendicular to the crank-arm, the effective force in the rod is 30 tons. Find the magnitudes of the twisting and maximum bending moments on the crank-pin. If three-quarters of the work is absorbed aft and one-quarter forward, find the same things. (*35.*)



3. State precisely how you would make a tensile test of a ductile material, such as wrought iron or mild steel, stating what measurements you would make and how you would reduce the observed results. What are the most important items to obtain from a commercial point of view, and what results would you expect in a specimen of good mild steel? (*30.*)

4. A plate web girder, of 80 feet clear span, is 6 feet deep and has booms which are 3 feet wide. If the total uniformly distributed load which the girder carries is 120 tons, obtain a suitable section for the booms at the middle of the span. Design also a suitable cover plate for one of the boom plates. The safe stress in tension, compression, and shear may be taken as 5 tons per square inch. (*45.*)

5. A cast-iron column, 5 inches internal and 7 inches external diameter, has a bracket attached to it, and a weight of 20 tons is carried by the bracket at a distance of 12 inches from the axis of the column. Estimate the maximum and minimum stresses induced in the column, and show, by means of a sketch, how the stress varies across the section. (*35.*)

6. Steel wire, $\frac{1}{8}$-inch diameter, is supplied in coils, 3 feet diameter. Find the bending moment and the stress induced by coiling the wire on a mandril 2 feet in diameter, keeping the direction of curvature unaltered. You may assume that Young's modulus is 13,000 tons per square inch. (*35.*)

7. A spiral spring is 2·9 inches diameter, centre to centre of the coils, and has 36 coils. The diameter of the wire is $\frac{1}{10}$ inch, and the modulus of rigidity of the material of which it is composed is 5,200 tons per square inch. Neglecting the inclination of the coils, find the stress induced and the elongation under the axial pull of $\frac{1}{2}$ lb. How would the problem be modified if the inclination of the coils could not be neglected? (*35.*)

8. Design a hollow steel tunnel shaft and coupling to transmit 10,000 horse-power at 120 revolutions per minute. There are eight bolts, and the

inside diameter of the shaft has to be ·6 times the outside diameter. Choose your own working stresses, &c. The shaft may be assumed subjected to pure twisting, and the ratio of the maximum to the mean twisting moment to be 1·2. (*45.*)

Section C.

1. Sketch any form of water accumulator with which you are acquainted, and explain the advantages it possesses over the arrangement in which the pumps pump water directly into the mains. In an accumulator the total force in lbs., due to cup leather friction, may be taken equal to $\cdot 047\, p\, d$, where p is the pressure in lbs. per square inch and d is the diameter of the ram in inches. The ascending pressure is 1,250 lbs. per square inch, and the diameter of the ram is 6 inches. Calculate the descending pressure, and deduce the efficiency of the accumulator as a storer of energy. (*30.*)

2. Define what is meant by the terms coefficient of velocity, contraction and discharge, in connection with the flow through orifices. A vessel containing water and having a circular orifice, 1 inch in diameter, in one of its sides is supported in such a way that when a jet of water issues from the orifice the displacing force can be very accurately measured. With a constant head of $2\frac{1}{4}$ feet of water above the orifice, the displacing force (at the centre of the orifice) was found to be ·9 lb. and the discharge $15\frac{1}{4}$ gallons per minute. Determine the three coefficients. (*35.*)

3. Experiments made by Froude to determine the skin resistance of planks in water give the following results:—

Speed in feet per minute = V, . . .	200	400	600	800
Total resistance per 100 sq. feet in lbs. = R,	3·28	11·7	24·6	41·7

Test whether the relation between R and V can be expressed by a law of the type R varies as V^n, and, if so, find the values of f and n in the formula $R = f S V^n$, in which S = wetted surface in square feet. (*30.*)

4. A fire engine supplies water at a pressure of 40 lbs. per square inch to a pipe 3 inches diameter. The pipe is led a distance of 100 feet to a nozzle $\frac{3}{4}$-inch diameter, at a height of 25 feet above the pump. Find the quantity discharged per second and the height to which the jet can rise—having given that the coefficient of friction of the pipe is ·02, that the frictional resistances in the nozzle can be neglected, and that the actual height the jet rises is three-quarters of the head due to the velocity of efflux. Find also the pumpi g horse-power. (*40.*)

5. When suction pumps are run at too high a speed, "shock" or "separation" takes place. Explain, as clearly as possible, the precise action and upon what the limiting speed depends. A single-acting suction and lift plunger pump, whose piston has a diameter of 12 inches and a stroke of 2 feet, with a suction pipe 33 feet long and 10 inches diameter, has a lift of 20 feet—the barometric head being 34 feet of water. Assuming that the piston moves in a simple harmonic manner, find the limiting number of revolutions per minute of the crank-shaft below which separation will not take place at the commencement of the stroke. (*40.*)

6. Describe, with the aid of sketches, any hydraulic pumping engine with which you are acquainted in which the delivery is the same for both strokes. Show, in particular, the arrangement of valves. (*35.*)

7. Discuss the advantages and disadvantages of turbines, pointing out what considerations determi Briefly describe the different ways by which the re to turbines is effected (*40.*)

8. Find the circumferential speed of the wheel which is required to lift 200 tons of water per min given that the hydraulic efficiency of the pump is through the wheel 4·5 feet per second, and that the wards so that the angle between their directions circumference of the wheel is 20°. (*45.*)

QUESTIONS SET BY THE BOARD OF EDUCATION AT THEIR ADVANCED AND HONOURS EXAMINATIONS ON APPLIED MECHANICS. ALSO, BY THE CITY AND GUILDS OF LONDON INSTITUTE AT THEIR TECHNOLOGICAL EXAMINATIONS IN MECHANICAL ENGINEERING, WHICH ARE NOT INCLUDED AT THE END OF ANY OF THE LECTURES IN EITHER VOLS. I. OR II.

Tools and Hydraulic Motors.

1. Describe any hydraulic motor you are familiar with, and obtain an expression for the gallons of water needed per H.P. hour, given the available water pressure. (C. & G., 1900, H., Sec. C.)

2. Show by sketches the construction of a hand plane for planing wood. Be particular as to the angle of the "plane iron." What is the use of the "top iron"? (B. of E. Adv., 1901.)

3. How many 6-inch pipes will be required to convey the water from a hydraulic central station, with a maximum possible output of 1,200 H.P.? The working water pressure to be maintained in the mains is 1,200 lbs. per square inch. Choose your own velocity of flow. Show by a sketch how the pipes in such mains are jointed together.
(C. & G., 1901, H., Sec. C.)

4. Describe with sketches only *one* of the following:—(*a*) The mechanism for giving an automatic feed to the cutting tool of a lathe or shaping machine, and how it is put in and out of action, and the amount of feed varied. (*b*) The construction and use of external and internal workshop gauges, by means of which the size of a spindle (say 2 inches diameter) and that of a hole into which it fits may be ensured within specified limits of accuracy. State any advantages due to this system of working. (*c*) Any tool used by riveters and worked by water pressure. (B. of E. H., Part I., 1902.) (*To answer* (*a*) *and* (*b*) *see my* "*Elementary Applied Mechanics.*")

5. Describe, with the aid of a sketch, Oldham's coupling. Prove that it transmits a constant angular velocity ratio between the two parallel shafts.
(C. & G., 1902, O., Sec. A.)

Strength of Materials.*

1. A cylindrical steel pin is used to couple together two rods, and the joint is arranged in such a way that the pin is subjected to double shear. If the total tensile force tending to pull the joint asunder is 18¾ tons, what diameter would you make the pin? Choose for yourself the working shearing stress which can be permitted. (C. & G., 1901, O., Sect. B.)

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* A few Lectures on these subjects where the (*) appears are now being prepared by me for Volume II. of this book to cover these questions, &c.—A. J.

Tests and Testing Machines.*

1. Suppose the vertical loads and supporting forces of a horizontal beam to be known, show how we find (1) the shearing force at the section, (2) the position of the neutral line, (3) the compressive stress at any part of the section, (4) the curvature of the beam, (5) the new shape of a portion of the section originally rectangular, its sides vertical and horizontal.
(B. of E. Adv. & H., Part I., 1900.)

2. A tie-bar 6 inches wide and $1\frac{1}{2}$ inches thick is curved in the plane of the width in such a way that the mean line of pull is 2 inches away from the geometrical axis of the bar in the centre of the length. If it is under a total pull of $22\frac{1}{2}$ tons, find the maximum and minimum stresses at the centre cross-section. (C. & G., 1900, H., Sec. A.)

3. Describe carefully the behaviour of a mild-steel bar tested gradually in tension up to rupture, and sketch a stress-strain curve for it. The following data were obtained in such a test:—Original diameter of bar $1\frac{1}{4}$ inches, final diameter at point of fracture $\frac{11}{16}$ of an inch, total load when limit of elasticity was reached 20·8 tons, total load at fracture 36·7 tons, total extension at fracture (on a length of 10 inches) 2·23 inches, elongation (of 10-inch length) under a total load of 10 tons ·005 inch. Deduce from these data the following:—(*a*) The modulus of tensile elasticity. (*b*) The reduction of area per cent. (*c*) The elongation at fracture per cent. (*d*) The limit of elasticity in tons per square inch. (*e*) The maximum load in tons per square inch (both for original and final area). (C. & G., 1901, H., Sect. A.)

4. What would be the probable breaking loads in tension of the following three bars:—(*a*) A cast-iron cylindrical rod, $1\frac{1}{2}$ inches in diameter; (*b*) a wrought-iron flat bar 2 inches by $\frac{5}{8}$ inch in section; (*c*) a mild-steel angle bar $3\frac{1}{2}$ inches by $3\frac{1}{2}$ inches by $\frac{1}{2}$ inch in section?
(C. & G., 1901, O., Sec. B.)

5. Answer only *one* of the following:—(*a*) Describe a laboratory experiment by which you could find E, Young's modulus of elasticity, for an iron wire 10 feet long and 0·05 inch diameter. How would you secure the upper end of the wire? How apply the load? And how measure the elongation? How would you plot your results, and how deduce the value of E? About how much elongation would you expect for a load of 15 lbs.?
(B. of E., H., Part I., 1902.)

6. Show that a load which, when applied gradually, induces an intensity of stress, f_0, in a bar of length, l, will induce, when dropped from a height, h, before stretching the bar, a stress given by

$$f_0\left\{1+\sqrt{1+\frac{2\,\mathrm{E}\,h}{f_0\,l}}\right\},$$

the whole energy of the blow being assumed expended in straining the bar.
(C. & G., 1902, H., Sec. A.)

7. In a tensile test of mild steel the original section was a rectangle of dimensions 1·96 inches by ·4 inch, and the breaking load was 25 tons. The extension at fracture, in a length of 8 inches, was 2·44 inches and the contracted dimensions were 1·41 inches by ·27 inch. Under a load of

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* See the previous footnote.

14 tons (within the elastic limit) the extension in 8 inches was ·0107 inch. Calculate the stress at breaking point, the percentage extension, the percentage contraction of area, and the value of Young's Modulus. Express an opinion whether the specimen is a good or a bad one.
(C. & G., 1902, O., Sec. B.)

Columns.*

1. A solid wrought-iron cylindrical strut is 8 feet 4 inches long, and has its ends solidly built in: assuming it has only a pure compressive load of 24 tons to support, what must the diameter be if the working load is only to be $\frac{1}{8}$th of the collapsing load? (C. & G., 1900, H., Sec. A.)

2. It is known that a solid cast-iron column of 4 inches diameter and 7 feet in length would collapse under a load of 17·5 tons per square inch: what total load could you allow the column to support with a factor of safety of 8? (C. & G., 1900, O., Sec. B.)

3. How much would a cast-iron column 12 feet long shorten under a load of $3\frac{1}{2}$ tons per square inch, if it is a hollow section 8 inches in external diameter and 1 inch thick? The modulus of elasticity of cast iron in compression is 12,500,000 lbs per square inch. What total compressive load will this column support under the above stress per square inch?
(C. & G., 1901, O., Sect. B.)

4. Explain why the resistance of a long strut depends more on the stiffness of the material than on its strength. Quote any formula which is used in the design of long struts. (C. & G., 1902, O., Sec. B.)

Harmonic Motion and Springs.*

1. What is the nature of the forces producing the simplest reciprocating motion? A body of 40 lbs. weight hangs from a spiral spring and vibrates vertically; what is the periodic time of its vibration if the stiffness of the spring is such that a weight of 10 lbs. would elongate it 0·1 foot? (S. and A. Adv., 1899.)

2. What is the nature of the forces producing the simplest reciprocating motion? A carriage of 322 lbs. moving without friction with this kind of motion on a horizontal table has a periodic time of 0·6 second. It moves to the distance of 1 foot on either side of a middle position; what are the forces acting upon it? *Ans.* 1,111·1 lbs. (B. of E. Adv., 1900.)

3. A body of 60 lbs. has a simple vibration, the total length of a swing being 3 feet; there are 200 *complete* vibrations (or double swings) per minute; calculate the forces which act on the body at the ends of a swing, and show on a diagram to scale what force acts upon the body in every position. (B. of E. Adv., 1901.)

4. A weight of 5 lbs. is supported by a spring. The stiffness of the spring is such that putting on or taking off a weight of 1 lb. produces a downward or upward motion of 0·04 foot. What is the time of a complete oscillation, neglecting the mass of the spring? (B. of E. Adv., 1902.)

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* See the previous footnote.

Crane Hooks.*

1. In a 21-ton crane hook, the radius of the eye is 2·75 inches and the horizontal section through the eye is a rectangle of dimensions 2·25 inches by 4·5 inches. Estimate the greatest tensile and compressive stresses at the section considered (C. & G., 1902, H., Sec. A.)

Collapsing Pressure of Cylindrical Tubes.*

1. A structure has a hollow circular section 10 inches outside diameter and 8 inches inside. The resultant of all the loads and supporting forces acting on one side of the section has a component of 30 tons normal to the section and it acts at 2 inches from the centre; find the maximum and minimum stresses in the section. (S. and A. H., Part I., 1899.)

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* See the previous footnote.

APPENDIX D.

THE INSTITUTION OF CIVIL ENGINEERS.

EXTRACTS FROM RULES AND SYLLABUS OF EXAMINATIONS FOR ELECTION OF ASSOCIATE MEMBERS.

Note.—The following extracts are simply printed here to show, how far my books upon *Applied Mechanics*, *Steam and Steam Engines*, as well as *Magnetism and Electricity* (including Munro & Jamieson's *Pocket-Book of Electrical Rules and Tables*), together with my "*Correspondence System of Electrical and Mechanical Engineering Science, as taught by Excercises, Drawings, and Instructions*" cover the Scientific and Practical Knowledge demanded by The Institution, under Part II., **Section A** (**1** and **2**), as well as **Section B** under (ii.) Hydraulics, (iii.) Theory of Heat Engines, and (viii.) Electricity and Magnetism.

N.B.—In and after October, 1904, the Associate Membership Examination will be held under Revised Regulations, as detailed herein three pages after this. I shall endeavour to keep my books in conformity with these new regulations.

In the new regulations for Students, which come into force in October, 1903, one of the new subjects which may be selected is that of Elementary Mechanics of Solids and Fluids. See my *Elementary Applied Mechanics* when studying this subject, and my *Elementary Magnetism and Electricity* book when reading that part of the Elementary Physics for admission of Students.

PART II.*—*Scientific Knowledge.*

SECTION A.

1. **Theoretical and Elementary Applied Mechanics** (two Papers, *time allowed*, 3 *hours for each*).

2. **Theory of Structures and the Strength and Elasticity of Materials** (two Papers, *time allowed*, 3 *hours for each*).

* Candidates may offer themselves for examination in Sections A and B of Part II. together; or they may enter for Section A alone, and, if successful, may take Section B at a subsequent examination. In the latter case, however, such candidates will not be allowed to present themselves for examination in Section B unless or until they are actually occupied in work as pupils or assistants to practising engineers. The Council may permit candidates who have attempted the whole of Part II. at one examination, and have failed in Section B only, to complete their qualification by passing in that section at a subsequent examination, subject to their being then occupied as above stated.

One of the following subjects (one Paper in any one subject, *time allowed,* 3 *hours*) :—(i.) Geodesy ; (ii.) **Hydraulics** ; (iii.) **Theory of Heat Engines** ; (iv.) Metallurgy ; (v.) Geology and Mineralogy ; (vi.) Stability and Resistance of Ships ; (vii.) Thermo- and Electro-Chemistry ; and (viii.) **Electricity and Magnetism.**

Mathematics.—The standard of Mathematics required for the Papers in Part II. of the examination is that of the mathematical portion of the Examination for the Admission of Students, though questions may be set involving the use of higher Mathematics.

The range of the examinations in the several subjects, in each of which a choice of questions will be allowed, is indicated generally hereunder :—

Section A.

1. Theoretical and Elementary Applied Mechanics :—*

Statics of solids and fluids ; statical problems connected with loaded structures ; gravity and attraction ; friction ; static, kinematic, and kinetic problems relating to machines ; conservation of energy ; theory of mechanisms.

2. Theory of Structures and the Strength and Elasticity of Materials :—*

Graphic and analytic methods for the calculation of bending moments and of shearing forces, and of the stresses in individual members of framework structures loaded at the joints ; plate and box girders ; incomplete and redundant frames ; theory of continuous girders and principal methods of calculation ; travelling loads ; riveted and pin-joint girders ; rigid and hinged arches ; strains due to weight of structures ; theory of earth-pressure and of foundations ; stability of masonry and brickwork structures.

Coefficients of elasticity ; elastic resistance to tension, compression, shearing, and torsion ; uniform and varying stress ; moment of stress and of resistance in beams of various sections ; distribution of shearing stress in beams ; stresses suddenly applied and effects of impact ; buckling of struts ; effect of different end-fastenings on their resistance ; combined strains ; calculations connected with statically indeterminate problems, as beams supported at three points, &c ; limit of elasticity, yield-point, and ultimate resistance of various materials as tested ; the plastic state ; principal forms of testing-machines and of appliances used in measurements for the determination of coefficients of elasticity ; calculation of extension, deflection, buckling, &c., within elastic limits, and of ultimate strength and ductility for ordinary materials of construction.

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* See my Elementary Book and Vols. I. and II. on *Applied Mechanics.*

Section B.

(ii.) Hydraulics:—*

The laws of the flow of water by orifices, notches, and weirs; laws of fluid friction; steady flow in pipes or channels of uniform section; resistance of valves and bends; general phenomena of flow in rivers; methods of determining the discharge of streams; tidal action; generation and effect of waves; impulse and reaction of jets of water; transmission of energy by fluids; principles of machines acting by the weight and pressure of water; theory and structure of turbines and pumps.

(iii.) Theory of Heat-Engines:—†

Thermodynamic laws; internal and external work; graphical representation of changes in the condition of a fluid; theory of heat-engines working with a perfect gas; air- and gas-engine cycles; reversibility, conditions necessary for maximum possible efficiency in any cycle; properties of steam; the Carnot and Clausius cycles; entropy and entropy-temperature diagrams, and their application in the study of heat-engines; actual heat-engine cycles and their thermodynamic losses; effects of clearance and throttling; initial condensation; testing of heat-engines, and the apparatus employed; performances of typical engines of different classes; efficiency.

(viii.) Electricity and Magnetism:—‡

The theory of the generation, storage, transformation, and distribution of electrical energy; continuous and alternating currents; electro-magnetism; polyphase systems; calculation of the losses in electrical processes, and of the efficiency of electrical machinery and apparatus; arc and incandescent lamps; electrical and magnetic measurement and measuring instruments.

The examination may, in the discretion of the Council, be taken by Students of the Institution who are not less than 21 years of age (see p. 2, Rule 2), before proposals in favour of their election into the Institution have been lodged with the Secretary.

In the discretion of the Council, arrangements may be made for the examination in October of persons residing in India or in the Colonies in whose favour proposals for election have been lodged (see p. 3, Rule 7).

* See my Elementary Book and Vols. I. and II. on *Applied Mechanics*.

† See my Elementary and Advanced Text-Books on *Steam and Steam Engines*, latest editions. Also, *Gas, Oil, and Air Engines*, by Bryan Donkin; *Marine Engineering*, by A. E. Seaton; *The Steam Engine and other Prime Movers*, by Prof. W. J. Macquorn Rankine; &c., as published by Charles Griffin & Co.

‡ This examination is really one involving a knowledge of certain parts of Electrical Engineering, in addition to the Scientific Principles of Electricity and Magnetism.

Engineers who desire to enter for the A.M.Inst.C.E. examinations should write *at once* to the Secretary, Great George Street, Westminster, S.W., for the Complete Rules, Syllabus, and Application Forms. They will find all the questions relating to the above mentioned subjects which have been set since these examinations commenced in 1897 in my Text-Books.

ELECTION OF ASSOCIATE MEMBERS.

OCTOBER EXAMINATION, 1904.

(*A Time-table of the Examination will be issued after February, 1904.*)

PART II.—*Scientific Knowledge*

SECTION A.

1. **Applied Mechanics** (one Paper, *time allowed*, 3 *hours*).

2. **Strength and Elasticity of Materials** (one Paper, *time allowed*, 3 *hours*).

3. *Either* (*a*) **Theory of Structures,**
or (*b*) **Theory of Electricity and Magnetism** (one Paper, *time allowed*, 3 *hours*).

SECTION B.

Two of the following nine subjects—not more than one from any group (one Paper in each subject taken, *time allowed*, 3 *hours for each Paper*):—Group i.—Geodesy, Theory of Heat-Engines, Metallurgy. Group ii.—Hydraulics, Theory of Machines, Thermo- and Electro-Chemistry. Group iii.—Geology and Mineralogy, Stability and Resistance of Ships, Applications of Electricity.

The range of the examinations in the several subjects, in each of which a choice of questions will be allowed, is indicated generally hereunder:—

SECTION A.

1. **Applied Mechanics:—**

Statics.—Forces acting on a rigid body; moments of forces, composition, and resolution of forces; couples, conditions of equilibrium, with application to loaded structures. The foregoing subjects to be treated both graphically and by aid of algebra and geometry.

Hydrostatics.—Pressure at any point in a gravitating liquid; centre of pressure on immersed plane areas; specific gravity.

Kinematics of Plane Motion.—Velocity and acceleration of a point; instantaneous centre of a moving body.

Kinetics of Plane Motion.—Force, mass, momentum, moment of momentum, work, energy, their relation and their measure; equations of motion of a particle; rectilinear motion under the action of gravity; falling bodies and motion on an inclined plane; motion in a circle; centres of mass and moments of inertia; rotation of a rigid body about a fixed axis; conservation of energy.

2. Strength and Elasticity of Materials—the same as detailed on a previous page.

3. (*a*) **Theory of Structures**—the same as detailed on a previous page.

(*b*) **Theory of Electricity and Magnetism:**—

Electrical and magnetic laws, units, standards, and measurements; electrical and magnetic measuring instruments; the theory of the generation, storage, transformation and distribution of electrical energy; continuous and alternating currents; arc and incandescent lamps; secondary cells.

SECTION B.

Hydraulics—the same as detailed on a previous page.

Theory of Machines:—

Kinematics of machines; inversion of kinematic chains; virtual centres; belt, rope, chain, toothed and screw gearing; velocity, acceleration and effort diagrams; inertia of reciprocating parts; elementary cases of balancing; governors and flywheels; friction and efficiency; strength and proportions of machine parts in simple cases.

Theory of Heat-Engines—the same as detailed on a previous page.

Applications of Electricity:—

Theory and design of continuous- and alternating-current generators and motors, synchronous and induction motors and static transformers; design of generating- and sub-stations and the principal plant required in them; the principal systems of distributing electrical energy, including the arrangement of mains and feeders; estimation of losses and of efficiency; principal systems of electric traction; construction and efficiency of the principal types of electric lamps.

☞ **Candidates should not forget, that their Applications, *duly completed*, *must* be in the hands of the Secretary of the Institution of Civil Engineers, Great George Street, Westminster, S.W., *before* 1st January for the February Examination, and *before* the 1st September for the October Examination. Candidates should, therefore, apply for the necessary "Forms," *at least six* months *before* these Examinations, to give them time to make due and proper Application, and to thoroughly *Revise* the subjects upon which they are to be examined.**

QUESTIONS SET BY THE INSTITUTION ENGINEERS FOR THE ASSOCIATE ME AMINATIONS FROM OCT., 1897, TO FEB., 1 ARE NOT INCLUDED AT THE END OF AI LECTURES IN EITHER VOLS. I. OR II.

STRENGTH AND ELASTICITY OF MATE

Tests and Testing Machines, with Stress-Strain

NOTE.—*Two or three Lectures are in Active Preparation for t of this Book, which, it is hoped, will cover the range of t In the meantime, students are referred to the already-m*

1. A bar 4 inches × 2 inches in cross-section is subjected to tension of 40 tons. Find the normal and shearing stresse inclined at 30° to the axis of the bar. (I.C.E., *Oct.*, 1897.)

2. Describe the method of conducting a tension test of steel. State what precautions should be taken in preparin and what measurements should be made. (I.C.E., *Oct.*, 189

3. Describe and sketch one form of testing machine. Sta conditions you would require in such a machine for testing t State how or to what extent its accuracy can be ascertained (I.C.I

4. Describe the methods of holding tension specimens, an special advantages or defects of any of these. (I.C.E., *Oct.*,

5. Sketch a stress-strain diagram for cast iron, mild steel, a Indicate on each what are the characteristic points. (I.C.E.

6. What is meant by the pressure of fluidity of a solid, a law governing the change of form when the pressure of fluidit Give a stress-strain diagram for a plastic solid cylinder sub pression. (I.C.E., *Oct.*, 1897.)

7. Give an account of the method of making and testing ceme Explain carefully what precautions have to be taken. Menti soundness. (I.C.E., *Oct.*, 1897.)

8. Describe carefully a form of micrometer for determining tensions of a bar. State how its accuracy can be tested. (I.C.I

9. Give a specification of tests of mild steel bars and pla work. (I.C.E., *Oct.*, 1897.)

10. Explain what is meant by Poisson's ratio. A cube of side has two simple normal stresses, p_1, p_2, on pairs of opposi the length of the sides of the cube when deformed by the stre (I.C.E

11. Find the relation between the coefficient of direct elas coefficient of rigidity. (I.C.E., *Oct.*, 1897.)

12. Describe the successive effects of an increasing pull st a specimen of mild steel, and illustrate your answer by referen strain diagram sketched to a scale. Define Young's modulus, its meaning by reference to a stress-strain diagram. (I.C.E.,

13. A specimen of mild steel 1 inch in diameter and 1 between the gauge-points gives the following results:—Yield-

per square inch; maximum load, 30 tons per square inch of original area; extension, 26 per cent. What results would you expect to obtain in testing two other specimens of the same quality of material, one ½ inch in diameter and 5 inches long and the other ½ inch in diameter and 10 inches long between gauge-points? Criticise in this connection the specification: "The steel is to have a strength of 28 tons per square inch and to give an elongation of 25 per cent. on a length of 8 inches." (I.C.E., *Feb.*, 1898.)

14. Sketch and describe one form of stress-strain indicator suitable for use in tension tests of iron or steel specimens. (I.C.E., *Feb.*, 1898.)

15. Explain how Poisson's ratio may be determined for a specimen of mild steel by experiments on torsion and bending, giving the formulæ required. (I.C.E., *Feb.*, 1898.)

16. The tensile strength of cast iron, as calculated by the ordinary formulæ from the results of experiments on the ultimate strength of beams, differs from that obtained by direct-tension experiments. Account for the difference, and state what you know of it quantitatively in any cases. (I.C.E., *Feb.*, 1898.)

17. When a 1-inch bolt of mild steel has a screw-thread cut upon its end, how will its strength and toughness be affected? And how will they be affected again if the shank is turned down to the diameter of the screw inside the thread? Explain these effects by reference to the stress-strain diagram. (I.C.E., *Oct.*, 1898.)

18. In testing materials under direct tension, describe the manner in which a stress-strain diagram is usually constructed, explaining what are the quantities represented by its co-ordinates; and sketch the typical form of such a diagram for a 10-inch test-bar of mild steel, with explanatory remarks. (It is not necessary here to describe the mechanical details of any particular apparatus.) (I.C.E., *Oct.*, 1898.)

19. Describe the different kinds of failure that would be observed under a direct crushing load, in cubes or short cylinders of the following materials:—Lead, cast iron, granite, mild steel, pine timber, and ash timber—the last two being set with the grain vertical. (I.C.E., *Oct.*, 1898.)

20. Having ascertained that a certain mixture of cast iron possesses an ultimate tensile strength of 9 tons per square inch, calculate the breaking weight of a standard bar 2 inches by 1 inch, placed upon supports 3 feet apart and loaded in the middle. In using for this purpose the ordinary theory of transverse flexure, add any comments that you may think necessary in regard to the result so obtained. (I.C.E., *Feb.*, 1899.)

21. Sketch a suitable form of briquette and clips for testing Portland cement in direct tension. Explain, also, how the result of the test would probably be affected by the age of the specimen and by the rapidity of loading. (I.C.E., *Feb.*, 1899.)

22. Describe some form of testing machine suitable for performing an experiment on stretching a test-piece till rupture occurs. Show how the results of such an experiment are exhibited graphically. Sketch the curve obtained for a specimen of mild steel, marking the most important points with the names commonly applied to them. (I.C.E., *Oct.*, 1899.)

23. State the characteristics of the "plastic state" of a material, and give examples of the "flow of solids." Describe an experiment on the crushing of a block of soft steel, and compare it with a similar experiment on a piece of cast-iron. (I.C.E., *Oct.*, 1899.)

24. Describe any experiment which has been made on the effect of repeating a load in the same or in the opposite direction; as for example, when a bar is bent alternately in opposite directions till it breaks. State the principal results of such experiments. (I.C.E., *Oct.*, 1899.)

25. Explain the terms "limit of elasticity," "modulus of elasticity." Describe the changes of form which occur when a piece of material is stretched or compressed within the elastic limit. Calculate these changes in fractions of an inch for a piece of wrought iron of circular section 2 inches in diameter, 6 feet long, under a pull of 15 tons, assuming probable values of the coefficients. (I.C.E., *Feb.*, 1900.)

26. If the line of action of the pull on a bar does not coincide with its geometrical axis, show that its strength is diminished. Find the deviation for a bar of circular section 2 inches in diameter when the strength is diminished 20 per cent. (I.C.E., *Feb.*, 1900.)

27. Describe fully the several stages of an experiment on the stretching of a piece of wrought iron till rupture occurs, stating the average results of such experiments. Also explain how the results are influenced by the form of test-piece adopted. (I.C.E., *Feb.*, 1900.)

28. State carefully the characteristics of plastic and non-plastic materials employed in construction, giving examples of each. Describe fully an experiment on the crushing of a block of non-plastic material, stating the circumstances which influence the crushing strength.

(I.C.E., *Feb.*, 1900.)

29. Explain what are meant by the terms "centre of resistance," "line of resistance," employed in the theory of blockwork structures. State fully the reasons why the deviation of the centre of resistance of a joint from its geometrical centre cannot generally exceed a certain fraction of its diameter. Determine that fraction for a circular area by the usual method. Name any exceptional cases in which the fraction (1) may be greater, or (2) must be less than that given by this rule

(I.C.E., *Feb.*, 1900.)

30. Find the greatest height to which a cylindrical column 4 feet diameter can safely be built so as to be stable under a wind pressure of 20 lbs. per square foot of a vertical diametrical section, the weight of the material being taken as 120 lbs. per cubic foot. Find the line of resistance in this case, and state the relation between the diameter of a column and its height for the same degree of stability under wind-pressure.

(I.C.E., *Feb.*, 1900.)

31. Mention some practical tests by which we can estimate or measure the ductility of metals, and some of the reasons which make it important to obtain a test of this property. (I.C.E., *Feb.*, 1901.)

32. Find the position of the neutral axis in a cast-iron beam whose section has the following dimensions:—Upper flange 4 inches wide and 2 inches deep; lower flange 10 inches wide and $2\frac{1}{2}$ inches deep; web 2 inches thick; total depth of section 20 inches. (I.C.E., *Feb.*, 1901.)

33. A solid square pillar 10 feet in height, with a section 2 feet by 2 feet, and weighing 50 cwts., is subjected to a horizontal wind pressure of 28 lbs. per square foot on one side. Assuming its material to be uniformly elastic, find the intensity of compressive stress at the leeward edge and at the windward edge of its base. (I.C.E., *Feb.*, 1901.)

34. If the elastic twist of a wire $\frac{1}{4}$ inch diameter and 4 feet long from the fixed end is 36° with a certain twisting moment, find the twist of the end of a wire of the same material $\frac{1}{8}$ inch diameter and 5 feet long from the fixed end, with the same twisting moment. (I.C.E., *Oct.*, 1901.)

35. Describe with the help of sketches the general arrangement of any testing-machine you are familiar with, suitable for tensile and compressive tests. Explain how the machine may be tested for accuracy.

(I.C.E., *Oct.*, 1901.)

36. Explain fully by means of stress-strain diagrams, the effects of a

tensile load applied suddenly to a tie-bar; (*a*) without impact; (*b*) with impact. (I.C.E., *Oct.*, 1901.)

37. A tie-bar 9 inches wide and 1½ inch thick is curved in the plane of its width. If there is a total tensile load on the bar of 30 tons, and if the mean line of pull passes 3 inches to one side of the geometrical axis at the middle of the bar, find the maximum and minimum stresses at the centre section of the bar. (I.C.E., *Oct.*, 1901.)

38. In a test of a cylindrical cast-iron beam on a 20-inch span it is found that with a load of 660 lbs. in the centre the deflection is 0·027 inch. If the diameter of the cross-section is 1·5 inch, what is the modulus of elasticity of the material? What hypothesis is assumed in deducing the formula you use in your calculation? Is it true for such a material as cast iron? (I.C.E., *Oct.*, 1901.)

39. Explain why in making tests of timber it is necessary to know the dryness condition of the wood at the time of the test. How would you proceed to ascertain exactly the dryness condition? Do the results obtained from transverse tests of small specimens differ materially from those obtained from large logs if the dryness condition is the same in both cases? (I.C.E., *Oct.*, 1901.)

40. Describe fully, with sketches of the apparatus used, Wöhler's and Bauschinger's experiments on the effect of repeated application of loads on the strength of any bar. What general conclusions can be drawn from these results, and how can they be utilised in designing machines and structures? (I.C.E., *Oct.*, 1901.)

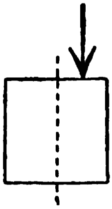
41. Find the greatest height to which a cylindrical stone column can be built if it is to be stable under the pressure of a wind which may reach a pressure of 45 lbs. per square foot on a flat surface normal to the wind direction. The weight of the material being 145 lbs. per cubic foot, find where the line of resistance will cut the base. (I.C.E., *Feb.*, 1902.)

42. Describe fully the behaviour of a piece of good mild steel during a tensile test from the first application of the load until it reaches the rupture value. Sketch the stress-strain curve for such a specimen. and mark on it the limit of elasticity, the yield point, the maximum load and the rupture load. (I.C.E., *Feb.*, 1902.)

43. Describe as fully as possible the various tests which are usually made to determine the quality of a sample of Portland cement. Sketch the form of the briquettes generally adopted, and of any type of testing-machine suitable for the tensile tests of such briquettes. (I.C.E., *Feb.*, 1902.)

44. Explain how a cube of cast iron can be tested to destruction by compression in a testing machine; how is it likely to fracture? Give your reasons. (I.C.E., *Oct.*, 1902.)

45. A cube of iron is in compression, the load is applied to one face on the centre line parallel to the sides in one direction, but is not at the centre of that line. Show that, if it deviates by one-sixth the length of the side from the centre of the face to which it is applied, there is no stress at the edge further from the load, and that, at the near edge the intensity of compression is twice the average intensity. (I.C.E., *Oct.*, 1902.)



46. In a body subject to stress, prove that the shearing stresses on planes at right angles to each other are equal. Show, by a sketch, how a cube subject to a shear on two pairs of faces would be distorted. (I.C.E., *Feb.*, 1903.)

47. Make a sketch of any form of autographic apparatus you are acquainted with, for causing a stress-strain diagram to be taken during the testing of a specimen to destruction in tension.

(I.C.E., *Feb.*, 1903.)

48. What quantities must be recorded in an autographic diagram when a specimen is being tested to destruction by torsion in order that the work done upon the bar may be found. Draw such a diagram for a steel bar ½ inch diameter and 8 inches long, and show how to find the work done on the bar. (I.C.E., *Feb.*, 1903.)

On Columns.

1. Explain the general relation between the formula for simple crushing, and Gordon's and Euler's formulæ for long columns. Putting l for the length and d for the diameter of a column; show by a sketch the relation of the results of applying the three formulæ for different values of $\frac{l}{d}$ for columns of the same section. (I.C.E., *Oct.*, 1897.)

2. State Gordon's formula for the strength of columns, and show how it is obtained. Apply it to find the breaking load for a cast-iron column 8 inches external diameter, 6½ inches internal diameter, and 22 feet high. The column has flat ends. Take $f = 80{,}000$, and $a = \frac{1}{600}$.
(I.C.E., *Feb.*, 1898.)

3. Prove Euler's expression for the limit of elastic stability of struts, and explain in how far you consider it applicable to such columns and other struts as are commonly used in engineering structures.
(I.C.E., *Feb.*, 1898.)

4. Prove that the deflection-curve of a slender elastic column, in the incipient stage of buckling, is identical with the curve of a flexible chain under a load whose varying intensity at different points is proportional to the depth of the chain below the horizontal chord-line joining the points of suspension. (I.C.E., *Oct.*, 1898.)

5. What conclusions may be directly drawn from Euler's formula, in regard to the relative strength of columns having the same dimensions but constructed of such different materials as cast iron, wrought iron, and steel? State generally what are the conditions under which the formula would be nearly correct, and those under which it would be quite at variance with experiment. (I.C.E., *Oct.*, 1898.)

6. Describe carefully the way in which a strut gives way when exposed to a gradually increasing crushing load, considering especially the differences due to difference in the ratio of length to diameter. Write down formulæ for the crushing load, (*a*) for very long, (*b*) for very short struts; also give any formula in common use for intermediate cases. Explain the effect of different ways of fixing the ends. (I.C.E., *Oct.*, 1899.)

7. A hollow cast-iron column is 9 inches in external diameter, its length is 12 feet, and its two ends are firmly built in. The compressive load it supports is 60 tons. What thickness must the metal be in order to have a factor of safety of 10? (I.C.E., *Oct.*, 1901.)

8. A solid cast-iron column is 5 inches in diameter and 9 feet 7 inches long. What total load will it support if the ends are firmly built in and the working load is not to exceed one-ninth of the crushing load?
(I.C.E., *Feb.*, 1902.)

9. Show how Rankine's formula for the breaking load of long columns is derived, and explain why a member of constant cross-section in compression is weaker as its length increases. (I.C.E., *Feb.*, 1903.)

Bicycles.

1. An ordinary bicycle, with, say, 6½-inch cranks, and 28-inch wheels geared to 56 inches, is lightly held in an upright position, with a crank in its lowest position. A person standing behind the machine pulls the crank horizontally towards himself. Will the bicycle go forwards or backwards? Explain on mechanical principles. (I.C.E., *Feb.*, 1901.)

On Shear Stresses.

1. Define what is meant by "principal stresses." If the stress at a point on one plane is inclined at an angle of 60° to that plane, and on a plane at right angles to the former the stress in a simple shear; find the principal stresses at the point and their directions. (I.C.E., *Oct.*, 1902.)

Hydraulics.—Miscellaneous.

1. Illustrate by sketches the distribution of velocities and depths in a river flowing through a flat alluvial country. (I.C.E., *Feb.*, 1899.)

2. A pipe of uniform diameter and internal condition, and 1 mile long, crosses a valley from an intake, A, and supplies a reservoir, B having a water-surface of 108 × 100 feet. The pipe is kept constantly full at the intake and enters the reservoir below the water-level. In virtue of the flow through the pipe, the water in the reservoir, B, was rising at the rate of 1 foot in 1½ hours, and had just reached a level of 20 feet below that of the intake, A, when a scour valve at the ½-mile distance, and 100 feet below the intake level, was suddenly opened and allowed to discharge water from the pipe at the rate of ½ cubic foot per second. Neglecting as unimportant change of momentum of the water, resistances at the intake, at the opening to the scour valve, and at the reservoir, what would be the effect at the reservoir, stated in rate of rise or fall per foot per hour? (I.C.E., *Feb.*, 1900.)

3. Sketch a good form of pilot valve to facilitate the opening of circular screw-down valves kept closed under heavy water pressure. (I.C.E., *Feb.*, 1901.)

4. Sketch and describe Duckham's hydraulic crane weighing machine. (I.C.E., *Feb.*, 1901.)

5. Describe the Davey differential valve gear as applied to town water supply pumping engines. The sketches used must be neatly and accurately made in fairly good proportion, and the action and the characteristic advantages of this gear as compared with others must be fully explained. (I.C.E., *Feb.*, 1901.)

6. What circumstances favour "stream line" and "eddying" motion? If you know the velocity and pressure at one point of the flow, what considerations would enable you to find their relative values at another point for each kind of motion? (I.C.E., *Oct.*, 1901.)

7. Two chambers with vertical sides, each 50 square feet in area, are connected by means of a rectangular sluice 3 feet by 2 feet near the bottom. One chamber contains water to a depth of 25 feet and the other a depth of 10 feet. If the sluice is opened, find how long it will be before the water is at the same level in the two chambers. (I.C.E., *Oct.*, 1901.)

8. Describe the working of a hydraulic passenger lift when the available working pressure is small and an intensifier is made use of. (I.C.E., *Oct.*, 1901.)

9. Give a general account, illustrated by ske
tidal phenomena in any estuary with which you ar

10. Criticise the following statement:—"Enginee
in attempting to utilise the energy at the Falls of
neglected the much greater amount available at
velocity is as great as 19 miles per hour." Assu
mean velocity at the rapids. Take the fall at the
calculate the horse-power available at each for a fl
per second. (I.C.E., *Feb.*, 1903.)

Rainfalls and Flow of Riv

1. The mean annual rainfall upon a steep m
covering impermeable rocks is ascertained to be
is likely to be the fall in the driest year and the an
three consecutive driest years? (*b*) What portion
two latter falls respectively would be gauged in a
flow from 1,000 acres of such an area, and what
difference? (*c*) What is the least flow and what i
second that you would expect to gauge in such a
why is the maximum flow per 1,000 acres affected b
of the drainage area of a stream? (I.C.E., *Feb.*, 19

2. (*a*) State the approximate relation between a
the surface of a stream and its mean velocity whe
1 foot per second. (*b*) Would there be any impor
relation by reason of the change of velocity being f
minute instead of from 1 foot to 2 feet per secon
difference. (I.C.E., *Feb.*, 1900.)

3. A river bed occurs in soft alluvial soil, the
unprotected from erosive action. Show by sket
relative velocities and directions of flow in plan
the consequent effect thereof, in the process of tin
bed of the stream. (I.C.E, *Feb.*, 1900.)

4. A river flowing through an alluvial plane, or "
either artificially or by a growth of trees or other v
great fluctuations of discharge, and in time of flood c
matter (detritus). Explain with sketches:—(*a*) W
assumes as its path through such a plane a permaner
line. (*b*) Why the outer bank at a bend is usually
bank at the same bend. (*c*) How these conditions b
growth of vegetation, and why in such changed c
the plane is often found to be higher as one approach

5. (*a*) Explain why in the process of time the me
plane such as that in (4) may become practically
standing that the river is untrained either artificial
growth of vegetation, and that it continues to flood th
carrying water. (*b*) If such a river ceased to brin
above, should you expect in the process of time to f
in the alluvial plane; and if so, in what manner
occur, and what ultimate result might be looked for?

Waves and Tides.

1. Describe the motion of the particles of water in a trochoidal wave. Suppose a circle whose radius is O Q rolls along a straight line Q R. Show how to find a point on the trochoid described by a point P at radius O P less than O Q. (I.C.E., *Oct.*, 1897.)

2. On what does the velocity of propagation of waves depend? What wave-pressures may be expected in open situations? (I.C.E., *Oct.*, 1898.)

3. Give a short explanation, by means of rough sketches, of the cause of the tides. (I.C.E., *Oct.*, 1898.)

4. Give the formula for the velocity of propagation of a rolling wave in deep water. To what depth does the disturbance caused by such a wave extend? (I.C.E., *Feb.*, 1899.)

5. Sketch, in one diagram, the diurnal variation in the height of a spring and neap tide on the west coast. How is the shape of the diagram affected at a point above a narrow neck in the estuary of a river? (I.C.E., *Feb.*, 1899.)

6. The lowest reach of a canal of rectangular section has a length of 10 miles in which the water is held up to a depth of 18 feet by gates opening inwards at the seaward end. A rising tide reaches the water-level of the canal at noon on a calm day and instantly opens the gates and continues to rise. (*a*) At what time will the first indication of the entering tide be observed at the next lock gates 10 miles distant? (*b*) What is the nature of the indication thus observed? (*c*) Is there any relation between the velocity at which this indication is propagated and the velocity acquired by the water at any cross-section of the canal during the filling of the canal by the rising tide? (I.C E., *Oct.*, 1899.)

7. The estuary of a river debouching upon a soft sandy foreshore consists wholly of sand and mud, and its axis at high water has a curvature of 20 miles radius and a length of 20 miles within tidal limits; its width at the narrow part of a trumpet-shaped mouth is 1 mile at high water, and at equal distances inland therefrom it is $1\frac{1}{2}$, 2, $1\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{10}$, and at the tidal limit $\frac{1}{20}$ of a mile respectively. The vertical range of a spring tide at the mouth is 25 feet. (*a*) Sketch a plan of such an estuary indicating probable low-water limits. (*b*) Sketch (to a much exaggerated vertical scale) a longitudinal section and, at the $1\frac{3}{4}$ mile width, a cross-section, showing the profiles of a spring tide at the time when high water and low water respectively occur at the mouth of the estuary. (*c*) Sketch to an enlarged scale part of the $1\frac{3}{4}$ mile section, including the portion occupied by water at low tide. (I.C.E., *Oct.*, 1899.)

8. What is meant by "spring" and "neap" tides? Give an account of the variation in the range of the tides from day to day at any port you are acquainted with. (I.C.E., *Oct.*, 1901.)

9. Taking for a line of reference the mean tide level at a port at which there is a considerable range of tide, sketch a diagram showing the hourly variation in level of a spring tide and a neap tide during 24 hours, dimensioning approximately the range in each case above and below the reference line. (I.C.E., *Feb.*, 1902.)

10. Illustrate by sketches the paths of the water particles in the shallow and deep water waves and in waves breaking on a shelving beach. What influence has the depth of the bottom upon the velocities of waves of different lengths? (I.C.E., *Oct.*, 1902.)

11. Give some account of the wave-pressures that may be expected in open situations under different circumstances. Find, in miles per hour, the velocity of propagation of a deep-sea wave 100 feet long.
(I.C E., *Feb.*, 1903.)

Miscellaneous Questions.

1. Explain the law of conservation of energy. A high-class engine is employed to light electric lamps by means of a dynamo. Discuss the losses of effective energy, giving roughly their percentage values, in the various transformations from the furnace to the lamps. (I.C.E., *Oct.*, 1900.)

2. Two elastic spheres whose coefficient of elasticity is e, of masses m_1 and m_2, move in opposite directions with velocities v_1 and v_2. Find an expression for their velocities after direct impact. (I.C.E., *Oct.*, 1901.)

3. A figure is made up of a square and an isosceles triangle on opposite sides of the same base. Find the relation between the altitude and base of the triangle in order that the centre of gravity of the whole figure may lie in the common base. (I.C.E., *Feb.*, 1902.)

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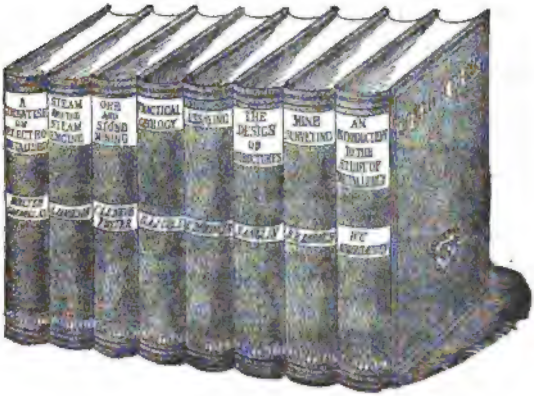

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